Lecture 8
Options Markets and Pricing

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Spring Semester, 2009

Part I

Assignment

Assignment

Part II

Introduction

Options Concept

An option is the right to do something, without the obligation to do it.

- A *call option* is the right to buy an asset at a fixed price, within a fixed time period.
- A *put option* is the right to sell an asset at a fixed price, within a fixed time period.

Options Concept
(Continued)

The price at which the exchange is made is called the *strike* or *exercise price*.

- Exercising the option involves exchanging cash for the underlying asset.
- When a call option is exercised by the holder, the underlying asset is purchased by paying the exercise price.
- When a put option is exercised by the holder, the underlying asset is sold for the exercise price.

Options Concept
(Continued)

We will develop the concept of options first without and then with a premium

- The premium is the amount paid for the contract itself
  - Later, we'll focus on this premium for a call option, also called the price of the call or P_C
- It offsets the cost and risk of writing the contract
- Someone could walk away from the options contract so the writer does not receive any compensation
- This differs from the futures contract which has no premium since the contract will – technically – be settled

Options Concept (Continued)

Options have a finite life.

- Upon expiration, the option contract is null and void.
- An *American option* can be exercised at any time prior to expiration.
- A *European option* can only be exercised at maturity, not before.
  - We will only consider European options

Options Concept (Continued)

Let's consider a call option first.

Scenario

Suppose Carla buys some land for $60,000 and immediately sells a 1-year European call option on it to Alex. Carla sells a call perhaps because she expects the price of land to fall, thus incurring a loss. The call protects her from this event. Assume the option has an exercise price of $65,000.

Options Concept
(Continued)

There are many possible results depending on the States-of-the World (SOW) that materialize. Consider just two...

1. The land value is $68,000 1 year later
   - Alex would gain $3,000 by exercising his option.
2. The land value is $62,000 1 year later
   - Alex's option would expire worthless.

The value of the call to Alex is...

$$V_C = max\left[\text{Market Value} - \$65,000, 0\right]$$

Options Concept
(Continued)

In general, the value of the call is...

$$V_C = max\left[P_0 - X, 0\right]$$

where...

$$\begin{aligned} P_0 &= \text{the price of the underlying today} \\ X &= \text{the exercise price} \end{aligned}$$

Options Concept (Continued)

Suppose Alex bought the call option from Carla for $2,000, the premium he pays Carla to cover her costs of writing the contract. Compute the dollar profit (or loss) for Carla and Alex at various land values at option maturity.

- The premium, $2,000, is the price of the call since Alex paid this
- The price is nonrefundable

Options Concept
(Continued)

Suppose the market value of the land is $55,000 when the option matures. What would Alex do? What is the result?

- Alex would not exercise the option.
- Thus, he would lose all the $2,000 he invested in the option for a -100% rate of return.

Options Concept
(Continued)

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Two Periods

Suppose the market value of the land is \$68,000 when the option matures.

- Alex would exercise the option:
- He would buy the land from Carla for \$65,000 and sell it in the open market for \$68,000.
- His net profit would be \$3,000 - \$2,000 or \$1,000.
- His rate of return would be $\frac{\$1{,}000}{\$2{,}000} = 50\%$

Binomial Model

There are different formulations depending on whether we are analyzing stocks or bonds

- We will discuss stocks first and then bonds
- Stocks are like the land Carla purchased in our example: the land itself has no fixed maturity date

Binomial Model
(Continued)

Assumptions. . .

- Frictionless and competitive capital markets
  - Riskless arbitrage opportunities exist
- Two possible SOWs so that $S = 2$ and $s = \{1, 2\}$
  - $s = 1$ is an upward movement in stock prices
  - $s = 2$ is a downward movement in stock prices
- The two states suggests a binomial approach

Binomial Model
(Continued)

Assume, for illustration, that...

$$
\begin{aligned}
P_{S0} &= \$20 = \text{current spot market price} \\
p &= 0.5 = Pr(s = 1) \\
q &= 1 - p = Pr(s = 2) \\
r_f &= 0.10 \\
u &= 1.2 = \text{stock price multiplier if s} = 1 \\
d &= 0.67 = \text{stock price multiplier if s} = 2
\end{aligned}
$$

