Foreign Exchange Intervention Redux

Roberto Chang*

Rutgers University and NBER

This Version: November 2017†

Abstract

Received wisdom posits that sterilized foreign exchange intervention can be effective by altering the currency composition of assets held by the public. This paper proposes an alternative channel: sterilization may (or may not) have real effects because it changes the net credit position of the central bank vis-à-vis financial intermediaries, thereby affecting external debt limits. This argument is developed in the context of an open economy model with a financial intermediation sector subject to occasionally binding collateral constraints. FX intervention has real effects if and only if it occurs when the constraints bind; at such times, a sterilized sale of official reserves relaxes the constraints by reducing the central bank’s debt to domestic banks, freeing resources for the latter to increase the supply of credit to domestic agents. Several novel implications for the analysis of FX intervention policy, official reserves accumulation, and the interaction between FX intervention and monetary policy are derived.

*Email: chang@econ.rutgers.edu
†Preliminary Draft. Prepared for the 2017 Central Bank of Chile Annual Conference. I am thankful to Luis Felipe Céspedes and seminar participants at Rutgers, ITAM, and Banco de Mexico for useful comments and suggestions. Of course, any errors or shortcomings are solely mine.
1 Introduction

Arguably, no issue in International Macroeconomics exhibits more dissonance between academic research and policy practice than foreign exchange intervention. The dominant view from academia is that sterilized foreign exchange (FX) intervention has a tiny, if any, impact on real variables, which makes it virtually useless as an independent macroeconomic policy tool. Indeed, a large body of empirical literature has struggled to find a consistent link between FX intervention and macroeconomic aggregates, including exchange rates.\footnote{For instance, citing Obstfeld (1982) and Sarno and Taylor (2001), Taylor (2014, p. 369) writes: "the evidence is often weak and a source of ongoing controversy". A more recent survey of the empirical literature is Menhoff (2013).} From a theory perspective, this is hardly surprising, especially since modern dynamic macroeconomic models often predict that FX intervention should be irrelevant (Backus and Kehoe 1989).

Policymakers, on the other hand, have ignored the prescriptions from research and have intervened, frequently and intensely, in the foreign exchange market. In advanced countries, such as Switzerland, FX intervention has become prominent and noticeable following the global financial crisis, while in emerging economies FX intervention was the norm already before the crisis, and even in countries committed to inflation targeting. Interestingly, central bankers reportedly believe that FX intervention is effective as a policy tool, and that it has been used successfully\footnote{See Chutasripinanish and Yetman (2015). Also, Adler and Tovar (2011).}

The purpose of this paper is to develop a recent perspective on FX intervention which, among other advantages, can help reconciling the contrasting views of academics and policymakers. Following Céspedes, Chang, and Velasco (2017), I adopt the view that FX intervention can and should be seen as a specific instance of the so called "unconventional" central bank policies reviewed, for example, in Gertler and Kiyotaki (2010). This view strongly indicates that a useful analysis of FX intervention requires a framework that allows for financial frictions and institutions, for otherwise unconventional policies turn out to be irrelevant (as in Wallace 1981 or, as already mentioned, Backus and Kehoe 1989).
Accordingly, I analyze FX intervention in an extension of Chang and Velasco’s (2017) model of a small open economy. In that economy, financial intermediaries or banks borrow from the world market and, in turn, extend credit to domestic households or the government, subject to an external debt limit. The model is standard and intended to be as simple as possible to help exposition, so as to isolate two features that turn out to be central. The first one is the specification of sterilized intervention. Sterilized FX interventions are operations in which the central bank buys (or sells) official reserves of foreign exchange, and at the same time it sells (or buys) an offsetting amount of securities, such as "sterilization bonds". This implies that the central bank issues sterilization bonds, or more generally reduces its net credit position, when it purchases reserves, and it cancels the bonds when it sells reserves.

The second aspect of our model is that domestic banks face an external debt limit that may or may not bind in equilibrium. This is key because, as I show, FX intervention has no impact on macroeconomic aggregates if it occurs at times at which that constraint limit does not bind. Conversely, as I also show, FX intervention does affect equilibrium real outcomes at times of binding financial constraints.

More precisely, sterilized FX intervention can affect equilibrium because the associated sterilization operations relax or tighten financial constraints. When the central bank sells foreign exchange, in particular, sterilization means that the central bank cancels sterilization bonds (or, more generally, increases its net credit position vis a vis domestic banks). If financial constraints do not bind, domestic banks accommodate this change by simply borrowing less from the world market, and equilibrium is left undisrupted. But when financial constraints do bind, the fall in the central bank’s demand for credit implied by sterilization frees resources for banks, allowing them to increase the supply of loans to the domestic private sector. The result is that loan interest rates fall, and aggregate demand expands.

This view of the mechanism through which sterilized FX intervention works differs sharply from alternative ones, and in particular from those of currently dominant portfolio balance models. Such models assume that domestic currency bonds and foreign currency bonds are
imperfect substitutes and, as a result, uncovered interest parity holds up to a risk premium that depends on the ratio of domestic currency bonds to foreign currency bonds in the hands of the public. Sterilized FX intervention affects that ratio and hence, the UIP risk premium, which in turn requires macroeconomic adjustments. In contrast, the mechanism proposed in this paper does not rely on imperfect asset substitutability nor on differences in currency denomination; in fact, I show that FX intervention can be an effective policy tool (when financial constraints bind) under perfect asset substitutability and even if the economy is "financially dollarized".

As a significant additional payoff, our exploration of the model highlights a close link between sterilized intervention and the cost-benefit analysis of official reserves accumulation. Under the natural assumption that the central bank cannot issue foreign currency, maintaining a large stock of foreign exchange enhances the ability of the central bank to stimulate the economy, by selling reserves, when financial constraints become binding. This is obviously beneficial and intuitive. But in this model there is also a cost of holding reserves, namely, that larger reserves also imply larger outstanding quantities of sterilization bonds, the financing of which put banks closer to their credit limits, making them more vulnerable to adverse exogenous shocks. In other words, the main tradeoff is that large amounts of official reserves allow the central bank to respond more effectively, via FX intervention, when financial constraints are hit, at the cost of those constraints being hit more frequently.

We use the model to derive several lessons for FX intervention rules and their relation to conventional monetary policy. Notably, we find that a policy of selling reserves when the exchange rate is weak and buying reserves when it is strong can relax financial constraints when they bind, but also that there is intervention when the constraints do not bind, which can be counterproductive. A policy of intervention based on credit spreads is superior, as it is only activated when financial constraints bind. Also, the question of whether sterilized FX intervention can be an independent policy tool that can complement conventional monetary policy has an affirmative answer in our model. But the fact that financial constraints bind only occasionally is crucial and means, in particular, that one must go beyond the analysis of linear
models or linear approximations around the steady state.

Finally, our approach yields several other appealing insights. Specifically, it is consistent with the empirical difficulty to find significant macroeconomic effects of FX intervention in the data, for our model implies that intervention matters only at times of binding constraints, which may be infrequent. It also indicates how FX intervention can be welfare improving. And it sheds light on the role of the so called *quasi fiscal* deficits that central banks derive from FX intervention.

