

# Target zone models with stochastic realignments: an econometric evaluation

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This paper provides empirical support for the second generation of target zone models with stochastic devaluation risk. I propose a simple non-linear framework with a time varying probability of exchange rate realignment. This model nests alternatives (i) with no devaluation risk; (ii) with constant devaluation risk; and (iii) the random walk. I reject these three in favor of a stochastic realignment model where devaluation risk varies with economic fundamentals. The model predicts 13 of 17 realignments for the Franc and Lira, including an out-of-sample episode in August 1993. (JEL F31).

Since March of 1979, the majority of European currencies have fluctuated within exchange rate bands.<sup>1</sup> Policy authorities in the European Monetary System (EMS) have coordinated their efforts so as to stabilize currency fluctuations. Participants in the Exchange Rate Mechanism (ERM) are obligated to intervene to help maintain an ecu central parity. While the UK and Italy suspended from the ERM in September of 1992, raising doubts about the Maastricht treaty's vision of monetary union, Belgium, Luxembourg, Denmark, France, Germany, Ireland, the Netherlands, Portugal, and Spain continue as members.

Theory caught up with these institutional realities with the work of Krugman (1991). In Krugman's model, the exchange rate varies non-linearly with a fundamental, usually the money supply. For tractability, the monetary authority is assumed to intervene only at the edges of the band. Furthermore, the bands are regarded as being perfectly credible; the central parity is never changed.

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Empirically, the Krugman model has found little support. Flood, Rose and Mathieson's (1991) study of the seven charter ERM currencies concludes that 'few of the relationships between the exchange rate and (a) interest differentials, (b) exchange rate volatility, and (c) exchange rate distributions seem to be in accord with existing theories.' The non-linear relationship between the exchange rate and fundamentals has also proved difficult to detect. Diebold and Nason (1990), Meese and Rose (1990, 1991) and Mizrach (1992) are all unable, using non-linear methods, to improve upon the random walk in forecasting the exchange rate. Lindberg and Soderlind (1991a) corroborate many of Flood *et al.*'s conclusions in their analysis of the Swedish krona.

The empirical difficulties with Krugman's model have motivated a second generation of target zone models, including contributions by Bertola and Caballero (1992), Bertola and Svensson (1991, 1992), Froot and Obstfeld (1991), and Svensson (1991, 1992). The critical extension is the assumption of imperfectly credible bands by allowing for devaluation risk.

A handful of recent papers have begun to explore the implications of the second generation models. Chen and Giovannini (1991), Lindberg, Soderlind, and Svensson (1993), and Rose and Svensson (1991) have extracted devaluation expectations from interest rate spreads. Edin and Vredin (1993) link devaluation risk to macroeconomic fundamentals. Lindberg and Soderlind (1991b) have estimated a model with imperfectly credible bands and intramarginal intervention.

This paper proposes an encompassing econometric framework to compare rival target zone models. The method has two parts: a simple structural model for interest differentials, and a model for expected exchange rates. The first part is quite standard, drawing on the interest parity condition. The second part allows the exchange rate to switch regimes. Unlike conventional Markov-switching models though, the probability of regime changes varies with financial market and macroeconomic fundamentals. This hybrid probit-Markov model is, to my knowledge, new to the literature.

The new framework nests the Krugman model, models with constant and time varying devaluation risk, as well as the random walk. Once I account for regime switches, I can reject the unit root in favor of mean reversion within the band. I also reject the Krugman model and the model with constant devaluation risk in favor of the model with a time varying probability of realignment.

The probit model for the regime switches almost always anticipates realignment episodes. The model captures five of six in-sample realignments for the French franc and seven of ten for the Italian lira. An out-of-sample confirmation of this approach came in August 1993 when the model accurately predicted the *de facto* devaluation of the franc.

The paper is organized as follows: Section I sketches a general target zone framework as derived by Bertola and Svensson (1993). Section II briefly surveys existing empirical evidence on the exchange rate models presented. Section III introduces a model for interest differentials that nests all the alternatives, and this model is tested in Section IV. An analysis of the realignment probabilities follows in Section V. A summary and conclusions are in Section VI.

### I. Target zone models with devaluation risk

The model of Bertola and Svensson (BS, 1993) generalizes the Krugman (1991) approach to target zone modeling. BS assume that the (log of) the spot exchange rate,  $s(t)$ , evolves (in continuous time) with a fundamental,  $f(t)$ , and a term proportional to the expected depreciation rate,

$$\langle 1 \rangle \quad s(t) = f(t) + \alpha E_t[ds(t)]/dt.$$

Corresponding to any given central parity,  $c(t)$ , is a range for the fundamental that will maintain the exchange rate on an interval  $[\underline{s}, \bar{s}]$ .

The process for fundamentals  $\{f(t)\}$  obeys the stochastic differential,

$$\langle 2 \rangle \quad df(t) = \mu_f dt + \sigma_f d\omega_f(t) + dL(t) - dU(t) + dc(t).$$

$\mu_f$  and  $\sigma_f$  are the instantaneous mean and standard deviation, and  $\{\omega_f(t)\}$  is a standard Wiener process. The processes  $\{L(t)\}$  and  $\{U(t)\}$  are, as in Svensson (1991), treated as regulators of the Brownian motion, applied in infinitesimal increments necessary to keep the exchange within its band.