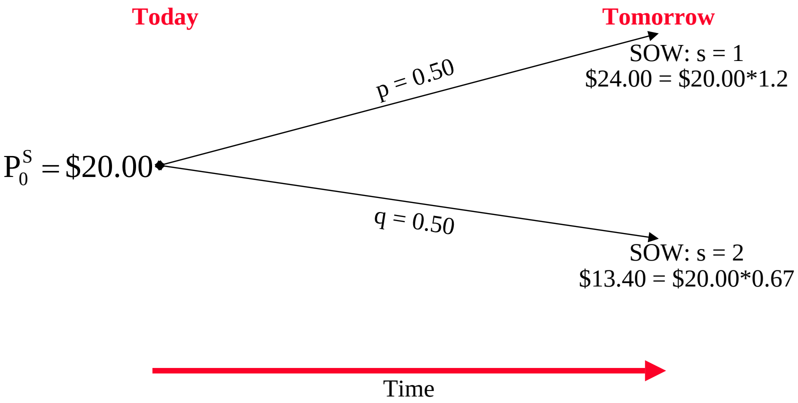
Binomial Model
(Continued)

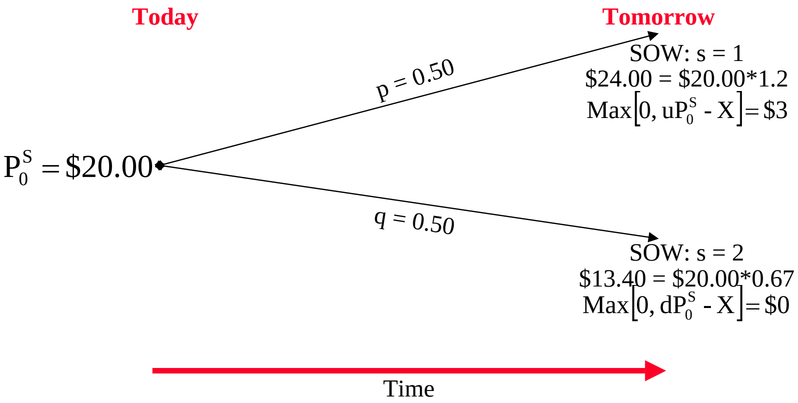
At the end of one period, the stock price could rise from $P_{S0} = \$20.00$ to $uP_{S0} = \$24.00$ if s = 1 with probability p, or fall to \$13.40 if s = 2 with probability q

The call values are...

Up World $V_{cu} = max\,[0; uP_{S0} - X] = \$3$

Down World $V_{cd} = max\,[0; dP_{S0} - X] = \$0$


Today
Tomorrow
SOW: s = 1
$24.00 = $20.00*1.2
p = 0.50
$P_0^S = \$20.00$
q = 0.50
SOW: s = 2
$13.40 = $20.00*0.67
Time



Today
Tomorrow
p = 0.50
q = 0.50
$P_0^S = \$20.00$
SOW: s = 1
$24.00 = $20.00*1.2
$\text{Max}\left[0, uP_0^S - X\right] = \$3$
SOW: s = 2
$13.40 = $20.00*0.67
$\text{Max}\left[0, dP_0^S - X\right] = \$0$
Time

Binomial Model
(Continued)

To find the call price, create a *hedging portfolio* – a portfolio that mimics the returns on the call – using stocks and bonds...

Up World $\$24S + \$110B = \$3$

Down World $\$13.40S + \$110B = \$0$

and solve for S and B simultaneously...

$$\begin{aligned} B &= -\frac{\$13.40 \cdot S}{\$110} = -0.1218S \\ S &= 0.2830 \end{aligned}$$

Binomial Model
(Continued)

Sanity check that these numbers work...

Up World	
Stocks	$\$24 \cdot 0.2830 = \$6.79$
Bonds	$\$110 \cdot (-0.0345) = -\$3.79$
Down World	
Stocks	$\$13.40 \cdot 0.2830 = \$3.79$
Bonds	$\$110 \cdot (-0.0345) = -\$3.79$

The cost of this hedging portfolio today is...

$$\begin{aligned} P_{HP,0} &= \$20 \cdot 0.2830 + \$100(-0.0345) \\ &= \$2.21 \end{aligned}$$

Therefore, by arbitrage...

$$P_{C0} = P_{HP,0}$$

So,...

$$P_{C0} = P_{S0} \cdot S + P_{B0} \cdot B = P_{HP,0}$$

Binomial Model
(Continued)

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Notice that...

$$
\begin{aligned}
S &= \frac{\$3}{\$24 - \$13.40} \\
&= \frac{\$3 - \$0}{\$24 - \$13.40} \\
&= \frac{V_{cu} - V_{cd}}{P_{S0} - P_{Sd}}
\end{aligned}
$$

