Modeling Open Economies

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Basic Modeling Decisions

- Large Economies versus Small Economies
- Commodity Structure: Exports, Imports, Nontraded Goods
- Relative Prices: Terms of Trade, Real Exchange Rate
- Financial Markets
- Nominal Rigidities: PCP versus LCP
- Optimal Monetary Policy

Basic Framework

We follow Gali and Monacelli (2005) and Gali (2008).

- The world is a continuum of small economies of zero measure
- Each economy has a representative household
- Each country produces varieties of a differentiated good

Preferences

Consider a single (Home) country. The Home agent's consumes an aggregate of a Home composite good and imports:

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- This implies that

$$C_H = (1-\alpha) \left(\frac{P_H}{P} \right)^{-\eta} C, \; C_F = \alpha \left(\frac{P_F}{P} \right)^{-\eta} C$$

  where P is the CPI:

$$P = \left[(1-\alpha) P_H^{1-\eta} + \alpha P_F^{1-\eta} \right]^{1/(1-\eta)}$$

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$$C_i = \left[\int C_i(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

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\begin{aligned}
C_H(j) &= \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} C_H \\
P_H &= \left[\int P_H(j)^{1-\varepsilon} dj\right]^{1/(1-\varepsilon)}
\end{aligned}
$$

Analogous expressions are derived for C_i, $C_i(j)$, P_F, and P_i

Budget Constraint

Assuming complete markets, the budget constraint is written as:

$$P_t C_t + E_t \left[Q_{t,t+1} D_{t+1} \right] \leq D_t + W_t N_t + T_t$$

where $Q_{t,t+1}$ is the *stochastic discount factor* for Home currency payoffs, which in equilibrium must satisfy

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}$$

Implications of Optimal Household Behavior

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2. The nominal interest rate is

$$\frac{1}{1+i_t} = E_t Q_{t,t+1} = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]$$

Some Definitions I

1. The Home *terms of trade* is given by P_F / P_H or, in log deviations from ss,
$$s = p_F - p_H$$
2. The CPI definition now gives (prove it!):
$$p = p_H + \alpha s$$
3. Hence the difference between CPI inflation and "domestic " inflation is
$$\pi_t = \pi_{Ht} + \alpha \Delta s_t$$
4. The *Law of One Price* says that the price of an imported good at Home equals its price in its country of origin, $P_i^i(j)$, corrected by the (nominal) exchange rate $\mathcal{E}_i$:
$$P_i(j) = \mathcal{E}_i P_i^i(j)$$

Some Definitions II

5. The bilateral *real exchange rate* between Home and country i is the ratio of their CPIs, $\mathcal{Q}_i = \mathcal{E}_i P_i^i / P$
6. See Gali or GM to see the appropriate multilateral definitions
7. To first order,

$$s = e + p^* - p_H$$

   where e is a nominal exchange rate index and p^* a world CPI index
8. The real exchange rate is related to the terms of trade by:

$$q = (1 - \alpha)s$$

International Risk Sharing

Perfect risk sharing is given by the familiar condition:

$$C_t = \vartheta_i C_t^i \mathcal{Q}_{it}$$

To first order, and assuming symmetry,

$$\begin{aligned} c_t &= c_t^* + \frac{1}{\sigma} q_t \\ &= c_t^* + \frac{1-\alpha}{\sigma} s_t \end{aligned}$$

Implications for Demand I

The demand for Home variety j is

$$\begin{aligned} Y(j) &= C_H(j) + \int C_H^i(j)di \\ &= \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} C_H + \int \left(\frac{P_H^i(j)}{P_F^i}\right)^{-\varepsilon} C_F^i di \end{aligned}$$

$$= \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C + \int \left(\frac{P_H}{\mathcal{E}_i P_F^i}\right)^{-\gamma} \alpha \left(\frac{P_F^i}{P^i}\right)^{-\eta} C_i di\right]$$