Devaluations are treated as discrete jumps in the exchange rate, the central parity, the fluctuation bands, and the fundamental. The probability of realignment in the finite interval  $dt$  is assumed to have a time varying intensity,  $p(t)$ . The exchange rate jump is also a random variable,  $q(t)$ . Bertola and Svensson (1993) make an assumption incorporating the two variables, defining a *stochastic devaluation process*,  $\{g(t)\}$ , by

$$\langle 3 \rangle \quad E_t[dc(t)]/dt = (p(t)E_t[q(t)]dt)/dt \equiv g(t).$$

For convenience, define the log deviation from central parity,

$$\langle 4 \rangle \quad x(t) \equiv s(t) - c(t).$$

Bertola and Caballero (1992) noted that interest differentials tend to widen when the exchange rate is in the weak half of the band,  $x(t) > 0$ . Under uncovered interest parity, the interest differential should equal the expected percentage change in the exchange rate which implies that  $g(t)$  and  $x(t)$  are correlated. Bertola and Svensson incorporate this potential interaction, setting analogously to  $\langle 2 \rangle$ ,

$$\langle 5 \rangle \quad dg(t) = \mu_g dt + \sigma_g d\omega_g(t), \quad d\omega_g(t)d\omega_f(t) = \rho dt, \quad |\rho| \leq 1.$$

The dynamics of the exchange rate are then governed by the expected depreciation within the band,  $E_t[dx]/dt$ , and the expected rate of devaluation,

$$\langle 6 \rangle \quad E_t[ds]/dt = E_t[dx]/dt + g(t).$$

Incorporating  $\langle 6 \rangle$ , BS then write

$$\langle 7 \rangle \quad s(t) = f(t) + \alpha g(t) + \alpha E_t[dx]/dt.$$

We can simplify the problem by defining the composite state variable,

$$\langle 8 \rangle \quad h(t) \equiv f(t) + \alpha g(t).$$

The sum of these two Brownian motions is itself a Brownian motion with differential

$$\langle 9 \rangle \quad dh(t) = \mu_h dt + \sigma_h d\omega_h(t),$$

where  $\mu_h \equiv \mu_f + \alpha\mu_g$  and  $\sigma_h \equiv \sqrt{\sigma_f^2 + \alpha^2\sigma_g^2}$ . The dynamics of the exchange rate now reduce to a problem in a single state variable,

$$\langle 10 \rangle \quad x(h) = h + \alpha E_t[dx]/dt.$$

A closed form solution for the exchange rate may be obtained as in Krugman (1991) and Froot and Obstfeld (1991),

$$\langle 11 \rangle \quad x(h) = h + \alpha\mu_h + A_1 e^{\lambda_1 h} + A_2 e^{\lambda_2 h},$$

with  $\lambda_1$  and  $\lambda_2$  the roots of the characteristic equation,  $\alpha\sigma_h^2\lambda^2/2 + \alpha\mu_h\lambda - 1 = 0$ . The 'smooth pasting conditions' determine the constants of integration,  $A_1$  and  $A_2$ .

Two important models of the exchange rate emerge as special cases of the Bertola–Svensson framework. The Krugman model follows by assuming that the bands are perfectly credible,

$$\langle 3' \rangle \quad E_t[dc(t)]/dt = 0, \forall t.$$

This implies that in the stochastic differential  $\langle 4 \rangle$ ,  $\mu_g = \sigma_g = 0$ , and that  $E[g] = 0$  as well.

Svensson (1991) incorporated devaluation risk into the Krugman model, but then assumed that the risk did not vary through time,

$$\langle 3'' \rangle \quad E_t[dc(t)]/dt = \bar{g}, \forall t.$$

The Svensson model implies that  $\mu_g = \sigma_g = 0$ , but that the mean of the stochastic differential for the devaluation process is non-zero,  $E_t[g(t)] = \bar{g}$ .

Evaluating these three models empirically, along with the naive unit root alternative, will be the focus of the first of two empirical sections of this paper. I begin by briefly reviewing the existing econometric evidence.

## II. Empirical evidence on target zone models

Since the appearance of Krugman's working paper in 1987, a number of authors have begun to examine its empirical implications. First, the model implies a non-linear relationship between the exchange rate and fundamentals. This particular point has been tackled by a number of authors. Diebold and Nason (1990), Flood, Rose, and Mathieson (1991), and Mizrach (1992) employ non-parametric procedures but fail to discover statistically significant non-linearities in EMS data. Non-linear specifications fail to improve upon linear models in forecasting the exchange rate. Lindberg and Soderlind (1991a) find similar results for the Swedish krona.<sup>2</sup>

Krugman's model further implies that the exchange rate should have a less than unitary elasticity with respect to the fundamentals,<sup>3</sup> *i.e.*  $ds(f(t))/df(t) < 1$ . Flood *et al.* find non-linearities in this functional relationship, but none that match existing theories. In fact, countries with more credible target zones like

the Netherlands (recall that the Krugman model implies they are perfectly credible) seem to have fewer non-linearities than a country like Italy that is among the least credible.

The basic target zone model also implies that the unconditional distribution of the exchange rate should have a bimodal U-shape. Intuitively, since the exchange rate will only be minimally reflected at the boundaries, it will spend the majority of the time at the extremes of the target zone. Conversely, the conditional variance will be an inverted U-shape since only very small changes will take place at the boundaries.<sup>4</sup> Lindberg and Soderlind reject both of these findings for the Krona; in fact, they cannot reject that their data are normally distributed. Flood *et al.* can reject the U-shaped unconditional density for the lira but not for the franc. While both the lira and the franc exhibit leptokurtosis and GARCH effects, none of these statistical regularities resembles those implied by Krugman's model.

