

A liquidity-in-advance model of the demand for money under price uncertainty*

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We develop a model of money demand in which the asset and goods markets open sequentially. Money balances are subject to price-level and consumption risk. Empirical evidence from a bivariate ARCH framework suggests both sources of risk reduced money demand in the 1970s. Inflation risk depresses money demand throughout our sample, 1962–1987. The econometric analysis resolves conflicting empirical estimates reported in the literature.

1. Introduction

Examinations of the demand for money have tended to focus on its role as a transactions asset. The Baumol (1952) and Tobin (1956) analysis, followed by the modern versions of Santomero (1979) and Frenkel and Jovanovich (1980), developed a transactions demand in an inventory-theoretic framework. However, this view treats money as a constant, generally zero, rate of return, riskless asset. In the presence of uncertain price movements, monetary economists have long recognized that these are clearly not tenable assumptions.

Proper specification of the demand for money requires the modeling of money's rate of return in an uncertainty framework. Eden (1976) was the first

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to attempt to develop such a model. Boonekamp (1978) and Fama and Farber (1979) extended this analysis, developing a more explicit treatment of the hedging motives associated with money holdings in a two-period model. Yet, as noted by McCallum and Goodfriend (1987), much more needs to be accomplished in order to develop a dynamic model of money demand with price uncertainty.

The empirical work in this area too requires additional work. Traditionally it has focused attention on the impact of inflationary risk in a utility framework where money provides 'services' like other durable goods. Here, Klein (1977) has argued that unexpected inflation will deteriorate the quality of cash balances, leading the consumer to hold more money. He finds a positive relationship between price variability and money demand in annual data over the period 1880–1972. Smirlock (1982) re-examines the empirical results, using several measures of inflation uncertainty in Goldfeld's money demand specification. He finds, contrary to Klein, that the term enters negatively. Smirlock, however, offers neither a theoretical explanation for his results, nor does he attempt to reconcile them with Klein's.

The analysis that follows investigates the theoretical and empirical work on money demand. In the theoretical sections, we first build a model of two asset demands, money and bonds, in which goods and asset markets are open simultaneously. Money balances are subject to inflationary risk like any nominally denominated asset. In this case, money demand depends only on the nominal interest rate and consumption, and as a consequence inflation risk can be perfectly hedged.

We then consider a model that incorporates the recent perspectives of Svensson (1985) and Giovannini (1989). Goods purchases are limited by the money balances obtained by the agent in the previous period. Money balances still bear inflationary risk, but there is also a precautionary demand related to the liquidity-in-advance constraint. In the traditional precautionary demand for money, price variability would lead to increased holdings of money balances for the risk-averse agent. In the dynamic portfolio model developed here, the agent may defer consumption to periods in which the goods market is less volatile. Weighing these competing effects is an empirical matter, so we derive from the Euler equations of the agent's problem a money demand equation involving the variability of inflation, consumption, and their covariance.

To examine this model empirically, we use a procedure developed by Engle (1982) to model the risk terms as conditional moments. They differ, often quite dramatically, from *ad hoc* sample measures of variability used in earlier studies. Furthermore, we show that the failure to instrument for the unobserved expectation leads to inconsistent estimates of the impact of the risk variables. These specification and estimation errors are helpful in resolving the contradictory results of Klein and Smirlock.

The liquidity in advance specification is supported by the data. Inflation and consumption risk enter significantly in the 1970s. Inflationary risk is significant for the sample period from the 1960s to the 1980s.

2. The initial two-asset model

We begin the analysis with a model in which goods and asset markets are open simultaneously. The agent chooses consumption and divides his wealth between money and bonds. Let M_t and B_t be nominal money balances and nominal bonds held at the end of period t . The bond is a one-period security sold at discount $(1 + R_t)^{-1}$, where R_t is the nominal interest rate, which pays one unit of money in period $t + 1$. Money balances are used for transactions purposes during the period and remain in the agent's wealth portfolio into $t + 1$. The real value of the agent's future wealth, $(M_{t+j} + B_{t+j})/P_{t+j} = m_{t+j} + b_{t+j}$, is uncertain because the price level, P_{t+j} , is stochastic.

We assume that the representative agent is attempting to maximize the expected present value of utility over an infinite horizon. In so doing the household selects consumption, c_t , and leisure, l_t , flows in each period, simultaneously with asset portfolio choice. The optimization problem facing the agent may be written as

$$\text{Max } U(c_t, l_t) + \sum_{j=1}^{\infty} \beta^j E[U(c_{t+j}, l_{t+j})], \quad 0 < \beta < 1. \quad (1)$$

To allow for a role to be played by money in such a model, one must assume that transactions are costly in some manner. Following the approach employed by McCallum and Goodfriend (1987), among others, we assume that labor is supplied inelastically, generating income y_t in each period but leisure must be expended to make transactions. The latter is assumed to depend negatively on the quantity of money held to facilitate such transactions. This approach, first suggested by Saving (1972) and Barro and Santomero (1976), results in a shopping cost function that may be written as

$$s_t = (1 - l_t) = \psi(c_t, m_t), \quad \psi_1 > 0, \quad \psi_2 < 0, \quad (2)$$

where the quantity of leisure time available is normalized at one. Transaction costs increase with consumption and decrease with money balances held. The latter formulation derives an indirect demand for money.

