A New Definition For Time-Dependent Price Mean Reversion in Commodity Markets

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Abstract: In this note we propose a *time-dependent* definition of mean reversion, and empirically compare our definition with two former definitions. We show that the incidence of mean reversion is approximately 30-40% less under the new and more robust definition.

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I. INTRODUCTION

The theoretical significance of price mean reversion in financial markets has long been recognized as crucial in discussions of predictive ability, arbitrage, hedging behavior, and derivative valuation.[1] One feature of empirical investigations to date which attempt to verify presence of mean reversion is that they all focus on *static* examinations of financial variables. In this note we recognize the potential significance of *time variation* in price reversion, and propose a *time-dependent* definition of mean reversion, which is based on the term-structure of prices. We empirically compare our definition with two other related definitions, and show that when mean reversion is allowed to be sample dependent[2], the incidence of mean reversion is approximately 30-40% less when our definition is used. We discuss reasons for this as well as pointing out a number of advantages of our definition.

II. DEFININING MEAN REVERSION

According to the theory of storage, futures prices with a delivery date of $t+m$ at time t, say $F_t(m)$,satisfy:

$$F_t(m) = P_t e^{(r_t(m) - c_t(m))}, \tag{1}$$

where P_t is the spot price, $r_t(m)$ is the interest rate over the remaining life of the futures contract, and $c_t(m)$ is the convenience yield net of storage costs. The spread between the interest rate and the convenience yield, $S_t = r_t - c_t$, can be thought as the *slope* of the term structure of futures prices. We take equation (1) as given in the sequel.

Many authors have investigated mean reversion in prices and returns, including Bollerslev and Mikkelsen (1996), Fama and French (1988a,b), and Park (1996). Bessembinder, Coughenour, Seguin, and Smoller (1995), for example, examine the presence of mean reversion in the above framework, where spot and future equilibrium prices are simultaneously determined in the absence of any arbitrage opportunities, and assuming that there is negative correlation between the futures risk premium and spot prices. This implies that anticipated mean reversion in spot prices is associated with a concave term structure, i.e. appreciation in intertemporal futures prices *decelerates* when spot prices rise. Accordingly, denoting the elasticity of futures price with respect to spot prices as $\varepsilon_{E(P)P,t}$, mean reversion is said to occur when $e_{E(P)P,t} < 1$, i.e.:

[1] See Miller, Muthuswamy and Whaley (1994), Park (1996), Poterba and Summers (1988), and the references contained therein for further related discussion.

[2] By sample dependence, we mean that alternative windows of historical data available at different calendar dates are used to assess whether there is mean reversion at any given point in time.

Definition 1: There is mean reversion if dS_t/dP_t is negative (or $\varepsilon_{FP,\,t} < 1$).[3]

This definition is appealing in a *static* context, and is consistent with the Samuelson (1965) proposition that futures prices vary less than spot prices. It also implies that temporary components of price shocks will be dampened over time when there is mean reversion. As Fama and French (1988a,b) suggest, offsetting effects of demand and supply on future spot prices will be stronger for longer horizons. Correspondingly, expected spot (or futures) price fluctuations reflect mainly *permanent* shocks, whereas spot prices are affected by both *permanent* as well as *temporary* shocks. Therefore, it could be argued that in the absence of other consecutive shocks, spot prices would follow futures prices over time. Accordingly, we might also define mean reversion as:

Definition 2: In the presence of both *temporary* and *permanent* shocks, mean reversion in spot prices occurs if either: (i) $P_t > F_t$ and $dP_t < 0$, or (ii) $P_t < F_t$ and $dP_t > 0$.

Assuming nonstationarity of prices, Definition 2 implies that P_t is not reverting to a stationary variable, but to a *nonstationary* one, F_t. Accordingly, the spot and futures series should be cointegrated.[4] In addition, parts (i) and (ii) of Definition 2 are both intuitively appealing as they involve assessing whether spot price changes follow changes in futures markets.