There is only a small empirical literature on the second generation models. Several papers look at the devaluation expectations in <3>. Lindberg, Soderlind, and Svensson (1993) and Rose and Svensson (1991) extracted these expectations from interest rate spreads. These papers support the view that devaluation risk does vary substantially over time. Chen and Giovannini (1991) show that target zone bands influence expectations of future exchange rates. Lindberg and Soderlind (1991b) have estimated a model with imperfectly credible bands and intramarginal intervention using simulated method of moments. They explain the positive correlation between exchange rates and interest differentials in Sweden through devaluation expectations.

While these papers are supportive of the second generation innovations, the literature lacks a formal econometric evaluation of the new models. I will attempt to fill this gap in the next two sections.

### III. Nesting the alternatives: a probit-Markov model

I now have presented four rival models of the exchange rate, three target zone models plus the random walk. This section nests the alternatives in a simple structural model of interest differentials with Markov realignments in the central parity. Econometric issues are postponed to Section IV.

#### III.A. A simple structural model of interest differentials

Consider nominal pure discount bonds maturing at date  $t + \tau$ . Let  $i_t^\tau$  denote the home currency interest rate and let  $i_t^{\tau*}$  denote the foreign (German) rate. Define the  $\tau$ -period interest differential,

$$\langle 12 \rangle \quad \delta_t^\tau \equiv i_t^\tau - i_t^{\tau*}.$$

I will assume<sup>5</sup> that the uncovered interest parity condition holds,

$$\langle 13 \rangle \quad \delta_t^\tau = E_t[\Delta s_{t+\tau}]/\tau.$$

Assume now that there are two states<sup>6</sup> of the world,  $j = 0, 1$ , with 1 indicating a devaluation of the central parity. If a realignment occurs during the interval

$t + \tau$  with probability  $p_t^\tau$ , it follows that

$$\langle 14 \rangle \quad \delta_t^\tau = \{(1 - p_t^\tau)E_t[\Delta s_{t+\tau}|j = 0] + p_t^\tau E_t[\Delta s_{t+\tau}|j = 1]\} / \tau.$$

Using the identity in  $\langle 4 \rangle$ ,  $\langle 14 \rangle$  can be written as

$$\langle 15 \rangle \quad \delta_t^\tau = \{(1 - p_t^\tau)(E_t[\Delta x_{t+\tau}|j = 0] + E_t[\Delta c_{t+\tau}|j = 0]) + p_t^\tau (E_t[\Delta s_{t+\tau}|j = 1])\} / \tau.$$

The second expectation will drop out of  $\langle 15 \rangle$ , since with no devaluation, the change in the central parity is zero,

$$\langle 16 \rangle \quad \delta_t^\tau = \{(1 - p_t^\tau)E_t[\Delta x_{t+\tau}|j = 0] + p_t^\tau E_t[\Delta s_{t+\tau}|j = 1]\} / \tau.$$

Using the variety of identifying assumptions, I will be able to compare the major target zone alternatives using this simple structural model.

### III.B Conditional expectations

To work with  $\langle 16 \rangle$ , I need to specify the two conditional expectations. Svensson (1991) shows that the expected deviation from parity given no realignment,  $E_t[\Delta x_{t+\tau}|j = 0]$ , is well approximated by a linear autoregression for many reasonable parameter values.

Relying on this approximation, I make the identifying assumption that the expectation can be modeled autoregressively,

$$\langle 17 \rangle \quad E_t[\Delta x_{t+\tau}|j = 0] = \beta_1 x_t.$$

Expression  $\langle 17 \rangle$  can tell us how rapidly policy intervention moves the spot rate towards parity. If the exchange rate within the band is a mean reverting process, one should find that  $\beta_1 < 0$ , indicating that a positive (negative) deviation from central parity will lead to a smaller (in absolute value) positive (negative) deviation in the future. If  $x_t$  is well approximated by a random walk,  $\beta_1$  should be close to zero.

Given an estimate of  $\langle 17 \rangle$ , one can infer, for a given devaluation size  $E_t[\Delta s_{t+\tau}|j = 1]$ , the probability of a realignment. This 'drift adjustment' of interest differentials for the expected depreciation within the band has been used by Lindberg, Soderlind, and Svensson (1993), Rose and Svensson (1991), and Bertola and Svensson (1993) to analyze Swedish and ERM devaluation risk. My method differs in that I try to estimate both the realignment probability and the size of the devaluation. This enables me to link the devaluation risk to economic fundamentals.

In the spirit of  $\langle 17 \rangle$ , I model the size of the jump autoregressively, letting it be proportional to the change in the spot rate during the previous realignment,

$$\langle 18 \rangle \quad E_t[\Delta s_{t+\tau}|j = 1] = \beta_2 + \beta_3 \Delta s_{-1}^{j=1}.$$

In order to discourage speculation, central banks would like the spot rate to remain unchanged when there is a devaluation. I can test for this tendency; if the spot rate tends to be unaffected by realignments, both parameters should be zero.

With both expectations identified, I can now turn to the probabilities agents assign to each state.

### III.C. Factors influencing realignment risk

The next step in bringing the model to the data is isolating factors which help predict realignments. I assume that the probability of devaluation is a function of an  $m$ -vector of state variables,  $z_t = (1, z_{2t}, z_{3t}, \dots, z_{mt})$ , including a constant term, and  $\gamma$  is a comfortable vector of parameters.