This paper is a contribution to a large literature on FX intervention. For useful surveys, see Sarno and Taylor (2001), Menkoff (2013), Adler and Tovar (2011), and Ostry, Ghosh, and Chamon (2016). Until the powerful critique by Backus and Kehoe (1989), the literature was dominated by models derived from the optimal portfolio choices of investors that viewed domestic currency assets and foreign currency assets as imperfect substitutes. Recently the portfolio balance approach has experienced a revival, led by Benes, Berg, Portillo, and Vavra (2015), and followed by Vargas, González and Rodriguez (2013), Montoro and Ortiz (2016), and Cavallino (2017).

The newer portfolio balance models are similar to ours in that FX intervention can have real effects because of the interaction of sterilization operations with financial frictions. They differ substantially along some important details, however. For example, Benes et al. (2015) and Vargas et al. (2013) impose that banks pay portfolio management costs similar to those in Edwards and Vegh (1997). They make assumptions about those costs that make domestic currency bonds and foreign currency bonds imperfect substitutes for the banks, which leads to the same kind of uncovered interest parity condition *cum* risk premium that was the hallmark of the older portfolio balance approach. This indicates that, while the newer models have been successful in providing satisfactory theoretical underpinnings to the portfolio balance approach, they still have to be reconciled with the same evidence as older models. In comparison, in the model of this paper, financial frictions only bite sometimes and not others, which makes a significant difference in the results. For one thing, under the assumption that financial con-
strains are not binding in the steady state, our model implies that FX intervention is irrelevant for shocks that are not large enough to drive the economy to the financially constrained region. This is consistent with the scarcity of empirical evidence of nontrivial effects of sterilized intervention on macro variables.

Section 2 of this paper presents the model that serves as the basis for our discussion. A baseline version of the model assumes complete price flexibility and financial dollarization. In that baseline version, Section 3 discusses FX intervention and reserves accumulation. Nominal price rigidities and, hence, a nontrivial role for monetary policy are introduced in Section 4. That section examines the interaction between monetary policy and FX intervention. Section 5 shows how the assumption of financial dollarization can be relaxed, with only minor changes in our arguments. Section 6 concludes.

2 A Model of FX Intervention and Reserves Accumulation

This section develops a basic open economy model that will serve to express our ideas regarding intervention policy. I extend the model of Chang and Velasco (2017) to a stochastic setting and, more importantly, to examine in detail the mechanism of sterilized intervention, and how FX intervention policy interacts with financial constraints that bind only occasionally.

In order to focus on the essentials, this section imposes very restrictive assumptions. In particular, it assumes perfectly flexible prices (so that conventional monetary policy has no bite) and that all financial assets are denominated in foreign currency (complete financial dollarization). These assumptions not only simplify the analysis but also serve to emphasize that the mechanism by which FX intervention works does not depend on the currency denomination of assets, or the interaction with other monetary policy tools. Of course, realistic models will require adding nominal price rigidities, powerful monetary policy, and differences in the currency denomination of assets. But these can be added at relatively little extra cost later, as
2.1 Commodities and Production

We consider an infinitely lived, small open economy. In each period there are two internationally traded goods, home and foreign. The price of the foreign good in terms of a world currency (called "dollar") is fixed at one.

The home good is the usual Dixit-Stiglitz aggregate of varieties, with elasticity of substitution $\epsilon$. There is a continuum of monopolistically competitive firms indexed by $i$ in $[0, 1]$. In period $t$, firm $i$ produces variety $i$ via $y_{it} = An_{it}$, where $n_{it}$ denotes labor input and $A$ a productivity term, assumed to be constant for ease of exposition. Firms take wages as given, so that nominal marginal cost in period $t$ is common to all, and given by:

$$MC_t = W_t/A \quad (1)$$

where $W_t$ is the nominal wage, that is, the wage expressed in terms of a domestic currency ("peso" hereon).

For now, we assume flexible prices, meaning that, in every period, all firms set a peso price for their produce. All varieties then carry the same price in equilibrium, given by the usual markup rule:

$$P_{ht} = \left(1 - \frac{1}{\epsilon}\right) MC_t \quad (2)$$

$P_{ht}$ is also the price of the domestic home aggregate good. That aggregate is sold at home and abroad. The foreign part of demand is given simply by a function $xe_t^x$ of its relative price, the real exchange rate:

$$e_t \equiv \frac{E_t}{P_{ht}}$$

with $E_t$ denoting the nominal exchange rate (pesos per dollar), and $x$ and $\chi$ positive parameters.

Home demand for the domestic aggregate good is related to the demand for final consump-
tion, which is denoted by $c_t$ and assumed to be a Cobb Douglas function of the domestic composite good and foreign goods. The Law of One Price is assumed to hold, implying that the peso price of foreign goods is given by $E_t$. Then the price of final consumption (the CPI) is

$$P_t = P_{h,t}^\alpha E_t^{1-\alpha}$$

where $\alpha$ is a parameter between zero and one.

The implied demand for the home aggregate is $c_{ht} = \alpha e_t^{1-\alpha} c_t$, and therefore the market clearing condition for home output is

$$y_t = \alpha e_t^{(1-\alpha)} c_t + x e_t^\chi,$$  \hspace{1cm} (3)

2.2 Banks

There is a large number of domestic financial intermediaries or banks, which borrow from the rest of the world to lend to either households or the government, subject to financial frictions.

A representative bank starts a period $t$ with an amount of capital or net worth of $k_t$ dollars. This amount is, as we will see, raised from domestic households in exchange for a share of the bank’s next period profits. Given $k_t$, the bank borrows $d_t$ dollars from foreigners, at a gross interest rate of $R_t^* \geq 0$, which the bank takes as given.

Because of financial frictions, external borrowing is restricted by a collateral constraint

$$d_t \leq \theta k_t$$

where $\theta$ is a constant. As noted in the literature, this kind of constraint can be rationalized in various ways.\hspace{1cm} \footnote{For example, one may assume that, after raising $d_t$, the banker can "abscend" with the funds at a cost of $\theta$ times equity. Knowing this, lenders will not extend more credit than $\theta k_t$.}
sector, \( l_t \), or to purchase bonds issued from the central bank, \( b_t \). For the time being, we assume that private loans and central bank bonds are perfect substitutes and carry the same interest rate, \( \varrho_t \), between periods \( t \) and \( t + 1 \).

Observe that we assume, for now, that loans and bonds, and the interest rate, are all denominated in dollars. This case, sometimes termed "financial dollarization", may be realistic for some countries and not for others. However, it is the simplest assumption to start with. More importantly, it emphasizes that the basic mechanism by which FX intervention works in our setting does not rely on differences in currency denomination. With that mechanism laid out, section 5 turns to its interaction with peso denominated loans and bonds.

Then, the bank’s balance sheet requires that:

\[
b_t + l_t = k_t + d_t.
\]

and the bank’s profits are given by

\[
\pi_{t+1} = (1 + \varrho_t)(l_t + b_t) - R^*_td_t,
\]

Note that, under our maintained assumptions, profits are realized in period \( t + 1 \) but they are known as of period \( t \). The bank’s problem, therefore, is simply to choose \( b_t, d_t, \) and \( l_t \) to maximize \( \pi_{t+1} \) subject to the collateral constraint.

