Professor Norman Swanson

Final Exam

1. Consider the model

   $x_t = \gamma\ x_{t-1} + \epsilon_{xt}, \mid \gamma \mid < \mid$

   $y_t = \beta\ x_t + u_t,\ u_t = \rho\ u_{t-1} + \epsilon_{yt}, \mid \rho \mid < 1$

and $E(\epsilon_{yt}\ | x_t,\ y_{t-1},\ x_{t-1}) = 0 \qquad E(\epsilon_{xt}\ |\ y_{t-1},\ x_{t-1}) = 0$

$$\begin{pmatrix} \epsilon_{y_t} \\ \epsilon_{xt} \end{pmatrix} \sim\in (0, \sum) \qquad \sum = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$$

$$\sigma_{12} = 0$$

1. (a) Does y Granger cause x?
   (b) Does χ Granger cause y?
   (c) Is x weakly exogenous for β?
   (d) Can we estimate β using a single equation with only x_t and y_t?

2. Outline carefully the augmented Dickey-Fuller test and the Phillips-Perron test, stating the null and alternative hypotheses, the decision rule, and how one may apply them in practice. Compare and contrast the two tests.

   (a) Give a brief definition and interpretation of cointegration between two I(1) economic variables.
   (b) Discuss carefully the Engle-Granger 2-step technique for testing for cointegration. (Include comments about the cointegrating vector, super consistency, bias, ...).
   (c) Outline the Johansen procedure for testing for cointegration.
   (d) Compare and contrast these two types of cointegration tests.

3. Suppose

$$x_t = (1 + \alpha_1 - \beta_1)\ x_{t-1} + \beta_1\ x_{t-1} - \alpha_1\ x_{t-3} + \epsilon_{1t}$$

$$y_t = \alpha_2\ y_{t-1} + \beta_2\ x_t + \epsilon_{2t}$$

where $\mid \alpha_2 \mid < 1$, and $\epsilon_{1t}\ \epsilon_{2t}$ are jointly independently and identically distributed. Show all work in the following:

1. (a) Is x_t I(0) or I(1)?
   (b) Is y_t I(0) or I(1)?
   (c) Are x_t and y_t cointegrated, and if so, what is the CI vector.

4. Provide very brief answers to the following questions.

   (a) If I ran a regression, say

$Y_t = \beta_1\ X_{1t} + \beta_2\ X_{2t} + u_t$, and I observe a high $\overline{R}^2$, but a very low Durbin-Watson statistic, and my data are macroeconomic time series, then:

1. (a) i. What might be causing this result?
   ii. Why?
   b. If two series X_t and Y_t are I(1), and I fit

$$\begin{cases} \widehat{\Delta X}_t = \widehat{\alpha}_1 + \widehat{\beta}_1 \Delta Y_{t-1} + \widehat{\gamma}_1 \Delta X_{t-1} \\ \widehat{\Delta Y}_t = \widehat{\alpha}_2 + \widehat{\beta}_2 \Delta Y_{t-1} + \widehat{\gamma}_2 \Delta X_{t-1}, \end{cases}$$

assuming that 1 lag is appropriate, what potential shortcomings of this model are worth noting if:

1. (a) i. I wish to carry out in-sample inference?
   ii. I wish to construct forecasts of X_t?