Realignments generally occur when the exchange rate is in the weak half of the band. Bertola and Caballero (1992) also note that interest differentials are positively correlated with the deviation from central parity when  $x_t > 0$ . These two facts motivate my use of the position of the spot rate within the band as my first explanatory variable. I set

$$\langle 19 \rangle \quad z_{2t} = (s_t - \underline{s}) / (\bar{s} - \underline{s}),$$

where  $[\underline{s}, \bar{s}]$  are the lower and upper bounds.

I also try to extract information from the domestic yield curve.<sup>7</sup> Anticipated devaluations impact the term structure through the interest parity condition  $\langle 13 \rangle$ . If the market knows that the franc will depreciate by 1 percent over the next quarter, three-month bills will carry a 4 percent annualized return differential,  $\delta^{3/12} = 1\% / (3/12)$ . As the interval  $\tau$  shrinks, the differential widens. In this example, a two-day bill would carry a differential of  $180\% = \delta^{(2/30)/12} = 1\% / 0.0055$ .

As a devaluation becomes imminent, very short-term rates can reach several hundred percent, and lead to a steep inversion of the term structure.<sup>8</sup> For example, on March 15, 1983, five days prior to realignment of the franc, the French one-month, two-day Eurorate spread was  $-315.00$ . The three-month, one-month spread was  $-46.00$ . Because the very shortest rates are often manipulated to artificial levels by the central bank to discourage speculation, I choose the three-month, one-month spread,

$$\langle 20 \rangle \quad z_{3t} = \log(1 + i_t^{3/12}) - \log(1 + i_t^{1/12}).$$

Edin and Vredin (1993) show that the probability of intervention by the central bank is also influenced by macroeconomic variables including the real exchange rate, the money supply, and real output. I incorporate measures of all three of these in my analysis as well, denoting them  $z_{4t}$ ,  $z_{5t}$ , and  $z_{6t}$ . These should affect the probability of realignment as they do in the standard monetary model. If German prices are rising faster than French prices, real depreciation makes French goods more competitive, decreasing the probability of a franc devaluation,  $\gamma_4 < 0$ . Increases in the French money supply should lead to inflationary pressure and weaken the franc,  $\gamma_5 > 0$ . If industrial output is rising, this should reduce the need to devalue on competitive grounds,  $\gamma_6 < 0$ .

To ensure that the probability remains on  $[0, 1]$ , I make a probit transformation,

$$\langle 21 \rangle \quad p_t^\tau = \int_{-\infty}^{\gamma z_t} (\sqrt{2\pi})^{-1} \exp(-t^2/2) dt \equiv \Phi(\gamma z_t).$$

Making these substitutions leads to the econometric specification,

$$\langle 22 \rangle \quad \delta_t^\tau = [\beta_1(1 - \Phi(\gamma z_t))x_t + \beta_2\Phi(\gamma z_t) + \beta_3\Phi(\gamma z_t)\Delta s_{-1}^{j=1}]/\tau.$$

This general functional form nests all the alternative models, and I can now turn to empirical issues.

#### IV. Estimation and hypothesis testing

After describing the data, I describe some of the subtler econometric issues in testing the three alternatives. The fully specified model is described in Section IV.A. Hypothesis testing is described in IV.B. Estimation issues and evaluation of the empirical modeling is discussed in Section IV.C.

##### IV.A Data and specification

For conciseness, I confine the work to the three predominant charter ERM currencies: the French franc, Italian lira, and German deutsche mark. The exchange rate data are nominal ecu exchange rates, collected daily at 2:30 pm Swiss time, for March 13, 1979 to September 11, 1992, over 3,400 observations. The data, through April 1992, were collected by Lars Svensson. I updated Svensson's sample through September with data compiled at the Federal Reserve Bank of New York.

For reasons related to the dominance of Germany in the European community, the ERM has in effect been a greater deutsche mark (DM) area. Giavazzi and Giovannini (1989) note: 'Often, the week which precedes the realignment is characterized by a fall of the dollar and by depreciation of some of the weak European currencies which move towards their maximum divergence limit relative to the DM' (p. 138). Potential entrants, during a trial period, try to stabilize their currency relative to the DM, as Portugal did with the Escudo. I converted the spot rates,  $s_t$ , using the DM as a base, into franc/mark (FF/DM) and lira/mark (IL/DM) exchange rates, since these are the rates implicitly targeted by the ERM. Central parities to form  $x_t$  are in Table 1.

The interest rates were drawn from Lars Svensson's database and again updated from New York Fed sources. I used annualized one-month, Euromarket rates,  $i_t^{1/12}$ , in decimal form, from France, Italy and Germany. I treat Germany as the foreign country and denote its rate with an asterisk. For the dependent variable in  $\langle 22 \rangle$ , I set

$$\langle 23 \rangle \quad \delta_t^{1/12} = \log(1 + i_t^{1/12}) - \log(1 + i_t^{1/12*}).$$

In the probit model, I also used 3-month Eurorates for France and Italy,  $i_t^{3/12}$ .

To coincide with the interest rate data, I model the risk of a realignment in the next 22 days (one month in daily data),  $p_t^{1/12}$ . I construct the real exchange rate,  $r_t$ , in the usual fashion as the spot rate times the ratio of the German to the French or Italian consumer price levels.<sup>9</sup> I also normalize the real rate back to one following each realignment. For the money supplies, I take an M2 equivalent for each country. I use industrial production as my output series. To



TABLE 1. Realignment dates and bilateral central DM rates in the EMS France and Italy.