The solution is simple. Combining the preceding two equations, profits can be written as

\[
\pi_{t+1} = R^*_tk_t + (1 + \varrho_t - R^*_t)(l_t + b_t)
\]

i.e. profits are a sum of a "normal" return on equity plus an excess return on domestic credit. Hence, if \( 1 + \varrho_t = R^*_t \), there are no supranormal returns, and the bank’s optimal policy is indeterminate as long as \( b_t + l_t = k_t + d_t \) and \( d_t \leq \theta k_t \). If \( 1 + \varrho_t > R^*_t \), on the other hand, the bank lends as much as it can. The collateral constraint then binds, so that \( d_t = \theta k_t \), and
\[ b_t + l_t = (1 + \theta)k_t. \]

Finally, the return to equity is denoted by \((1 + \omega_t)R_t^* \equiv \pi_{t+1}/k_t\), and given by:

\[
\frac{\pi_{t+1}}{k_t} = R_t^* + (1 + \rho_t - R_t^*)(1 + \theta) \equiv (1 + \omega_t)R_t^*
\]

2.3 Central Bank, Intervention, and Reserves Accumulation

The essence of sterilized FX intervention is that a central bank sells or buys foreign exchange and, at the same time, buys or sells a matching amount of securities. This can be implemented in a myriad ways, and the menu of alternatives depends in practice on institutional aspects of each economy, such as the kind of securities that are involved in sterilization. But, and as emphasized in the literature, the crucial aspect of sterilized intervention is that it involves a simultaneous change in official reserves and the net credit position of the central bank.

Accordingly, in what follows we assume that sterilized FX intervention means that the central bank buys or sells official reserves (dollars) and, at the same time, issues or retires a corresponding quantity of its own bonds (which we will refer to as *sterilization bonds*). While highly stylized, this assumption is the same as in the recent papers of Benes et al. and Vargas et al. It also corresponds closely to actual practice in some countries. For example, Vargas et al. explain the Colombian experience in some detail, and how the practice of FX intervention led Colombia’s government to issue sterilization bonds. The same specification is incorporated in modern textbooks, e.g. Feenstra and Taylor (2014).

In our model, it will become apparent, FX intervention can affect equilibria because the matching sterilizing operation may relax or tighten the external credit constraint. This argument was stressed in Céspedes, Chang, and Velasco (2017) and differs from older ones, in particular with the traditional portfolio balance view. That view relied on the assumption that sterilization operations involved securities denominated in domestic currency, so that FX intervention must change the ratio of foreign currency assets to domestic currency assets in private hands. If, in addition, securities denominated in different currencies were imperfect substitutes,
restoring equilibrium would require a change in relative rates of return. Such an argument is obviously inapplicable in our model, as we have assumed that all securities are denominated in dollars and are perfect substitutes. But this is only to emphasize that the mechanism by which FX intervention works is not a portfolio balance one.

It should be noted that we assume that sterilization bonds are held by domestic agents, banks in this case. This assumption is natural and realistic, and no different from what is usually imposed in the literature. But it is a crucial part of our argument. If the central bank could freely sell sterilization bonds to the rest of the world, then the economy as a whole would effectively face no external collateral constraint. Under our assumptions here, it is key that sterilization bonds add to the economy’s overall debt, which has a limit. One can presumably adapt our analysis to alternative scenarios as long as they imply that sterilization bonds interact with financial frictions (in fact, this is the case of Benes et al. and Vargas et al.).

As mentioned, central bank bonds are assumed to yield the same interest rate as private loans, \( r_t \). Reserves, on the other hand, are assumed to be invested abroad, at the external interest rate \( R_t \). In this setup, the central bank can make operational losses (the so-called quasifiscal deficit) if \( 1 + r_t > R_t \). For the time being we assume that such losses, if any, are financed via a lump sum tax on households; one implication is that the net worth of the central bank will be constant, and normalized here to zero for convenience. These assumptions are prevalent in the literature and necessary to proceed further. Having said this, I also stress that they are not trivial either for the theory or in practice. Here further research is clearly warranted; I offer further thoughts on this issue in the closing section.

Our maintained assumptions now ensure that, if \( f_t \) denotes the central bank’s international reserves, the central bank’s balance sheet is simply given by \( f_t = b_t \), and that the central bank’s quasifiscal deficit in period \( t \) is given by

\[
T_t = (1 + r_{t-1} - R_{t-1}^*) b_{t-1}
\]
Hence there is a tight link between foreign exchange intervention and the amount of central bank bonds: selling foreign exchange reserves is a fall in $f_t$, which then amounts to a reduction in $b_t$; reserves accumulation is an increase in $b_t$.

Finally, it seems natural to assume that the central bank cannot issue international currency. In this setting, this requires imposing that official central bank reserves have a lower bound, which we assume to be zero: $f_t = b_t \geq 0$.

### 2.4 Households

The economy has a representative household with preferences that depend on consumption and labor effort, and given by the expected value of $\sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$, with

$$U(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\phi} n^{1+\phi}$$

where $\sigma$ and $\phi$ positive parameters.

In each period $t$, the household decides how much to consume and to work, how much to borrow from domestic banks, and much much equity to send to the banks. The period’s budget constraint, expressed in dollars, is:

$$e_t^\alpha c_t + k_t - l_t = (1 + \omega_{t-1})R_{t-1}^s k_{t-1} - (1 + \rho_{t-1})l_{t-1} + e_t^\alpha w_t n_t + v_t + z_t - T_t,$$

where $w_t = W_t/P_t$ is the real wage, $v_t$ denotes (dollar) profits from domestic firms and banks, and $T_t$ denotes to the lump sum taxes needed to finance the central bank’s quasifiscal deficit. Finally, $z_t$ is an exogenous endowment of foreign goods (dollars), which can be thought of as income earned from the ownership of a natural resource, as oil or commodities.

Finally, we follow Chang and Velasco (2017) in assuming that there is an exogenous limit, 4And as usual, if $\sigma = 1$, $u(c) = \log(c)$.
referred to as the domestic *equity constraint*, to how much bank equity the household can hold:

\[ k_t \leq \tilde{k} \]

with \( \tilde{k} > 0 \) is some constant. The *equity constraint* reflects, presumably, some domestic distortions that we do not model here.