Using the perfect risk sharing condition $C_t = C_t^i \mathcal{Q}_{it}$ and loglinearizing, this can be rewritten as:

$$y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$$

where

$$\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$$

Implications for Demand II

Recalling that, from perfect risk sharing, $c_t = c_t^* + \frac{1-\alpha}{\sigma} s_t$, one gets

$$y_t = c_t^* + \frac{1}{\sigma_\alpha} s_t$$

where

$$\sigma_\alpha = \frac{\sigma}{1 - \alpha + \alpha\omega}$$

Implications for the Dynamic IS

Linearizing the Euler equation gives

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho)$$

Since $c_t = y_t - \frac{\alpha\omega}{\sigma} s_t$,

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) - \frac{\alpha\omega}{\sigma} E_t \Delta s_{t+1}$$

This can be rewritten as

$$y_t = E_t y_{t+1} - \frac{1}{\sigma_\alpha}(i_t - E_t \pi_{H,t+1} - \rho) + \alpha(\omega - 1) E_t \Delta y^*_{t+1}$$

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- The risk sharing condition plays a very important role here.

The Supply Side

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- Calvo pricing

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- Traditional assumption is *producer currency pricing (PCP)*: firm j sets price for both domestic sales and exports in terms of its own currency
- Alternative: *local currency pricing (LCP)*, under which firms set prices in terms of the currency where output will be sold.

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- Under LCP, a nominal depreciation does not change the relative prices faced by consumers, so there is no expenditure switching
- Instead, a depreciation raises the Home currency value of exports and export profits (Betts and Devereux)

Inflation and Marginal Costs

The Calvo assumption results in the same relation between domestic inflation and marginal costs as in the closed economy:

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \lambda mc_t$$

But here

$$\begin{aligned} mc_t &= w_t - p_{Ht} - a_t \\ &= w_t - p_t + p_t - p_{Ht} - a_t \\ &= \sigma c_t + \varphi n_t + \alpha s_t - a_t \\ mc_t &= \sigma c_t^* + \varphi y_t + s_t + (1 - \varphi) a_t \end{aligned}$$

$$mc_t = \sigma c_t^* + \varphi y_t + s_t + (1 - \varphi) a_t$$

Recalling $y_t = c_t^* + \frac{1}{\sigma_\alpha} s_t$,

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- This tells us how marginal cost is affected by foreign variables, here summarized by c_t^*
- Under flexible prices, real (producer) marginal cost is constant, so the above gives natural output:

$$y_t^n = \Gamma_a a_t + \Gamma_* c_t^*$$

with $\Gamma_a = (1+\varphi)/(\sigma_a + \varphi)$ and $\Gamma_* = -\alpha(\omega - 1)\sigma_\alpha/(\sigma_\alpha + \varphi)$

The New Keynesian Phillips Curve

One can now derive

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa_\alpha (y_t - y_t^n)$$

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1. Openness affects dynamics by affecting the slope of the Phillips Curve
2. Foreign shocks (to consumption) affect the natural rate of output

Dynamics Under Interest Rate Rule

Three equation system:

$$
\begin{aligned}
\pi_{Ht} &= \beta E_t \pi_{Ht+1} + \kappa_\alpha \tilde{y}_t \\
\tilde{y}_t &= E_t \tilde{y}_{t+1} - \frac{1}{\sigma_\alpha}(i_t - E_t \pi_{H,t+1} - r_t^n) \\
i_t &= \phi_\pi \pi_{Ht} + \phi_y \tilde{y}_t + v_t
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- As exercise, derive r_t^n
- Analysis of dynamics in Gali
- In practice, interest rate rules can depend on π_t, not π_{Ht}

Optimal Monetary Policy

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- In an open economy, there is another distortion: the so called *terms of trade externality* (Corsetti and Pesenti)
- Home country policy affects the relative price of its exports
- This implies the existence of an "optimal tariff " motive.