Date	Central Parity	
	FF/DM	IL/DM
March 13, 1979	2.30950	457.314
September 24, 1979	2.35568	466.460
November 30, 1979	2.35568	466.460
March 23, 1981	2.35568	496.232
October 5, 1981	2.56212	539.722
February 22, 1982	2.56212	539.722
June 14, 1982	2.83396	578.574
March 21, 1983	3.06648	626.043
July 22, 1985	3.06648	679.325
April 7, 1986	3.25617	699.706
August 4, 1986	3.25617	699.706
January 12, 1987	3.35386	720.699
January 8, 1990	3.35386	748.217
September 14, 1992	3.35386	802.488
September 17, 1992	3.35386	Suspension

transform these data to a daily frequency, I interpolate from monthly series. Because of the induced autocorrelation, I take differences, and lag the series by one month.<sup>10</sup>

Having now fully described the data and specified the model, I next describe some econometric issues regarding hypothesis testing in the switching model and then proceed to estimation.

#### IV.B Comparing the alternatives

The Svensson (1991) model assumes a constant devaluation process  $\bar{g}$ . This implies that the probability of realignment and the expected change in the central parity are constant. Because devaluations can occur anywhere within the target zone, though, the expected change of the position within the band after realignment need not always be the same. By the identity <4>, this indicates that the expected change in the spot rate need not be a constant. Hence, the Svensson alternative does not place restrictions on the Markov portion of our model, and can be tested by setting all but the constant term,  $\gamma_1$ , in the probit equal to zero,

$$\langle 24 \rangle \quad \delta_i^\tau = \left[ \beta_1(1 - \Phi(\gamma_1))x_i + \beta_2\Phi(\gamma_1) + \beta_3\Phi(\gamma_1)\Delta s_{-1}^{j=1} \right] / \tau.$$

Testing the Krugman alternative is somewhat more tricky. With a perfectly credible target zone, the probability of realignment is zero. In our model, this implies  $\gamma_1 = -\infty$ , and  $\gamma_2 = \gamma_3 = 0$ . Testing this hypothesis is non-standard though.

Note that in <22> if  $\Phi(\cdot) = 0$  for all  $t$ , the (nuisance) parameters  $\beta_2$  and  $\beta_3$  are not identified. The scores of any likelihood function would be identically zero with respect to those parameters. Test statistics, such as the Wald and the likelihood ratio, do not have the usual  $\chi^2$  distribution because of the non-standard conditions.<sup>11</sup>

Hypothesis testing can proceed using numerical procedures (see *e.g.* Davies, 1987). Here, I can do it with a simple reparameterization of <22>. I begin by linearizing the probit expression.

$$\langle 25 \rangle \quad \Phi(\gamma z_t) \cong \Phi(0) + \Phi'(0)\gamma z_t + \dots$$

Substituting this into <22>, and setting  $m = 2$  for expository purposes, I obtain

$$\langle 26 \rangle \quad \delta_t^\tau = [\theta_1 + \theta_2 x_t + \theta_3 z_{2t} + \theta_4 \Delta s_{-1}^{j=1} + \theta_5 z_{5t} x_t + \theta_6 z_{2t} \Delta s_{-1}^{j=1}] / \tau,$$

where  $\theta_1 = \beta_2 \Phi(0) + \beta_2 \Phi' \gamma_1$ ,  $\theta_2 = \beta_1 (1 - \Phi(0) - \Phi' \gamma_1)$ ,  $\theta_3 = \beta_2 \Phi' \gamma_2$ ,  $\theta_4 = \beta_3 (\Phi(0) - \Phi' \gamma_1)$ ,  $\theta_5 = \beta_1 \Phi' \gamma_2$ , and  $\theta_6 = \beta_3 \Phi' \gamma_2$ . Noting that  $\Phi(\gamma_1 = -\infty, \gamma_2 = 0) = \Phi(\gamma_1 = \gamma_2 = 0) - 0.5$ , the Krugman alternative can be tested with the restrictions,

$$\langle 27 \rangle \quad H_0: \theta_1 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0.$$

Under these assumptions, a standard Wald test can be performed on <27>.<sup>12</sup>

Allowing for both heteroscedasticity and autocorrelation in the errors, denote the asymptotic distribution of the parameters as

$$\langle 28 \rangle \quad \sqrt{T} \left( \begin{bmatrix} \hat{\beta} \\ \hat{\gamma} \end{bmatrix} - \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \right) \sim N(0, \sigma^2 \Omega^{-1}),$$

where  $\Omega^{-1}$  is a symmetric, positive-definite matrix. Consider a  $q$ -vector of restrictions on the parameters,  $H(\beta)$ . If we have estimated the covariance matrix of the  $\beta$ 's in a consistent manner, it follows that

$$\langle 29 \rangle \quad W \equiv H(\hat{\beta}) \left[ \hat{\sigma}^2 \left( \frac{\partial H(\hat{\beta})}{\partial \beta'} \right) \hat{\Omega}^{-1} \left( \frac{\partial H(\hat{\beta})}{\partial \beta} \right) \right]^{-1} H(\hat{\beta}) \sim \chi^2(q).$$

#### IV.C Parameter estimates and tests

I ruled out maximum likelihood estimation in this framework. Throughout the paper, I have tried to use procedures robust to the departures from normality that are a defining aspect of exchange rate behavior. Specifying a likelihood function in the present context would have been quite arbitrary. I chose instead to use non-linear least squares and construct hypothesis tests from Wald restrictions rather than likelihood ratios.