The household’s optimal plan is straightforward. Optimal labor supply is given by

\[ w_t c_t^{-\sigma} = \eta n_t^\phi \quad (4) \]

Assuming that the household borrows a positive amount, which will be the case in equilibrium, the usual Euler condition must hold:

\[ c_t^{-\sigma} = \beta E_t c_{t+1}^{-\sigma} R_{t+1} \]

where we have defined the consumption interest rate by

\[ R_{t+1} = (1 + \varrho_t) \left( \frac{e_{t+1}}{e_t} \right)^\alpha \]

Finally, the equity constraint binds in period \( t \) if and only if the return on equity, \( (1 + \omega_t) R_t^* \), exceeds the cost of domestic loans, \( 1 + \varrho_t \). As the reader can check, in equilibrium this will be case if and only if \( 1 + \varrho_t > R_t^* \). But this means that the equity constraint and the bank’s external debt constraint must bind under exactly the same circumstances. This simplifies the analysis considerably, since it allows us to impose, without loss of generality, that \( k_t = \tilde{k} \) always, and that the constraint \( d_t \leq \theta k_t = \theta \tilde{k} \) binds if \( 1 + \varrho_t > R_t^* \) and is slack if \( 1 + \varrho_t = R_t^* \).
2.5 Equilibrium

We assume that parameter values are such that financial frictions do not bind in the non-stochastic steady state. As is well known, in order to be able to apply approximation techniques around that steady state, we need to make some assumption to ensure stationarity (Schmitt Grohe and Uribe 2017). Here we assume that the external rate of return, $R_t^*$, is given by a world interest rate, denoted by $\bar{R}^*$ and taken as exogenous and constant (for simplicity), plus a spread term that depends on the amount of bank credit $l_t = k_t + d_t - b_t$:

$$R_t^* = \bar{R}^* + \bar{\Psi}(e^{h-l} - 1) = \bar{R}^* + \bar{\Psi}(e^{d_t-b_t-(d-b)} - 1)$$

where $\bar{l}$, $\bar{d}$ and $\bar{b}$ are the steady state values of domestic loans, external, debt and reserves, respectively, and $\bar{\Psi}$ is an elasticity coefficient.

Two brief comments on the above specification are warranted. First, that the $R_t^* - \bar{R}^*$ spread increases with domestic loans implies that it increases with the economy’s external debt net of reserves. This seems defensible: in fact, the (negative of the) quantity $d_t - b_t$ corresponds to measures of international liquidity emphasized in Chang and Velasco (2000) and elsewhere. Second, we will assume that $\bar{l}$ is given exogenously. This differs somewhat from the literature, which usually imposes an exogenous $\bar{d}$. This is because we want to examine FX intervention policies involving the management of reserves and central bank debt, with implications for the steady state value of reserves $\bar{b}$. It will become apparent that the assumption of an exogenously given $\bar{l}$ is more transparent than an exogenous $\bar{d}$ for our analysis. Since this part of the model is only important for technical reasons, we stick with exogenous $\bar{l}$.

Under flexible prices, one can combine the optimal markup rule (2) and the labor supply condition (4) to arrive at the equilibrium aggregate supply condition:

$$e_t^{-(1-\alpha)} c_t^{-\sigma} = (1 - \frac{1}{\epsilon}) \eta \phi / A^{1+\phi}$$
In turn, the external resource constraint can be written as

\[(1 - \alpha)e_t^{\alpha} c_t - [z_t + \pi e_t^{\chi-1}] = d_t - b_t - R^*_t (d_{t-1} - b_{t-1})\]

which says that the trade deficit in period \(t\) must be financed by increasing external debt or reducing central bank debt, i.e., selling international reserves. As emphasized by Chang and Velasco (2017), this constraint is a key aspect of the model, given that the collateral constraints require

\[d_t = \theta \hat{k} \quad \text{if} \quad 1 + \rho_t > R^*_t\]

\[\leq \theta \hat{k} \quad \text{if} \quad 1 + \rho_t = R^*_t\]

Note that the RHS of the external resource constraint can be expressed as

\[(d_t - d_{t-1}) + \Delta_t - r^*_t (d_{t-1} - b_{t-1})\]

where we have defined \(r^*_t = R^*_t - 1\) and \(\Delta_t = -(b_t - b_{t-1}) = f_{t-1} - f_t\) is the size of foreign exchange sales of the central bank in period \(t\). This emphasizes that sales or purchases of foreign exchange are just given changes in official reserves and sterilization bonds. Also, if reserves cannot be negative, \(\Delta_t \leq b_{t-1}\), that is, foreign exchange operations in each period are limited by the inherited level of reserves.

Equilibrium is now pinned down once we specify a rule for the evolution of \(b_t\), that is, a foreign exchange intervention policy. We now turn to the analysis and implications of alternative policies.
3 Reserves Accumulation and Intervention

3.1 General Considerations

As in Céspedes, Chang, and Velasco (2017), FX intervention in this model is irrelevant unless it occurs at times of binding collateral constraints (or make financial frictions bind if they would have not). For a more precise statement, fix any equilibrium, which we will denote with carets. It can be checked that all of the equilibrium conditions, except the collateral constraints, can be expressed in terms of a vector of variables that excludes \( \hat{d}_t \) and \( \hat{b}_t \). In turn, the collateral constraints can be rewritten as:

\[
\hat{l}_t = (1 + \theta)\tilde{k} - \hat{b}_t \quad \text{if} \quad 1 + \hat{\varrho}_t > R^*_t \\
\hat{l}_t \leq (1 + \theta)\tilde{k} - \hat{b}_t \quad \text{if} \quad 1 + \hat{\varrho}_t = R^*_t 
\]

Finally, the bank’s balance sheet requires:

\[
\hat{d}_t = \hat{b}_t + \hat{l}_t - \tilde{k}
\]

Consider any FX policy that implies an alternative process \( \{b'_0, b'_1, \ldots\} \) for official reserves that coincides with \( \{\hat{b}_0, \hat{b}_1, \ldots\} \) at all times except at some given date \( t \). If the collateral constraint did not bind at \( t \) in the original equilibrium, and does not bind under the new policy (i.e. \( \hat{l}_t \leq (1 + \theta)\tilde{k} - b'_t \)), then the policy does not affect equilibrium. The only adjustment is that external debt adjusts to offset the change in reserves (that is, \( d'_t = b'_t + \hat{l}_t - \tilde{k} \)). Conversely, to affect equilibria, a change in FX intervention policy must imply a change in \( b_t \) at some \( t \) in which either the collateral constraint binds or a nonbinding constraint becomes binding under the new policy.

It also becomes apparent that, when collateral constraints bind, the central bank can stimulate the economy by selling foreign exchange. By doing so, it redeems central bank bonds, making room for domestic banks to increase credit to households. In this sense, and as emphas-
ized by Céspedes, Chang, and Velasco (2017), sterilized foreign exchange intervention "works" because sterilization relaxes the external collateral constraint.

The way FX intervention works in this model is similar to that in Benes et al. and Vargas et al. As in those papers, FX intervention changes the amount of sterilization bonds that domestic banks must hold. Importantly, however, both Benes et al. and Vargas et al. impose a financial transaction technology that implies that an increase, say, in the supply of sterilization bonds must induce domestic banks to also increase the supply of loans to households, resulting in a fall in the interest cost of domestic loans. As a consequence, as noted by Vargas et al., central bank FX purchases must always be expansionary in their model. In the model here, in contrast, central bank FX purchases either leave the supply of loans unchanged (if financial constraints do not bind) or increase it (if they do). And, crucially, the circumstances under which FX purchases stimulate domestic credit are exactly those in which the economy is credit constrained.

The alert reader might recognize that the propositions just stated extend those in Backus and Kehoe (1989). For a large class of models, Backus and Kehoe identified conditions under which sterilized intervention would not affect equilibria. But they also allowed for the possibility that sterilized intervention might not be irrelevant if those conditions were not met. Our analysis proceeds further, by asking what are the implications of intervention when they can matter.\footnote{In contrast, Backus and Kehoe stopped the analysis after stating that, when FX intervention can matter, its real effects depend on accompanying assumptions about fiscal policy. Here we make progress by making specific assumptions on the quasi fiscal deficit.}

Intuitively, the economy will benefit if the central bank sells foreign exchange reserves when financial constraints bind. This provides a rationale for the accumulation of official reserves, if (as we have assumed) foreign exchange reserves cannot be negative. In other words, our analysis of FX intervention has implications for the discussion of observed reserves accumulation in emerging economies and elsewhere.