Before proposing our definition of mean reversion, we first outline the predictions of Definitions 1 and 2 under all current spot and futures price combination scenarios. In particular, Table 1 summarizes all of the possible price combinations and displays the implications of using Definitions 1 or 2 to define mean reversion. A comment on the organization of the table will be useful before summarizing the results. That is, note that the sign of dS_t is not given for Cases 1-4. This is because dS_t must be greater than zero for Cases 1 and 3 and less than zero for Cases 2 and 4. This in turn follows directly from equation (1), and from the price ratios and price trends given in columns 2 and 3 in the table. On the other hand, dS_t can be both positive and negative whenever both spot and futures prices are increasing or decreasing (Cases 5-8). Thus, we have partitioned these cases into parts a and b, depending on whether

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[3] This follows by first defining $p_t(m) = \ln[E_t(P_{t+m})/F_t(m)]$, where $p_t(m)$ is the bias in the futures spot price prediction, when that price is predicted using $F_t(m)$. Then, it follows that the current price can be written as $P_t = \dfrac{E_t(P_{t+m})}{e^{r_t(m)+\pi_t(m)-c_t(m)}}$, and that $e_{E(P)P,t} = 1 + P_t[r'_t(m) + p'_t(m) - c'_t(m)]$, and $e_{FP,t} = 1 + P_t[r'_t(m) - c'_t(m)]$. Thus, for example, a finding that $r'_t(m) - c'_t(m) < 0 \Leftrightarrow e_{FP,t} < 1$ is a *sufficient* condition for inferring that $\varepsilon_{E(P)P} < 1$, provided that $p'_t(m) < 0$.

[4] Engle-Granger (1987) two-step and Johansen (1988, 1991) cointegration tests were performed for all spot-future price combinations. In all combinations the null hypothesis of no-cointegration was rejected. For further discussion of cointegration and the implications thereof on prediction, see Christoffersen and Diebold (1997). For further discussion of the integration

prices are increasing or decreasing. To illustrate, note that in Case 4, dS_t >0 and P_t is decreasing, so that dS_t/dP_t is negative, implying mean reversion according to Definition 1. In addition, mean reversion is clearly implied by Definition 2 in this case. Thus, both definitions correctly find mean reversion when prices are evolving according to Case 4, given that spot and futures prices are clearly converging over time.

Table 1: Characterization of Mean Reversion Using Definitions 1 and 2

	Price Ratios	**Price Trends**	**Definition 1 (D1)**	**Definition 2 (D2)**	**Comment**
Case 1:	$F_t > P_t$	$F_t \uparrow$ $P_t \downarrow$	⇒ Mean Reversion	⇒ No Mean Reversion	*Misdiagnosed* by (D1)
Case 2:	$F_t < P_t$	$F_t \downarrow$ $P_t \uparrow$	⇒ Mean Reversion	⇒ No Mean Reversion	*Misdiagnosed* by (D1)
Case 3:	$F_t > P_t$	$F_t \downarrow$ $P_t \uparrow$	⇒ Mean Reversion	⇒ Mean Reversion	
Case 4:	$F_t < P_t$	$F_t \uparrow$ $P_t \downarrow$	⇒ Mean Reversion	⇒ Mean Reversion	
Case 5a: $dS_t < 0$	$F_t > P_t$	$F_t \uparrow$ $P_t \uparrow$	⇒ Mean Reversion	⇒ Mean Reversion	
Case 5b: $dS_t > 0$	$F_t > P_t$	$F_t \uparrow$ $P_t \uparrow$	⇒ No Mean Reversion	⇒ Mean Reversion	*Misdiagnosed* by (D2)
Case 6a: $dS_t < 0$	$F_t < P_t$	$F_t \uparrow$ $P_t \uparrow$	⇒ Mean Reversion	⇒ No Mean Reversion	*Misdiagnosed* by (D1)
Case 6b: $dS_t > 0$	$F_t < P_t$	$F_t \uparrow$ $P_t \uparrow$	⇒ No Mean Reversion	⇒ No Mean Reversion	*Misdiagnosed* by (D1,D2)
Case 7a: $dS_t > 0$	$F_t > P_t$	$F_t \downarrow$ $P_t \downarrow$	⇒ Mean Reversion	⇒ No Mean Reversion	*Misdiagnosed* by (D1)
Case 7b: $dS_t < 0$	$F_t > P_t$	$F_t \downarrow$ $P_t \downarrow$	⇒ No Mean Reversion	⇒ No Mean Reversion	*Misdiagnosed* by (D1,D2)
Case 8a: $dS_t > 0$	$F_t < P_t$	$F_t \downarrow$ $P_t \downarrow$	⇒ Mean Reversion	⇒ Mean Reversion	
Case 8b: $dS_t < 0$	$F_t < P_t$	$F_t \downarrow$ $P_t \downarrow$	⇒ No Mean Reversion	⇒ Mean Reversion	*Misdiagnosed* by (D2)