I minimized the sum of squared residuals,

$$\langle 30 \rangle \quad \sum_{t=1}^T [\delta_t^{1/12} - f(\beta, \gamma, s_t, z_t, x_t)]^2,$$

using an iterative procedure, the Davidon–Fletcher–Powell algorithm. I correct the standard errors using the Newey–West estimator with 22 lags to account for the overlap in the data. Results are in Table 2.

TABLE 2. Non-linear least squares estimation Markov regime changes.

Dep. var.											Wald Tests		
	$\beta_1$ Mean rev.	$\beta_2$ Jump con.	$\beta_3$ % jump	$\gamma_1$ Prob. con.	$\gamma_2$ Band pos.	$\gamma_3$ Term str.	$\gamma_4$ Real fx	$\gamma_5$ Money	% Output	$\bar{R}^2$	Unit Rt.	Krugman	Svensson
FF	-0.106 (3.56)	0.014 (6.59)	0.504 (12.19)	-1.532 (10.15)	0.939 (3.98)	-11.525 (5.28)	-3.734 (1.09)	8.141 (2.17)	-0.238 (0.65)	0.711	12.65	1947.95	86.48
IL	-0.143 (14.37)	0.025 (2.23)	0.495 (2.11)	-1.664 (7.32)	1.508 (6.42)	-11.133 (2.69)	-2.888 (2.63)	0.268 (0.40)	-0.524 (1.46)	0.437	206.57	606.69	41.95

Note: The dependent variable is the 1-month interest differential as defined in expression (23). Heteroscedasticity and autocorrelation consistent *t*-statistics are in parentheses. The Wald statistic is distributed  $\chi^2(q)$ . For the unit root alternative, I test  $H_0: \beta_1 = 0, q = 1$ . For the Svensson hypothesis, I test  $H_0: \gamma_{*1} = 0, q = 5$ . The Krugman test utilizes the reparameterization in (26),  $H_0: \theta_{*2} = 0, q = 15$ .

Agents appear to anticipate reversion to the central parity,  $\beta_1 = -0.106$  for the franc, and  $\beta_1 = -0.143$  for the lira. The parameter estimates imply that 11–14 percent of the current deviation from central parity will be eliminated over the month.

The spot rate for the franc is expected to experience, upon realignment, an average 2.31 percent depreciation,  $E[\beta_2 + \beta_3 \Delta s_{-1}^{j=1} = 0.014 + 0.504 \times \Delta s_{-1}^{j=1}]$ . After realignment, agents expect the lira to depreciate 2.79 percent over the month, with  $\beta_2 = 0.025$  and  $\beta_3 = 0.495$ .

The position of the exchange rate within the band also is quite important for the probability of realignment. The movement of the exchange rate from the center to the weak edge of the band raises the probability of realignment by 18 percent for the franc,  $\Delta\Phi(\gamma_2 \Delta z_{2t} = 0.939 \times 0.5)$ , and by 30 percent for the lira,  $\Delta\Phi(\gamma_2 \Delta z_{2t} = 1.664 \times 0.5)$ .

The yield curve is highly significant for both currencies. If the one-month rate rises by 10 percent relative to the three-month rate, the franc's devaluation risk rises by 38 percent,  $\Delta\Phi(\gamma_3 \Delta z_{3t} = -11.525 \times -0.1)$ , and the lira's by 37 percent,  $\Delta\Phi(\gamma_3 \Delta z_{3t} = -11.133 \times -0.1)$ .

The macro variables enter with their anticipated signs, but are not particularly significant. Only the money supply for France and the real exchange rate for Italy are statistically significant. As a group though, I can reject at the 5 percent level for France and Italy that the macro variables have no impact on the probability of realignment.

I now turn to direct comparisons of the target zone alternatives. I construct six tests in Table 2 comparing the time varying risk formulations against the Krugman and Svensson models and the unit root. In each case they are handily rejected. The Krugman model, in particular, is clearly unacceptable on statistical grounds. The Wald statistics are enormous, almost 2,000 for the franc, and over 600 for the lira. This is a decisive rejection of the assumption that agents behave as if the bands are never going to be realigned. This result reaffirms the empirical evidence presented in Section II.

The data are also clearly at odds with the assumption of a constant probability of realignment. As seen in Table 3, this probability rises sharply near devaluation episodes. The only way for the Svensson model to explain these episodes is to raise the unconditional probability. This tends to overestimate the probability on any given day though, leading to a rejection of this hypothesis.

The unit root also is strongly rejected in favor of mean reversion within the hand. Apparently, market participants know when the currency is likely to be successfully returned back to the center of its band, and when a realignment is more likely to ensue. Grouping these states together, without accounting for possible regime shifts, is what seems to obscure the evidence against the unit root.<sup>13</sup> Through *endogenous* sample selection of credible regimes, the model finds strong evidence of mean reverting behavior.

## V. Estimates of devaluation risk

I can back out from the model agent's implicit beliefs about devaluation risk. In this part, I discuss the fitted probability estimates,  $\hat{p}_t^{1/12}$ . Table 3 looks at

TABLE 3. Realignment probabilities from structural model.