Binomial Model
(Continued)

And...

$$
\begin{aligned}
B &= \frac{\$13.40S}{\$110} \\
P_{B0}B &= \frac{\$13.40S}{\$110} \cdot P_{B0} \\
&= \frac{\$13.40S}{\$100(1+r_f)} \cdot P_{B0} \\
&= \frac{\$13.40S}{1+r_f} \\
&= \frac{P_{Sd}S - 0}{1+r_f}
\end{aligned}
$$

Binomial Model
(Continued)

Therefore,...

$$
\begin{aligned}
P_{C0} &= SP_{S0} + BP_{B0} \\
&= \left[\frac{V_{cu} - V_{cd}}{P_{Su} - P_{Sd}}\right] P_{S0} - \frac{P_{Sd}S - V_{cd}}{1 + r_f} \\
&= \frac{\left[\frac{V_{cu} - V_{cd}}{P_{Su} - P_{Sd}}\right] P_{S0}(1 + r_f) - \frac{P_{Sd}S - V_{cd}}{P}_{Sd} + V_{cd}}{1 + r_f}
\end{aligned}
$$

Binomial Model (Continued)

Simplifying notation with...

$$\begin{aligned} \widetilde{p} &= \frac{(1+r_f)-d}{u-d} \\ \widetilde{q} &= 1-\widetilde{p} = \frac{u-(1+r_f)}{u-d} \end{aligned}$$

we get...

$$P_{C0} = \frac{\widetilde{p}V_{cu} + \widetilde{q}V_{cd}}{1+r_f}$$

Binomial Model
(Continued)

From basic number theory, we have...

$$
\begin{array}{lcl}
1 & \Rightarrow & {}_0C_0 \\
1\ 1 & \Rightarrow & {}_1C_0\ {}_1C_1 \\
1\ 2\ 1 & \Rightarrow & {}_2C_0\ {}_2C_1\ {}_2C_2 \\
1\ 3\ 3\ 1 & \Rightarrow & {}_3C_0\ {}_3C_1\ {}_3C_2\ {}_3C_3
\end{array}
$$

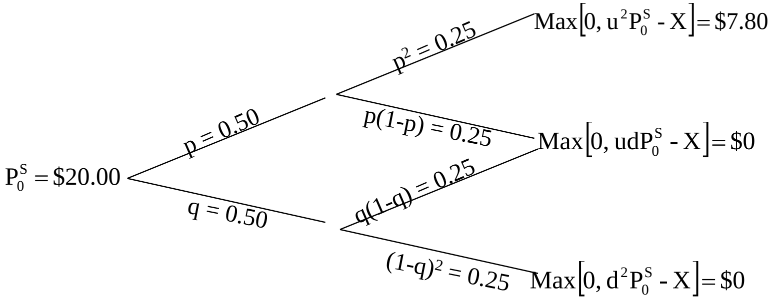
where...

$$
{}_nC_r = \frac{n!}{r!(n-r)!}
$$

Binomial Model
(Continued)

We can now write...

$$\begin{aligned} P_{C0} &= \frac{\widetilde{p} V_{cu} + \widetilde{q} V_{cd}}{1 + r_f} \\ &= \frac{1 \cdot \widetilde{p} V_{cu} + 1 \cdot \widetilde{q} V_{cd}}{1 + r_f} \\ &= \frac{{}_1C_0 \cdot \widetilde{p} V_{cu} + {}_1C_1 \cdot \widetilde{q} V_{cd}}{1 + r_f} \\ &= \frac{\sum_{r=0}^{1} {}_1C_r \widetilde{p}^r \widetilde{q}^{1-r} max\left[0; u^r d^{1-r} P_{S0} - X\right]}{1 + r_f} \end{aligned}$$



P0S = $20.00
p = 0.50
q = 0.50
p2 = 0.25
p(1-p) = 0.25
q(1-q) = 0.25
(1-q)2 = 0.25
Max[0, u2P0S - X] = $7.80
Max[0, udP0S - X] = $0
Max[0, d2P0S - X] = $0

Binomial Model
(Continued)

We can now have...

$$P_{C0} = \frac{\sum_{r=0}^{2} {}_2C_r \widetilde{p}^r \widetilde{q}^{2-r} max \left[0; u^r d^{2-r} P_{S0} - X\right]}{(1 + r_f)^2}$$