However, overall examination of the entries in Table 1 reveals that despite its intuitive appeal, Definition 1 exhibits some shortcomings when used to define mean reversion. For instance, note that in Cases 1 and 2, $e_{FP,t} < 1$. Thus, Definition 1 suggests that mean-reverting behavior is present in these cases, which is clearly a counterintuitive outcome, as noted in the final column of the table. Likewise, Cases 6a and 7a also correspond to mean-reverting behavior according to Definition 1, even though F_t and P_t are moving *apart* from each other in these cases. For example, in Case 6a, where $dS_t < 0$, which implies that P_t is growing faster than F_t , note that F_t is initially lower than P_t (see column 2). Since both prices are increasing over time (see column 3), this in turn suggests that spot

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properties of asset prices, the reader is refered to Bollerslev and Mikkelsen (1996), Ghysels and Chernov (1999), and the references contained therein.

prices are increasing faster than futures prices, and as futures prices are initially lower, spot and futures prices must be moving apart, or diverging, over time – so that mean reversion in this case is counterintuitive.

It should perhaps be stressed that these counterintuitive findings arise because we are evaluating our baseline definition (Definition 1) in a dynamic setting rather than in the static setting. Notice also that Definition 2 erroneously finds mean reversion in Cases 5b and 8b, although Definition 1 does not. Noting the obvious complementarity between the two definitions, it should not come as a surprise that virtually all misdiagnosed cases (with the exception of Cases 6b and 7b) can be avoided by defining mean reversion as the intersection of Definitions 1 and 2. In particular, consider:

Definition 3: There exists mean reversion among anticipated spot price returns *if*

(1) $e_{FP,t} < 1$, and,

(2) either: (i) $P_t > F_t$ and $dP_t < 0$, or (ii) $P_t < F_t$ and $dP_t > 0$.

It follows directly that Definition 3 correctly diagnoses all possible cases, except 6b and 7b. Interestingly, based on our data (see below), it turns out that price dynamics at any given point in time fall into Cases 6b or 7b less than 5% of the time, suggesting that our definition is rather accurate, particularly relative to the baseline definition (Definition 1). Furthermore, it follows that:

Proposition 4: Assume that equation (1) holds, and S_t and P_t are twice continuously differentiable with respect to P_t. Then the following relationships exist between Definitions 1 and 2:[5]

(i) Mean Reversion (Definition 1) $\Leftrightarrow$ $dS_t/dP_t < 0$, and

(ii) Mean Reversion (Definition 2) $\Leftrightarrow$ $(dS_t/dP_t)(dS_t/S_t) > 0$.

Part (i) of Proposition 4 states that the term structure becomes *flatter* as spot prices increase. In order to satisfy Definition 2 in this case, S_t must be negative. However, note that Definition 2 also holds when the term structure becomes steeper, as long as the rate of growth of the basis is positive, and spot prices follow futures prices. In addition, note that under the same conditions as those stated in Proposition 4, the intersection between Definitions 1

[5] For the proof of Part (i), see Footnote 3. Part (ii) of Proposition 4 can be proven by noting that Part (2) of Definition 3 can be stated equivalently as $(F_t - P_t)dP_t > 0$. However, from the definition of S_t it follows that the sign of $S_t dP_t$ is the same as that of $(F_t - P_t)dP_t$. Again by definition, substituting ($dF_t / F_t - dP_t / P_t$) for S_t into the RHS of Part (ii) of Proposition 4 yields: $\frac{(dF_t - dP_t)^2 S_t dP_t}{(F_t - P_t)^2 (S_t dP_t)^2} > 0$, which is clearly positive as long as $S_t dP_t > 0$. As $S_t dP_t > 0$ is implied by Definition 2, the *if* part

and 2, i.e. Definition 3, implies that $dS_t/S_t < 0$. Thus, Definition 3 holds when the market is in *contango* ($S_t > 0$) or in *backwardation* ($S_t < 0$), as long as the basis is shrinking. In other words, it implies that for mean reversion to exist, the absolute value of the *basis* should be *shrinking* over time, which is a condition that is consistent with the fundamentals of the theory of futures markets.