Date	Month before percentage	Day before percentage	Day after percentage
<i>French franc</i>			
September 24, 1979	18.19	22.93	13.99
October 5, 1981	22.57	50.02	10.30
June 14, 1982	26.56	57.73	7.02
March 21, 1983	17.11	99.46	6.66
April 7, 1986	21.21	31.67	11.94
January 12, 1987	19.39	29.88	10.16
August 2, 1993	19.49	29.07	21.48
Mean = 16.25	SD = 8.01	95% = 26.31	
<i>Italian lira</i>			
September 24, 1979	10.90	17.26	12.49
March 23, 1981	29.47	38.05	18.04
October 5, 1981	20.06	34.79	15.11
June 14, 1982	27.54	27.25	10.13
March 21, 1983	17.64	51.19	12.00
July 22, 1985	27.09	31.29	11.37
April 7, 1986	21.47	22.97	14.78
January 12, 1987	16.10	32.72	14.51
January 8, 1990	26.70	35.86	18.56
September 14, 1992	36.30	58.67	
Mean = 19.35	SD = 7.79	95% = 31.53	

these estimates around realignment dates, and Figures 1 and 2 graph these probabilities for the entire ERM experience for both currencies.

On an average day, agents think there is a 16.3 percent chance of a realignment of the franc over the next month. The average probability is about one-fifth higher for the lira, 19.4 percent. These probabilities vary considerably through time, peaking near realignments. As little as one month before each formal devaluation, the probability is generally near the unconditional mean. With the franc, the probability doubles or even triples by the day before the episode, spiking back down to the average immediately after the realignment. The extremes for the lira are less pronounced, but the average probability rises about 20 percent in the month preceding realignment. It then drops back well below the pre-crisis estimate within a day.

The model is quite accurate in predicting devaluations. In five of the six French franc realignments in the estimation period, the realignment risk exceeds a 95 percent confidence level ( $\hat{p}_t > 26.31\%$ ) prior to the formal devaluation. The only episode not predicted by the model is the very first devaluation in September of 1979.

The model works nearly as well for the lira. In seven of ten cases, the risk reaches a 95 percent level ( $\hat{p}_t^{1/12} > 31.53\%$ ) prior to devaluation. The model also works well in the ERM crisis of September 1992. The day prior to the

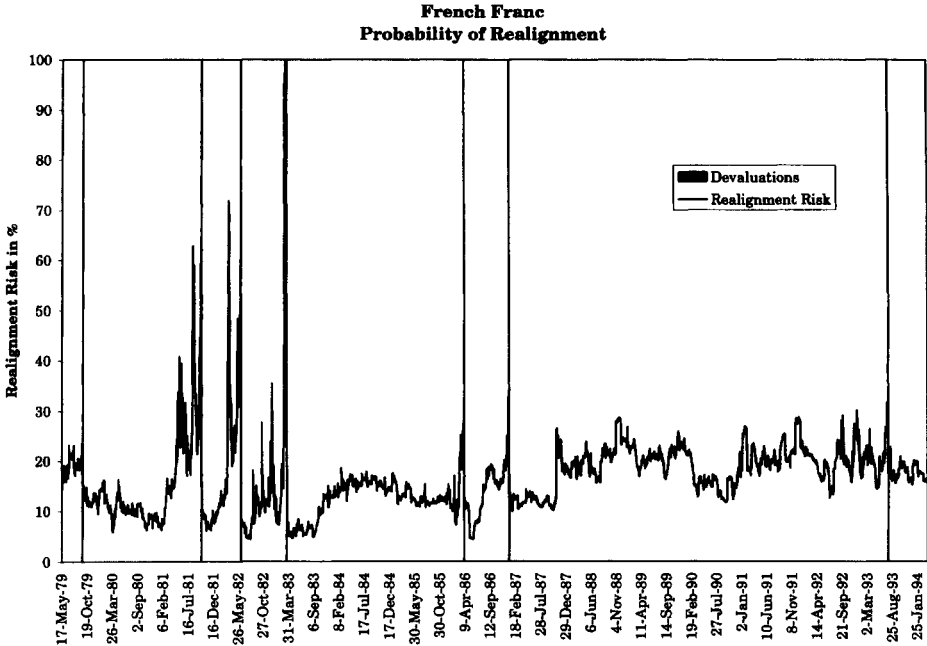


FIGURE 1.