One implication, in fact, relates to the costs of accumulating reserves. Why would the central bank not accumulate a very large amount of foreign exchange in normal times, so as to
be ready to act if financial constraints suddenly bind? Our model highlights two aspects of the answer. The first one is that, to finance the accumulation of reserves, the central bank borrows from domestic banks, with an interest cost that adds to the quasifiscal deficit. In our model, however, in normal times (i.e. when financial constraints do not bind), the interest cost is fully offset by the interest earned on reserves. There is a second, more significant cost: central bank reserves accumulation induces domestic banks to increase their own external debt and, hence, place themselves closer to their foreign credit limit. This means that the economy becomes more likely that, in response to exogenous shocks, that limit becomes binding.

In short, our model features a key tradeoff in reserves accumulation. Larger official FX reserves are necessary for the central bank to be ready to stimulate the economy at times of binding financial constraints; but the financing of those reserves induces domestic banks to increase international borrowing, making the economy less resilient.

3.2 Numerical Illustrations

We illustrate the main ideas in a calibrated version of the model. From the outset I stress that the objective of this subsection is to expand and clarify our discussion, rather than empirical realism. Hence we choose some of our parameter values just because of simplicity and convenience.

A period is a quarter. In steady state, the world interest rate is four percent per year. The steady state values of \( y, e, \) and \( c \) are all one, and the trade surplus to GDP ratio is one percent. In the absence of foreign exchange intervention, these choices require that a steady state debt to (annual) GDP ratio of twenty five percent, which accords well with usual values in the literature (see e.g. Schmitt Grohe-Urube 2017).

The final important aspect of the calibration is the debt limit \( \theta \tilde{k} \). For purposes of our discussion here I set it at a very stringent value, so that in steady state the economy is not financially constrained, but close to being so. This is because my purpose is to illustrate the workings of the model, with emphasis on the role of financial constraints.
Having calibrated the model, finding numerical solutions requires some nontrivial approximation procedures. For the experiments reported here, I solved the model via the remarkably useful \texttt{occbin} code due to Guerrieri and Iacovello (2015). \texttt{occbin} adapts the popular \texttt{dynare} package to approximate our model regarded as having different regimes, given by times of binding and nonbinding constraints. In response to exogenous shocks, the transition between regimes is endogenous and part of the computation. See Guerrieri and Iacovello (2015) for details, as well as commentary on the accuracy of the resulting approximations.

To obtain a feel for the model in the absence of foreign exchange intervention. Figure 1 displays impulse responses to a purely temporary fall in endowment income $z$. The broken green lines give the responses if there were no financial constraints. In that case, as clear from the figure, a purely temporary fall in $z$ would be accommodated primarily by borrowing from the rest of the world. This would allow the economy to spread the cost over time, smoothing the response of consumption. The exchange rate would depreciate, but only by a small amount. Finally, the interest rate on loans ($\rho$) would essentially remain the same (it increases minimally only because the increase in the debt raises the spread $R^*_t - \bar{R}^*$ through the debt elastic mechanism, which is negligible).

With occasionally binding financial constraints, the impulse responses are given by solid blue lines. External debt increases to the credit limit, which binds for thirteen periods. The binding constraint implies that, in response to the fall in $z$, consumption contracts substantially more than without the constraint. As households would like to borrow more, the real interest rate of domestic loans increases substantially. For this to happen, there is both a large increase in the loan interest rate and a large real depreciation.

Hence the model implies that binding financial constraints can exacerbate the real impact of adverse external shocks. It bears mentioning that the assumption in Figure 1 is that the fall in $z$ is large enough that the debt constraint becomes binding. If it does not, the impulse responses just coincide with the ones without financial constraints (in the figure, the solid and dashed lines coincide if the fall in $z$ is small enough).
Figure 1: Responses to a Fall in $z$
Figure 2: A Simulation

Figure 2 shows the first one thousand periods of a typical simulation. The figure illustrates two aspects of the calibration. First, the value of the debt limit, given by $\theta \tilde{k}$, combines with the stochastic process for exogenous shocks to give the frequency with which financial constraints bind. For the figure, I assume i.i.d. shocks with standard deviation of one percent. Then $\theta \tilde{k}$ is set so that the collateral constraint binds about one fourth of the time. This may be too frequent for realism, but again our purpose here is to illustrate the workings of the model.

Second, the figure emphasizes that times of binding constraints are also times of high volatility in consumption, the real exchange rate, and interest spreads.

We turn to the impact of intervention policy. To start, let us assume that intervention is given by the simple autorregresive process:

$$b_t = Max\{0, (1 - \rho_b)\bar{b} + \rho_b b_{t-1} + \varepsilon_{bt}\}$$
where $\varepsilon_{bt}$ is an iid process, which could be interpreted as an unanticipated central bank purchase of reserves. Here, $\bar{b}$ is the steady state stock of reserves. For ease of exposition, we assume that $0 < \bar{b} < (1 + \theta)\bar{k} - \bar{l}$, that is, we assume a policy such that foreign reserves are strictly positive and the external constraint does not bind in the steady state.

Under the above assumption on $\bar{b}$, and intuitively, small FX operations (i.e. values of $\varepsilon_{bt}$ of small absolute value) do not affect real equilibria, and there are matched one for one by changes in $d_t$. The impact of large $\varepsilon_{bt}$ is asymmetric. A large negative $\varepsilon_{bt}$ prescribes a large sale of official reserves. But reserves are bounded below by zero, so the central bank runs out of reserves. This is the only important impact in the model, however: the fall of reserves is completely offset by a decrease in external debt, leaving domestic credit untouched.

In contrast, a sufficiently large unanticipated purchase of reserves brings the economy to the financially constrained region. Figure 3 depicts such a possibility. As in the previous figures, the dashed green lines depict impulse responses to a positive $\varepsilon_{bt}$ in the absence of financial constraints. In this case, as shown in the figure, the accumulation of reserves would be exactly matched by an increase in the external debt of the banks, with no other real effect. In contrast, the solid blue lines are the responses taking into account financial constraints. The central bank intervention requires an increase in the amount of stabilization bonds, leading domestic banks to borrow abroad up to the credit limit. In this case, the economy remains financially constrained for two periods. Because of the credit limit, loans to domestic households must fall, which explain the fall in consumption, the increase in the loan interest rate, and the real exchange rate depreciation. Finally, the real depreciation is responsible for the output increase on impact. In short, the large purchase of FX reserves leads to the exhaustion of external credit and a domestic credit crunch.

