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Solution to Final Exam Practice Problems

1. $\left(\begin{array}{c} y_t \\ x_t \end{array} \middle| f_{t-1} \right) \sim N \left(\begin{array}{c} \beta\gamma y_{t-1} + \beta\gamma x_{t-1} + \delta y_{t-1} \\ \gamma y_{t-1} \end{array}, \Omega \right)$

(a) and $\quad$ D(x_t | x_{t-1}) $\neq$ D(x_t | x_{t-1}, y_{t-1})

$\Longrightarrow y_t$ $\quad$ G.C. x_t $\quad$ w.r.t. x_{t-1}, y_{t-1}

1. b. The same holds the opposite way

$\Longrightarrow x_t$ $\quad$ G.C. y_t $\quad$ w.r.t. x_{t-1}, y_{t-1}

(i.e., y_t | x_{t-1}, $y_{t-1} \sim$ IN$((\beta\gamma + \delta)y_{t-1} + \beta\gamma x_{t-1}, w)$ for some w and this is not the same conditional distribution as that of y_t | y_{t-1}).

1. c. Notice using the formula from class that

y_t | $x_t \sim$ IN$\Big((\beta\gamma + \delta)y_{t-1} + \beta\gamma x_{t-1} + \frac{\beta\sigma_{22}+\sigma_{12}}{\sigma_{22}}(x_t - \gamma y_{t-1}), w\Big)$

$w = \sigma_{11} - \frac{\sigma_{12}^2}{\sigma_{22}}$

So β cannot unambiguously be obtained from the conditional distribution, unless $\sigma_{12} = 0$. If $\sigma_{12} = 0$, then we can estimate the conditional model (given above), i.e., regress y_t on x_t, x_{t-1} and y_{t-1} given the original equation

$y_t = \beta x_t + \beta\gamma x_{t-1} + \delta y_{t-1} + \epsilon_{yt}$.

2. (a) $\Delta y_t = \epsilon_{2t} \Longrightarrow y_t$ is I(1)

$\Longrightarrow x_t$ is I(1).

1. b. $x_t - \alpha y_t = \epsilon_{1t} \Longrightarrow$ they are cointegrated with cointegrating vector (1, -α).

c. $x_t = \alpha(y_{t-1} + \epsilon_{2t}) + \epsilon_{1t}$

$= (x_{t-1} - \epsilon_{1t-1}) + \alpha\epsilon_{2t} + \epsilon_{1t}$

$$\text{VAR}\begin{cases} x_t = x_{t-1} + \alpha\epsilon_{2t} + \epsilon_{1t} - (x_{t-1} - \alpha y_{t-1}) \\ y_t = y_{t-1} + \epsilon_{2t} \end{cases}$$

$$\Longrightarrow \text{VEC}\begin{cases} \Delta x_t = -z_{t-1} + (\alpha\epsilon_{2t} + \epsilon_{1t}) \\ \Delta y_t = \epsilon_{2t} \end{cases} \qquad z_{t-1} =$$

$(x_{t-1} - \alpha y_{t-1})$

A more useful solution strategy is

In general, say have an equation with contemporaneous variables in it. Say, for example, that $x_t = \alpha x_{t-1} + \beta y_t + \gamma y_{t-1} + \epsilon_{xt}$ then (1-αB)$x_t = (\beta + \gamma B)y_t + \epsilon_{xt}$

$\Longrightarrow x_t - \frac{(\beta+\gamma B)}{(1-\alpha\beta)}\ y_t = \frac{\epsilon_{xt}}{(1-\alpha B)}$

and assuming $|\ \alpha\ | < 1 \Longrightarrow \frac{\epsilon_{xt}}{(1-\alpha B)}$ has finite variance $\Longrightarrow x_t - \frac{(\beta+\gamma B)}{(1-\alpha B)}\ y_t$ is stationary and at B = 1 we get the CI vector

$\Longrightarrow$ CI vector is $\left(1, -\frac{(\beta+\gamma)}{(1-\alpha)}\right)$

3. $y_t \mid X_t,\, f_{t-1} \sim N(X_t\beta + \rho_1 u_{t-1} + \rho_2 u_{t-2}, \sigma_\epsilon^2)$

$u_{t-1} = y_{t-1} - X_{t-1}\beta$
$u_{t-2} = y_{t-2} - X_{t-2}\beta$
$\Longrightarrow y_t \mid X_t,\, f_{t-1} \sim N((X_t - \rho_1 X_{t-1} - \rho_2 X_{t-2})\beta + \rho_1 y_{t-2} + \rho_2 y_{t-2}, \sigma_\epsilon^2)$
$H_0 : \rho_1 = \rho_2 = 0$

Under H_0, $y_t \mid X_t,\, f_{t-1} \sim N(X_t\beta,\, \sigma_\epsilon^2)$ so Step 1 is: Regress y_t on X_t, construct u_t.

Now $g(\cdot) = (X_t - \rho_1 X_{t-1} - \rho_2 X_{t-2})\beta + \rho_1 y_{t-1} + \rho_2 y_{t-2}$
$g\beta \mid_{\rho_1=\rho_2=0} = X_t$
$g\rho_1 \mid_{\rho_1=\rho_2=0} = (X_{t-1}\beta - Y_{t-1}) = u_{t-1}$
$g\rho_2 \mid_{\rho_1=\rho_2=0} = (X_{t-2}\beta - Y_{t-2}) = u_{t-2}$

So Step 2. Regress $\widehat{u}_t$ on X_t, $\widehat{u}_{t-1}$, $\widehat{u}_{t-2}$ and $TR^2 \sim \chi_2^2$ from this regression, where T = the number of observations used.

Also, as in class, note that
$\xi_{LM} = S(y; \widetilde{\theta})\; I^{-1}(\,\widetilde{\theta})\; S(y; \widetilde{\theta})$ and you should try to derive these scores and informations of the test statistic directly in addition to the work above.