III. EMPIRICAL RESULTS AND CONCLUSIONS

We calculate frequencies of mean reversion by examining daily spot and futures prices for four mineral (crude oil, gold, silver and platinum), two agricultural (orange juice and live cattle), and two financial (treasury bonds and S&P500) markets. In particular, we determine whether mean reversion exists using window sizes of 60, 120, and 240 days for each day between 4/1/82 and 10/31/95. Entries reported in Table 2 correspond to the percentage of times mean reversion is found according to Definitions 1-3, at the end of each day in the sample period. All data were obtained from the CRB Commodity database.[6]

A number of clear results emerge upon inspection of the percentages given in Table 2. First, differences in *window size* do not have an appreciable impact on the incidence of mean reversion, suggesting that our findings are robust to the length of the sample period used. Second, Definition 1 implies that mean reversion holds 90% of the time in some cases, with most figures falling in the 60-80% range. However, mean reversion percentages are somewhat lower based on Definition 2, and substantially lower for Definition 3, which is consistent with our observation that Definition 1 *misdiagnoses* some market situations as being “mean-reverting”. In particular, percentages based on Definition 2 are concentrated around 50%, regardless of commodity and nearby contract. This finding suggests that *temporary* shocks account for as much as 50% of spot price changes. In addition, Definition 3 finds mean reversion only around 30-40% of the time. Finally, it is worth noting that, percentages clearly vary the least across nearby contract and across commodity when Definition 3 is used. Thus, we conclude that Definition 3 is not only more intuitively appealing than Definitions 1 and 2, but it also leads to a more robust and substantially reduced estimate of the incidence of mean reversion.

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of the proposition follows. The *only if* part of the proof follows by writing the RHS of Part (ii) as $\frac{(dS_t)^2}{S_t dP_t}$, which is greater than zero only if $S_t dP_t > 0$.

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[6] Except the S&P500 index contract which began trading on 4/21/1982 and the crude oil contract began trading on 3/30/1983. Crude oil and platinum price data are from the NYMEX, gold data are from the COMEX, silver and Treasury bond data are from the CBT, live cattle and the S&P500 index data are from the CME, and orange juice data are from the NYCE.

Table 2: Mean Reversion Frequencies By Definition, Data Window, and Commodity[7]

Nearby	Definition 1 Win 1	Definition 1 Win 2	Definition 1 Win 3	Definition 2 Win 1	Definition 2 Win 2	Definition 2 Win 3	Definition 3 Win 1	Definition 3 Win 2	Definition 3 Win 3
GOLD									
2	45	44	43	49	48	48	27	27	25
3	47	46	47	50	49	49	28	27	28
4	50	49	50	50	50	50	29	29	29
5	51	51	51	50	50	51	28	28	29
6	52	52	53	50	50	51	29	29	30
7	54	54	54	50	50	50	30	30	30
8	55	55	55	50	49	50	30	30	31
SILVER									
2	63	63	63	51	51	52	33	34	34
3	64	64	63	52	52	52	34	34	34
4	65	65	65	52	52	52	34	35	35
5	66	67	67	52	52	52	35	36	36
6	68	68	68	52	51	52	36	35	36
7	70	71	71	51	52	51	36	37	37
PLATINUM									
2	62	65	68	49	49	50	32	33	35
3	63	65	66	50	49	50	32	33	34
4	65	66	67	50	50	50	33	34	34
5	90	91	92	53	53	53	49	49	50
CRUDE OIL									
2	73	74	75	49	49	49	36	37	37
3	76	76	77	49	49	48	38	38	38
4	78	78	78	47	47	46	36	36	36
5	79	79	80	47	47	47	38	37	38
6	80	80	80	47	47	47	38	38	38
7	80	80	80	48	47	47	39	39	38
8	80	81	81	48	47	47	38	39	39
LIVE CATTLE									
2	75	74	74	50	50	51	38	37	38
3	81	82	82	48	49	49	39	40	40
4	86	86	85	49	49	49	42	43	43
5	87	87	87	50	49	49	43	43	43
6	86	86	86	49	49	49	43	43	43
ORANGE JUICE									
2	72	73	72	51	50	50	38	38	37
3	77	77	76	50	50	50	40	39	38
4	80	79	78	50	50	50	40	40	39
5	81	80	80	51	51	50	42	41	41
6	83	83	82	50	50	50	41	41	41
7	92	93	93	52	52	52	48	49	49
TREASURY BONDS									
2	50	52	52	51	51	51	26	28	26
3	52	54	55	51	52	52	27	28	28
4	70	70	71	50	50	50	35	35	35
5	56	57	57	50	51	51	27	29	29
6	56	57	57	50	51	51	27	29	29
7	57	58	59	50	51	51	28	29	30
8	60	62	63	49	49	50	28	30	31
S&P500 INDEX									
2	55	55	52	53	53	54	29	29	28
3	56	55	54	53	53	53	30	30	29

[7] Frequencies are calculated by counting the incidence of daily mean reversion between 4/1/82-10/31/95. The number of trading days used for each successive daily calculation was fixed at 60 (Win 1), 120 (Win 2), or 240 days (Win 3). All figures can be interpreted as percentages, e.g. 50 = mean reversion for 50% of the days examined.