This discussion illustrates the main tradeoff associated with the average level of reserves $\bar{b}$. A low $\bar{b}$ raises the possibility that the central bank runs out of reserves. A high $\bar{b}$, on the other hand, requires external credit and uses up some of the country’s credit limit, making the economy more likely to fall into the financially constrained region in response to exogenous
Figure 3: An Excessive Purchase of Reserves
To illustrate further, Figure 4 shows how the response of external debt to an unanticipated purchase of reserves depends on the average value of reserves $\bar{b}$. The dashed line corresponds to a lower average level of reserves (a lower value of $\bar{b}$) than the solid line. In each case, the figure shows the response of debt relative to its steady state value, which depends on $\bar{b}$ (since $d = \bar{l} - \bar{b}$). The purchase of reserves is of the same magnitude and results in the external constraint binding in both cases. However, as shown, with lower $\bar{b}$, external debt can expand by more before hitting the credit limit. In addition, the economy exits the constrained region faster than with higher $\bar{b}$.

The above considerations shed light on the implications of intervention rules that respond to endogenous variables, such as exchange rates. Consider, for instance, an intervention rule of the form:

$$b_t = \text{Max}\{0, (1 - \rho_b)\bar{b} + \rho_b b_{t-1} - v_e(e_t - \bar{e})\}$$  \hspace{1cm} (5)$$

This argument is reminiscent of that of Alfaro and Kanczuk (2006) in the context of sovereign debt. In their model, increased official reserve levels may reduce the amount of sovereign debt that is sustainable. The mechanisms in that paper, however, are quite different to ours, and they do not bear on the issue of FX intervention.
where \( v_e \geq 0 \). According to this rule, the central bank buys foreign exchange when the exchange rate is stronger than its steady state value, and sells it when the exchange rate is abnormally weak, with the size of the response given by the coefficient \( v_e \).

It should be clear how the policy may help stabilization in the face of shocks that make financial constraints bind. In that case, the central bank sells reserves in response to the real depreciation. The resulting fall in the quantity of stabilization bonds frees domestic banks to extend additional credit to households, who want it in order to smooth consumption. This is depicted in Figure 5. In the figure, the dashed green lines are impulse responses to a fall in \( z \) assuming that \( v_e = 0 \), that is, that FX intervention does not respond to the exchange rate. In that case, there is no FX intervention at all. The shock is assumed to be large enough that the economy hits the credit constraint. Domestic credit increases, but not enough to satisfy the increased demand for credit. Consumption then falls, the exchange rate depreciates, and the interest rate on loans goes up. The solid blue lines assume that \( v_e > 0 \). Since the fall in \( z \) leads to depreciation, the central bank sells reserves. As it does so, it retires stabilization bonds, freeing resources for domestic banks to increase loans to households. As a result, the fall in consumption is less acute, and the adjustments in the real exchange rate and interest rates less sharp.

While an FX intervention rule of the form is beneficial in stimulating the economy when financial constraints become binding, it also has pitfalls. In particular, it prescribes that there is intervention in response to exchange rate movements even when financial constraints do not bind, which is at best ineffective and, at worst, can be detrimental.

To see this, suppose that financial constraints do not bind, and the economy is hit by an unanticipated increase to \( z \). This should be beneficial: the economy can afford more consumption, and the exchange rate tends to appreciate. The intervention rule then prescribes that the central bank will accumulate reserves. If the accumulation of reserves is small, because either the shock is small enough or the elasticity \( v_e \) is small, then the economy remains financially unconstrained. But if the increase in reserves is large enough, the financial constraint will bind.
Figure 5: Exchange Rate Based FX Intervention
In order to accommodate the sterilization bonds of the central bank, domestic banks must then reduce loans to households. In other words, the financing of FX intervention can crowd private credit out.

This is depicted in Figure 6. As before, dashed lines are impulse responses when there are no financial constraints. An unanticipated increase in $z$ induces the representative household to consume more and, at the same time, to borrow less. The exchange rate appreciates, as expected. In response, the central bank buys foreign exchange; the increase in the quantity of sterilization bonds more than compensates for the fall in the private demand for credit, and external debt increases. In the absence of financial constraints, increased external borrowing does not affect the cost of domestic loans.

In our model, however, the FX intervention rule implies that the economy hits the external constraint, which remains binding for a number of periods. To accommodate the increase in central bank sterilization bonds, domestic credit falls by more than in the absence of financial constraints. This means that domestic consumption must fall, which is associated with an increase in the interest rate on domestic loans. Note that the exchange rate appreciates by less than in the unconstrained case. Hence the FX intervention policy looks like it succeeds at stabilizing the exchange rate. But, as it becomes evident, this is the case only because it also creates a credit crunch.

The disadvantage of a FX intervention rule that responds to the exchange rate is, therefore, that it prescribes intervention even it is not justified by binding financial constraints. This suggests that there is a superior strategy: intervention should occur if interest rate spreads widen, as in:

$$b_t = \text{Max}\{0, (1 - \rho_b)\bar{b} + \rho_b b_{t-1} - v_e(1 + \rho_t - R_t^*)\}$$

with $v_e \geq 0$ giving the elasticity of central bank sales to widening spreads. This rule implies that the central bank sells foreign exchange, relaxing financial constraints, when the loan interest rate increases above the cost of international credit (which signals that financial constraints
Figure 6: Pitfalls of Exchange Rate Based FX Intervention
bind). When financial constraints do not bind, however, the spread is zero in our model, so that no intervention is called for (over and above what is required to bring the level of reserves back to its steady state value $\bar{b}$).

Responses under this rule to a fall in $z$ are given in Figure 7. The fall in $z$ raises the households’ demand for credit, which banks try to accommodate by borrowing abroad. As the credit limit is hit, the spread of the domestic loan rate over the foreign interest rate widens. The intervention rule then implies that the central bank sells reserves, reducing $b_t$ and allowing domestic credit to expand further. This helps stabilizing credit spreads, consumption, output, and the exchange rate.

The shape of the responses in Figure 7 is similar to the ones in Figure 6, and the intuition is also very close, the main difference being the variable to which FX intervention reacts to (the exchange rate in Figure 6, credit spreads in Figure 7). But this difference is crucial when financial constraints do not bind, in which case there is active FX intervention with the exchange rate-based policy, but none with the spreads-based policy.
4 Nominal Rigidities and Monetary Policy

To allow for the study of conventional monetary policy, we now drop the assumption that domestic producers adjust nominal prices every period. Following the recent literature and adopt the well known Calvo protocol. Because this specification is well known, we only give a brief description here, and refer interested readers to Gali (2015) for details.

In any given period, an individual producer can set a new price for her product only with some probability $(1 - \theta) < 1$. Because producers cannot set prices every period, they do not set the static optimal markup when they can, and equation (2) is dropped. Instead, producers able to change prices choose them so that the markup over marginal cost that is optimal, on average, for the random period of time until they can change prices again. As shown in Gali (2015), to a first order approximation, domestic inflation, denoted by $\pi_{ht} = \log P_{ht} - \log P_{ht-1}$, is determined by

$$\pi_{ht} = \beta E_t \pi_{h,t+1} + \lambda (\log mc_t - \mu)$$ (7)
where $mc_t = MC_t / P_{ht}$ denotes marginal cost in terms of domestic goods, $\mu = \log(1 - \frac{1}{\epsilon})$ is its steady state value (in logs), and the coefficient $\lambda$ is given by

$$\lambda = \frac{(1 - \theta)}{\theta} \left( 1 - \beta \theta \right)$$

Domestic inflation now depends on current and future real marginal costs. In turn, marginal costs in our model are determined by technology, as given by (1), and optimal labor supply (4). These conditions imply:

$$mc_t = \frac{MC_t}{P_{ht}} = \frac{(W_t/A_t)}{P_{ht}} = \eta e^{1-\alpha} c_t^\alpha \gamma_t^\phi / A^{1+\phi}$$