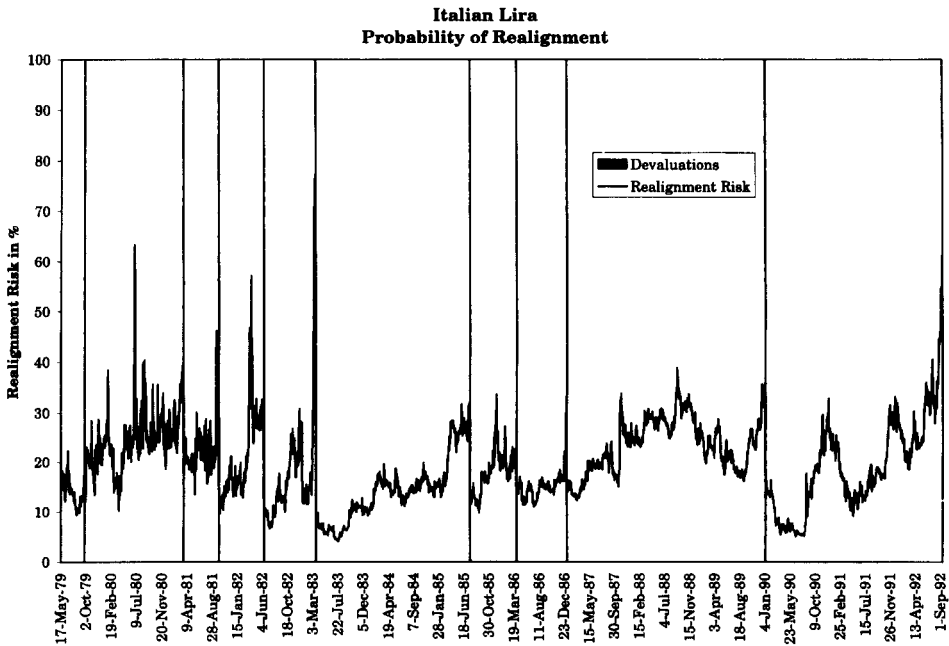


FIGURE 2.

lira's devaluation and three days prior to their withdrawal from the ERM, the estimated probability of realignment was 58.7 percent, the highest in more than 7 years.

Since the first version of this paper was written, the ERM has experienced an additional *de facto* realignment on August 2 1993 when the target zone bands were widened to  $\pm 15$  percent. The franc and the other major currencies drifted outside their old fluctuation bands. This event provided an out-of-sample test which proved to be very supportive of the probit-Markov model. As can be seen in Table 3, the risk again exceeded the 95 percent confidence prior to the policy change.

## VI. Conclusion

A framework for analyzing target zone exchange rate models was presented. The structural model was derived from the interest parity condition, and it nested four alternatives, Krugman's (1991) model with no risk of realignment, Svensson's (1991) model with constant devaluation risk, Bertola and Svensson's (1993) model with time varying risk, as well as the random walk.

Previous empirical studies had found little support for target zone models. Many papers had trouble distinguishing exchange rates from a random walk. This paper provided strong corroboration of the second generation models of target zone models with stochastic devaluation risk.

A model of Markov switches with a probit specification of transition probabilities was introduced to estimate the structural model. The probit-Markov estimates sharply favor the model with time varying devaluation risk over the other target zone models. I also provide strong evidence against a unit root in favor of mean reversion within the band.

The model produces extremely credible estimates of the probability of ERM realignments. It predicts, using economic fundamentals, 13 of the 17 realignment episodes from the inception of the ERM through the widening of the target zone bands in August of 1993. These may prove useful to policy makers if they can continue to provide some early warning signals.<sup>14</sup>

The framework presented is quite flexible. Natural extensions include expanding the probit specification to look at other risk factors like central bank intervention. With tools like those developed here, empiricists may be able to contribute to improvements in the next generation of target zone models.

## Notes

1. The bands were  $\pm 2.25$  percent for most countries. The exceptions are the Portuguese escudo and the Spanish peseta which traded in a  $\pm 6.0$  range. From March 1979 until the devaluation of January 7, 1990, the Italian lira also had a  $\pm 6.0$  band, as did the British pound between its entry to the ERM in October 1990 and its suspension in September 1992.
2. The krona was kept in a band defined by an index of 15 currencies with weights varying according to the Swedish trade basket. From August 1977 to June 1985, the band was  $\pm 2.25$  percent subsequently narrowed to  $\pm 1.5$  percent. In mid-November 1992, however, Stockholm agreed to let the krona float in the wake of speculative pressure.

3. Because the target zone makes the exchange rate less responsive to the fundamental, this theoretical implication is sometimes called the 'honeymoon effect.'
4. Both of these results are formally established in Lindberg and Soderlind (1991a, p. 4).
5. Svensson (1992) works out a representative agent asset pricing model where the spot exchange rate takes Poisson jumps with constant intensity. He concludes that 'disregard [ing] the risk premium seems warranted, at least for narrow target zones.' One could also readily obtain the relation under risk neutrality.
6. Allowing for the possibility of more than two states is a straightforward extension. In our sample though, there are no cases of appreciation. Attesting to the strength of the DM, Giavazzi and Giovannini (1989, p. 141) note that 'all EMS realignments resulted in some appreciation of the DM relative to other EMS currencies.'
7. Lindberg, Soderlind, and Svensson (1993) model the realignment risk using a range of short-term interest rates. For a given maturity  $t$ , they estimate the cumulative realignment probability during the interval  $t + \tau$  and an expected time to devaluation.
8. I thank Vivek Moorthy for this observation.
9. I also tried using unit labor costs and wholesale prices. While all three measures yielded similar results, the consumer price indices were more consistent across countries.
10. All the data will share this autocorrelated component because of the month-long overlapping expectations in the data. I use heteroscedasticity and autocorrelation consistent estimates to correct the standard errors.
11. This difficulty commonly arises in threshold regression. The problem can be avoided any time the transition probabilities are smooth though. See Luukkonen, Saikkonen, and Terasvirta (1988) for additional discussion.
12. In the full six variable probit, the Krugman alternative imposes 15 zero restrictions: all five fundamental factors, their cross products with the  $x_t$  and  $\Delta s_{-1}^{j=1}$  variables, and a constant. Because the  $x$  and  $z_2$  variables are proportional, there is one redundant restriction.
13. For more discussion, see Mizrach (1993a) where I find that conventional unit root tests are not sufficiently powerful to reject the unit root alternative.
14. For some demonstration of this capability, see Mizrach (1993b).

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