Bubbles and the Current Crisis

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- A good illustration of the issues we discussed last time
- Novelty: bubbles can be expansionary.
- Main idea: plegeable income of entrepreneurs depends on the terminal value of capital, so a bubble enables them to borrow and invest more. This increases investment efficiency and can more than compensate for the fact that bubbles absorbes savings.

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- Risk neutral

Firms

Each firm j that can produce in period t has access to technology

$$F(l_{jt}, k_{jt}) = k_{jt}^{\alpha} l_{jt}^{1-\alpha}$$

A consequence is that the wage is given by the marginal product of labor:

$$\begin{aligned} w_t &= (1-\alpha)(k_t/l_t)^{\alpha} \\ &= (1-\alpha)k_t^{\alpha} \end{aligned}$$

the last equality holding because $l_t = 1$. (k_t is the aggregate capital/labor ratio).

Investment

For existing firms in period t, the investment technology is the usual one:

$$k_{j,t+1} = Z_{jt} + (1 - \delta) k_{jt}$$

New firms in period t (which can only start producing in $t + 1$) have

$$k_{j,t+1} = \pi_t Z_{jt} + (1 - \delta) k_{jt}$$

where $\pi_t > 1$ and can be random ("investment efficiency").
Note that efficiency would entail that *all* investment should be done by new firms.

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- Anyone can buy old firms and operate them

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- $E_t R_{t+1} =$ "the " interest rate at t
- Collateral constraint of agent j:

$$\begin{aligned} R_{t+1} f_{jt} &\leq \phi_{t+1} \left[F(l_{j,t+1}, k_{j,t+1}) - w_{t+1} l_{j,t+1} + V_{j,t+1} \right] \\ &= \phi_{t+1} \left[r_{t+1} k_{jt+1} + V_{j,t+1} \right] \end{aligned}$$

where $f_{jt} =$ amount borrowed, $\phi_t =$ "financial friction", $V_{j,t+1} =$ price of the firm acquired by j, and $r_t = \alpha k_t^{\alpha - 1}$

Optimality conditions

If non-entrepreneurs are willing to lend to entrepreneurs and to buy old firms, then

$$E_t R_{t+1} = \max \frac{E_t \left[r_{t+1} k_{jt+1} - R_{t+1} f_{jt+1} + V_{jt+1} \right]}{V_{jt} + Z_{jt} - f_{jt}}$$

where $r_t = \alpha k_t^{\alpha - 1}$ and the max is s.t.

$$k_{j,t+1} = Z_{jt} + (1 - \delta) k_{jt}$$

Likewise, if entrepreneurs are willing to start new firms, then

$$E_t R_{t+1} \leq \max \frac{E_t \left[r_{t+1} k_{jt+1} - R_{t+1} f_{jt+1} + V_{jt+1} \right]}{Z_{jt} - f_{jt}}$$

where $r_t = \alpha k_t^{\alpha - 1}$ and the max is s.t.

$$k_{j,t+1} = \pi_t Z_{jt} + (1 - \delta) k_{jt}$$

and

$$R_{t+1} f_{jt} \leq \phi_{t+1} \left[r_{t+1} k_{jt+1} + V_{j,t+1} \right]$$

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- The conjecture assumes that there is no bubble

Solution

- Entrepreneurs only invest in new firms and borrow as much as they can. The collateral constraint then becomes:

$$f_{jt} = \frac{\pi_t E_t \phi_{t+1}}{1 - \pi_t E_t \phi_{t+1}} w_t$$

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- Capital evolves according to:

$$k_{t+1} = \left[1 + \frac{(\pi_t - 1)\,\varepsilon}{1 - \pi_t E_t \phi_{t+1}} \right] (1 - \alpha) k_t^{\alpha}$$

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- Capital evolves as in the typical OG model, with shocks to "technology "
- MV: This model seems insufficient to understand the crisis.
- Main argument: what was the particular shock that caused such a severe downturn?

Bubbly Equilibria

- Now let us conjecture that there are bubbles in equilibrium:

$$\begin{aligned} E_t R_{t+1} &= r_{t+1} + (1-\delta) = \alpha k_{t+1}^{\alpha-1} + (1-\delta) \\ V_{jt} &= (1-\delta) k_{jt} + b_{jt} \end{aligned}$$

The entrepreneurs ' collateral constraint becomes

$$f_{jt} = \frac{\pi_t E_t \phi_{t+1}}{1 - \pi_t E_t \phi_{t+1}} w_t + \frac{E_t \phi_{t+1} b^N_{jt+1}}{(1 - \pi_t E_t \phi_{t+1}) E_t R_{t+1}}$$

This is the same as before except for the bubble component.

Also, for any existing firm,

$$E_t R_{t+1} = \frac{E_t b_{jt+1}}{b_{jt}}$$

i.e. the expected growth rate of bubbles must equal the interest rate.

Capital Accumulation with Bubbles

The capital accumulation equation becomes

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k_{t+1} &= \left[1 + \frac{(\pi_t - 1)\,\varepsilon}{1 - \pi_t E_t \phi_{t+1}}\right](1-\alpha)k_t^{\alpha} \\
&\quad + \frac{(\pi_t - 1)}{1 - \pi_t E_t \phi_{t+1}} \frac{E_t \phi_{t+1} b_{t+1}^N}{\alpha k_{t+1}^{\alpha-1} + (1-\delta)} - (b_t + b_t^N)
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Note that now bubbles can have two opposite effects on the accumulation of capital:

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Note that now bubbles can have two opposite effects on the accumulation of capital:

1. Usual crowding out effect
2. They may relax financial constraints, leading to *faster* capital accumulation.

The bubbles growth condition becomes

$$E_t b_{t+1} = \left[\alpha k_{t+1}^{\alpha-1} + (1-\delta)\right] \left(b_t + b_t^N\right)$$

The aggregate bubble grows faster than the interest rate because of the creation of new (bubbly) firms

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- Assume $\text{Prob}\{z_{t+1} = B | z_t = F\}$ is negligible

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- $\pi_t = \pi, \phi_t = \phi$
- $\delta \approx 1$
- Define $x_t = b_t / \left[(1-\alpha)k_t^{\alpha}\right]$ = bubble as share of savings
- Then

$$x_{t+1} = \frac{\left[\alpha(1+n)/(1-\alpha)(1-p)\right] x_t}{1 + \frac{(\pi-1)\varepsilon}{1-\phi\pi}\left(\frac{(\pi-1)\phi n}{1-\phi\pi} - 1\right)(1+n)x_t}$$

We also need

$$x_t \leq \frac{1 - \phi\pi - \varepsilon}{1 - \phi(\pi - n)} \frac{1}{1 + n} \equiv \bar{x}$$

Given path for x_t, capital accumulation is given by

$$k_{t+1} = \left(\left[1 + \frac{(\pi - 1)\,\varepsilon}{1 - \pi\phi} + \frac{\phi(\pi - 1)n}{1 - \phi\pi} \right] (1 + n)x_t \right) (1 - \alpha) k_t^{\alpha}$$