Solving the model now requires one more equation, which is given by a monetary policy rule. Our model is cashless but, as discussed by Woodford (2003), this is not an issue if monetary policy is given by an appropriate interest rate rule of the Taylor type. As advocated by Romer (2000), here we assume that the central bank sets policy in order to steer the expected real interest rate:

$$i_t \equiv E_t R_{t+1} = E_t(1 + \varrho_t) \left( \frac{e_{t+1}}{e_t} \right)^\alpha$$

Then we posit a Taylor rule such as:

$$i_t = \log R_t^\pi + \phi \pi_t + u_{mt}$$

To get a sense of the implications, Figure 8 displays impulse responses to a contractionary monetary shock (positive $u_{mt}$) that is large enough to place the economy in the financially constrained region. The dashed green lines assume no financial constraints, while the solid blue lines take binding constraints into account. The shock directly raises the expected consumption based interest rate (by assumption), and therefore consumption growth. In response,
consumption must fall on impact. Households attempt to cushion the blow by borrowing from domestic banks, so domestic loans increase. But this mechanism is limited if there are financial constraints. As the figure shows, the limit on external credit is reached on impact: if there had been no constraints (dashed lines), consumption would fall less and external debt would increase more than in the presence of constraints (solid lines). To ration credit in the case of binding constraints, the interest rate on domestic loans, $1 + \varphi_t$, rises above and over the world interest rate.

Importantly, the exchange rate appreciates in response to this kind of shock, but binding financial constraints limit the extent of the appreciation. This means that domestic inflation and output fall by less and that the policy rate increases than more than in the absence of financial constraints.

This exercise emphasizes that not only monetary policy is powerful in this model, but also that binding financial constraints can exacerbate the real impact of monetary shocks.

We might now ask about the role of FX intervention. Our first observation is that, as in

---

7NB For this experiment I assumed that the CRRA is 2. Under log utility, there is no impact on the level of debt.

32
the model with flexible prices, FX intervention cannot have real effects if it occurs at times of nonbinding constraints. The argument is virtually the same as in subsection 3.1 except that the relevant system of equilibrium equations excludes (2) and includes (7), (8), and (9), and the intuition is unaltered: if the collateral constraint does not bind at $t$, any change in $b_t$ (which leaves the constraint still not binding) is offset one for one by a change in $d_t$, without any impact on equilibrium.

A notable implication is that, independently of monetary policy, FX intervention policy does not impact real allocations for small enough shocks, that is, shocks that do not make financial constraints bind. This is clear under intervention rules such as (5) or (6). In fact, if FX intervention is triggered by abnormally high credit spreads, as in (6), there is no intervention at all as long as constraints do not bind. If FX intervention responds to the exchange rate, shocks that do not imply binding financial constraints do trigger sales or purchases of reserves, but ones that are fully accommodated by changes in external debt $d_t$, with no impact on equilibrium.

For large enough shocks, financial constraints bind and, as we have stressed, FX intervention can have real effects, and can complement conventional monetary policy. To illustrate, Figure 9 displays responses to a fall in $z$, assuming a Taylor rule like (9), and either no active FX intervention (dashed green lines) or FX intervention that responds to spreads (solid blue). Without an active FX intervention response, the shock would raise the domestic demand for private loans. Banks would then borrow abroad up to the credit limit, and the loan interest rate would increase to ration credit. The exchange rate would depreciate, leading to an increase in the foreign demand for domestic output, which explains the output increase. As a consequence, domestic inflation increases. Then the Taylor rule prescribes an increase in the policy interest rate.

With an FX rule as (6), the increase in spreads prompts the central bank to sell reserves. As discussed, the corresponding fall in sterilization bonds allows for domestic loans to increase by more than in the absence of FX intervention. For this calibration, the FX rule has negligible effects on the impact response of consumption, although it implies a smoother transition back
to the steady state. More notably, the active FX rule moderates the exchange rate depreciation, and hence the increases in output and domestic inflation.

This analysis reveals that, in the presence of financial frictions, the question of whether sterilized FX intervention can be an independent policy instrument has an unambiguously positive answer. But the answer differs substantially from others offered in the recent literature. Sterilized FX intervention is ineffective locally: it cannot benefit for shocks small enough that financial constraints do not bind. Intervention can help when the constraints bind, and in that case it works by alleviating the external credit limit.

On the other hand, nonlinearities are essential. Hence proper analysis of FX intervention requires going beyond currently fashionable approaches that restrict attention to local approximate behavior around the steady state.
5 The Role of Financial Dollarization

Our models so far have studied an economy that is "financially dollarized", in which all financial instruments are denominated in dollars. This was partly because some actual economies are financially dollarized, and partly because we wanted to emphasize that the arguments presented here do not depend on currency mismatches or debt denomination.

In many economies, however, some securities are denominated in pesos along with others that are dollar denominated. In this section we argue that, while some additional effects are introduced in the model through that alternative assumption, our line of reasoning remains largely untouched.

We assume now that domestic loans and central bank bonds are denominated in pesos, paying a gross interest rate $R^n_t$ between periods $t$ and $t + 1$. What is crucial is that $R^n_t$ is determined in period $t$: the arguments of previous sections obviously apply if return on peso securities are indexed to, say, the dollar. In that case, the dollar return on loans and bonds between $t$ and $t + 1$ depends on the realized rate of depreciation, and is given by

$$
R^d_{t+1} = R^n_t \frac{E_t}{E_{t+1}}
$$

Observe the notation: the subscript on $R^d_{t+1}$ emphasizes that it is a random variable that becomes known only at $t + 1$. This requires amending our analysis of the decision problems of domestic agents.

To simplify things, now we assume simply that domestic banks belong to households, which provide equity $\tilde{k}$. Then the typical bank’s problem is to maximize the discounted expected value of dollar profits:

$$E_t M_{t+1} \pi_{t+1}$$

where

$$\pi_{t+1} = R^d_{t+1} (l_t + b_t) - R^n_t d_t$$

35
subject to $b_t + l_t = \tilde{k} + d_t$ and the collateral constraint $d_t \leq \theta \tilde{k}$, where $M_{t+1}$ is the household’s discount factor for dollar payoffs, which we derive shortly.

The first order conditions to this problem imply that the collateral constraints now can be written as:

$$
\begin{align*}
    d_t &= \theta \tilde{k} \quad \text{if } E_t M_{t+1} (R_{t+1}^d - R_t^*) > 0 \\
    \leq \theta \tilde{k} \quad \text{if } E_t M_{t+1} (R_{t+1}^d - R_t^*) = 0
\end{align*}
$$

Note that these conditions are very similar to the ones we derived earlier, in the case of financial dollarization.

The analysis of the central bank is the same as before, observing only that the quasifiscal deficit in period $t$ is now given by

$$
T_t = (R_t^d - R_{t-1}^*) b_{t-1}
$$

and hence it depends on the realized rate of depreciation.