Ramsey Policy

Let us consider a simpler specification of demand for home goods:

$$Y_t = (1-\alpha)\left(\frac{P_{ht}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P^*_{ht}}{P^*_t}\right)^{-\eta} C^*_t$$

where P^*_t is the world price level and

$$P^*_{ht} = P_{ht}/\mathcal{E}_t$$

This can be rewritten as:

$$Y_t = (1-\alpha)p_{ht}^{-\eta} C_t + \alpha p_{ht}^{-\eta} Q_t^{\eta} C^*_t$$

where $p_{ht} = \frac{P_{ht}}{P_t}$ and

$$Q_t = \frac{\mathcal{E}_t P^*_t}{P_t}$$

is the real exchange rate.

Recalling:

$$P = \left[(1-\alpha) P_H^{1-\eta} + \alpha P_F^{1-\eta} \right]^{1/(1-\eta)}$$

and $P_F = \mathcal{E}\mathcal{P}^*$, it follows that

$$1 = \left[(1-\alpha) p_h^{1-\eta} + \alpha Q^{1-\eta} \right]^{1/(1-\eta)}$$

so we can write

$$p_h = g(Q) = \left[\frac{1 - \alpha Q^{1-\eta}}{(1-\alpha)} \right]^{1/(1-\eta)}$$

Finally, recall the risk sharing condition:

$$C = C^* Q^{1/\sigma}$$

The *Ramsey problem* is to maximize $u(C) - v(N)$ subject to risk sharing, $p_h = g(Q)$, and

$$AN = (1-\alpha) p_h^{-\eta} C + \alpha p_h^{-\eta} Q^{\eta} C^*$$

Ramsey Optimality Condition

You should show that

$$\frac{1}{\sigma} C u'(C) = N v'(N) \epsilon_Q^D$$

where ϵ_Q^D is the total elasticity of demand wrt the real exchange rate

$$D = (1-\alpha) g(Q)^{-\eta} C^* Q^{1/\sigma} + \alpha g(Q)^{-\eta} Q^{\eta} C^*$$

Flexible Price Outcome

Under flexible prices,

$$\begin{aligned} P_h &= \mu MC \\ &= \mu(1-v)(W/A) \end{aligned}$$

This leads to (supply the details):

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$==>$ The flex price outcome will be optimal only if $\frac{\mu(1-\upsilon)C}{g(X)AN} = \epsilon_Q^D$

Optimal Feasible Policy

In order to make progress, one can derive a second order approximation to utility of the form

$$-E\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}v_t'L_v v_t + v_t'L_a\hat{a}_t + \frac{1}{2}l_\pi\pi_{ht}^2\right]$$

where $v_t = (\hat{y}_t, \hat{c}_t, \hat{p}_{ht}, \hat{x}_t)'$, L_v and L_a are matrices, and l_π is a scalar (Benigno and Benigno 2005).

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- This contains no linear terms. Then one can use the *first order* approximation to the model equations to find optimal policy.

Targets

Actually, in this model one can express all the variables in v_t as a function of output:

$$v_t = N\hat{y}_t + N_e a_t$$

Replacing, the objective function can be rewritten as

$$-E\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}l_y(\hat{y}_t - \tilde{a}_t)^2 + \frac{1}{2}l_\pi \pi_{ht}^2\right]$$

for suitable scalars l_y and l_π.
Here, $\tilde{a}_t$ is a linear transformation of a_t (or of other shocks, if there were).
Using the same representation for v_t one can write the Phillips curve as

$$\pi_{ht} = \lambda_y\hat{y}_t + \lambda_a a_t + \beta E_t\pi_{ht+1}$$

Optimal Policy, finally

The FOC for optimality can be written as

$$\pi_{ht} + \frac{l_y}{\lambda_y l_\pi}(\Delta\hat{y}_t - \Delta\tilde{a}_t) = 0$$

This can be seen as a "flexible inflation targeting" prescription.

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- Hence optimal policy "looks like" the closed economy one
- However, it must be stressed that open economy aspects of the model show up in the weight $\frac{l_y}{\lambda_y l_\pi}$ and the properties of $\tilde{a}_t$.
- This is not the only way to express the optimal policy. One could express it in terms of e.g. inflation and a real exchange rate target.