Lastly, the household’s problem is solved just as before, but now we need to take into account that the dollar interest rate on loans taken at $t$ is $R_{t+1}^d$ instead of $1 + \varphi_t$, and hence it is uncertain as of period $t$. The Euler condition for loans then becomes:

$$
\begin{align*}
    c_t^{-\sigma} &= \beta E_t c_{t+1}^{-\sigma} R_{t+1}^d \left( \frac{e_{t+1}}{e_t} \right)^\alpha \\
    \text{or}
\end{align*}
$$

$$
1 = E_t M_{t+1} R_{t+1}^d
$$

which identifies the dollar discount factor as:

$$
M_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \left( \frac{e_{t+1}}{e_t} \right)^\alpha
$$
The expected consumption-based real rate is $E_t R^d_{t+1} \left( \frac{c_{t+1}}{e_t} \right)^\alpha$. With these modifications, we can retrace the analysis above, without significant change. To illustrate, Figure 10 presents impulse responses to a fall in $z$. The figure assumes a Taylor rule of the form (9), and an FX intervention rule similar to (6) but with $E_t R^d_{t+1} - R^*_t$ as the relevant spread. In the absence of financial constraints (dashed green lines), the shock would be accommodated by increased household borrowing, and an increase in the banks’ external debt, without noticeable impact on real variables or inflation. Given the policy rules, the central bank does not change the policy interest rate nor intervenes in the foreign exchange market.

The shock is assumed to be large enough for external debt to hit the credit limit, however. As discussed before, adjustment then entails a larger fall in consumption, which requires an increase in the real interest rate. This is accomplished via a relatively large devaluation and, in this model, an increase in the nominal peso interest rate on loans. The monetary policy rate increases in response to rising inflation, and reserves fall because credit spreads widen.

6 Final Remarks

This paper was advanced an alternative view of the way sterilized FX intervention works, and derived several implications for theory and policy. As promised in the introduction, this perspective can help reconcile theory and practice in compelling, intuitive ways.

As for the theory, I have shown that occasionally binding financial constraints imply that sterilized FX intervention can have real effects, but only at some times, that is, if it relaxes the financial constraints when they bind. This result is quite consistent with standard theory, but it implies that sterilized FX intervention is not always irrelevant. And in fact, our analysis suggests that FX intervention can be powerful when it matters the most.

Our analysis also suggests that FX intervention may be irrelevant much, or even most, of the time. In this sense, it is no surprise that empirical evidence for significant effects of FX intervention has been elusive. Future empirical research should examine whether the impact of
Figure 10: A Fall in $z$ with peso securities
FX intervention depends on the incidence of financial constraints.

More generally, our analysis stresses that the impact of sterilized FX intervention may depend on the degree of financial frictions as well as the nature of financial institutions. This suggests that, empirically, the effectiveness of FX intervention should differ across countries, according to their degree of financial development.

For exposition, we made very specific assumptions along several dimensions. One of them was that the central bank used its own sterilization bonds in sterilization operations. A little thought should convince the reader that this assumption is much less restrictive than it appears. For instance, suppose that, for sterilizing FX purchases, the central bank sells government debt instead of its own debt. The impact of this operation would be exactly the same as the one in this paper, assuming that government debt has to be absorbed by domestic banks. In fact, one profitable way to look at this alternative is to think of the "central bank" in this paper as a consolidated entity encompassing the central bank plus the fiscal authority.

In this regard, it was mentioned that our analysis was developed under a specific assumption on the central bank quasifiscal deficit. If that assumption were to be dropped, one would have to supply further details about how the quasifiscal deficit is financed and, further, what determines the evolution and management of the central bank’s net worth. These are not trivial issues, but best left for future research. Let us only remark that this question is related to the more general claim that unconventional policy may matter if there are frictions in the links between the central bank and fiscal authorities. See, for example, Benigno and Nistico 2015).

This paper has focused on the transmission mechanism behind FX intervention, and suggested some ways in which intervention may be beneficial in terms of welfare. But it has not attempted to characterize welfare maximizing policy, which is a promising avenue for future studies. A related question is that of optimal reserves management in our model. Our discussion has stressed a novel trade-off in holding large stocks of reserves, namely that the central bank is in a better position to deal with suddenly binding financial constraints, against the fact that the financing of the reserves may imply that the constraints bind more often. This
indicates that an optimal reserves policy may have to take into account precautionary motives of the kind stressed in the recent macroprudential policy literature. But this is a substantial task, and hence delayed to further research.
Appendix

Here we provide details on the calibration used for the examples and illustrations. I assume that there is a steady state in which the external constraint does not bind. (It should be noted that this assumes that FX intervention policy is consistent with such a steady state.)

We denote steady state values with overbars. Then, $1 + \bar{\rho} = \bar{R}^*$ (which here denotes the steady state value of both $\bar{R}_t^*$ and $R_t^*$) because financial constraints do not bind. The Euler condition then requires that $\beta \bar{R}^* = 1$, as usual.

The steady state values of $y, c,$ and $e$ must satisfy:

$$
\bar{y} = \alpha \bar{e}^{(1-\alpha)} \bar{c} + x \bar{e}^\chi
$$

$$(1-\alpha) \bar{e}^{-\alpha} \bar{c} - [\bar{\xi} + \chi \bar{e}^{\chi-1}] = -\bar{r}^*(\bar{d} - \bar{b})$$

$$1/\eta(1 - 1/\varepsilon) = \bar{e}^{(1-\alpha)} \bar{y}^\phi \bar{c}^\sigma / A^{1+\phi}$$

where $\bar{r}^* = \bar{R}^* - 1$

For calibration, I impose that the steady value of $e$ be one, and that the trade balance surplus be one percent of output (it is common to impose balanced trade in the steady state, but Schmitt Grohe and Uribe 2017 argue in favor or a surplus of two percent of GDP; as a compromise, I impose one percent). Now, from the definition of trade surplus, this requires:

$$[\bar{\xi} + \chi] - (1-\alpha) \bar{c}$$

$$= \bar{\xi} + \bar{y} - \bar{\xi}$$

$$= 0.01 y$$

the second equality following from market clearing ($\bar{y} = \alpha \bar{c} + \chi$)

Optimal labor supply reduces to

$$\Theta = \bar{c}^\sigma \bar{y}^\phi / A^{1+\phi}$$
where

$$\Theta = 1/(1 - \frac{1}{\varepsilon})\eta$$

I choose parameters so that $\bar{y} = \bar{c} = 1$ as well. For the market clearing condition to be satisfied, this will require $\varkappa = 1 - \alpha$. Also, for optimal output,

$$A = \Theta^{-1/(1+\phi)} = \left(1 - \frac{1}{\varepsilon}\right)^{1/(1+\phi)}$$

and

$$\bar{z} = 0.01$$

Finally, for the country budget constraint to hold, we need that

$$0.01 = \bar{z} = \bar{r}^* (\bar{d} - \bar{b})$$

This restricts $(\bar{d} - \bar{b})$. The usual assumption is that $\bar{b} = 0$; if so, $\bar{d} = \bar{z}/\bar{r}^*$. If we assume $\bar{r}^* = 0.01$, then $\bar{d} = 1$. (Note that this is the ratio of debt to quarterly output. So, it corresponds to 0.25 in terms of the usual debt/annual GDP ratio, and so it is in the ballpark.)
References