The agent faces budget constraints each period of the form

$$y_t = c_t + m_t - (1 + \pi_t)^{-1} m_{t-1} + (1 + R_t)^{-1} b_t - (1 + \pi_t)^{-1} b_{t-1}, \quad t = 0, 1, \dots, \infty, \quad (3)$$

where all terms have been previously defined with the exception of π_t which is the inflation rate over the period [$\pi_t = (P_t - P_{t-1})/P_{t-1}$].

The maximization of eq. (1) subject to (2) and (3) results in the period t first-order conditions for consumption, shopping time (or equivalently leisure), and money and bond holdings:

$$U_1 - \phi_t \psi_1 - \Gamma_t = 0, \quad (4a)$$

$$-U_2 + \phi_t = 0, \quad (4b)$$

$$-\phi_t \psi_2 - \Gamma_t + \beta E[\Gamma_{t+1}(1 + \pi_{t+1})^{-1}] = 0, \quad (4c)$$

$$-\Gamma_t(1 + R_t)^{-1} + \beta E[\Gamma_{t+1}(1 + \pi_{t+1})^{-1}] = 0, \quad (4d)$$

where ϕ and Γ are the Lagrange multipliers for constraints (2) and (3).

Eqs. (4) suggest that money is related to current consumption, lagged money and bonds, and the current and future values of interest rates and inflation. Working with the latter two Euler equations one may derive an expression, analogous to that which was obtained by Fama and Farber (1979), for the equilibrium conditions for money and bonds,

$$-U_2 \psi_2 / (U_1 - \psi_1 U_2) = R_t / (1 + R_t). \quad (5)$$

Eq. (5) indicates the nature of the agent's portfolio decision. In equating the marginal utility between a unit of bonds and a unit of money, the nominal interest rate fully captures the opportunity cost. Within this framework both portfolio assets are nominally denominated and subject to identical inflationary risk.

McCallum and Goodfriend (1987) refer to this type of expression as a portfolio-balance relationship, which may be written in implicit form as

$$f(m_t, c_t, R_t) = 0. \quad (6)$$

Note that everything relating to price uncertainty in this setup is incorporated into the nominal interest rate term contained in eq. (6). Assuming this can be solved for m_t , we have a structural form similar to the traditional money demand function

$$M_t/P_t = L(c_t, R_t). \quad (7)$$

3. A liquidity-in-advance model of asset choice

This section considers the case in which the goods and asset markets open sequentially. As in the work of Svensson (1985) and Giovannini (1989), there exists a precautionary demand for money in that consumption is bounded by liquidity obtained prior to the goods market trading period. In this dynamic framework, intertemporal substitution of consumption is an additional hedge.

Formally, the model adds another constraint to eqs. (1), (2), and (3) above. Specifically, we assume that money balances used for consumption are chosen at the end of the period for use in the subsequent period,

$$c_t \leq (1 + \pi_t)^{-1} m_{t-1}. \quad (8)$$

Notice that this formulation is a type of cash-in-advance constraint but without unitary velocity. Shopping time now decreases with $\psi(c_t, M_{t-1}/P_t)$, and the budget constraint can therefore be rewritten as

$$y_t + (1 + \pi_t)^{-1} m_{t-1} - c_t = m_t + (1 + R_t)^{-1} b_t - (1 + \pi_t)^{-1} b_{t-1}. \quad (3')$$

The maximization proceeds as follows. The agent is perceived to maximize eq. (1) subject to (2), (3'), and (8). Defining the Lagrangian multiplier of the liquidity constraint as λ , the new first-order conditions are

$$U_1 - \phi_t \psi_1 + \lambda_t - \Gamma_t = 0, \quad (9a)$$

$$-U_2 + \phi_t = 0, \quad (9b)$$

$$-\Gamma_t + \beta E \left[\Gamma_{t+1} (1 + \pi_{t+1})^{-1} \right] - \beta E \left[\lambda_{t+1} (1 + \pi_{t+1})^{-1} + \phi_{t+1} \psi_2 \right] = 0, \quad (9c)$$

$$-\Gamma_t (1 + R_t)^{-1} + \beta E \left[\Gamma_{t+1} (1 + \pi_{t+1})^{-1} \right] = 0. \quad (9d)$$

Notice that the symmetry that was previously obtained between money and bonds in eqs. (4c) and (4d) above is broken. Bonds serve a different role in this model. They remain a contract to deliver a certain quantity of money balances in the next period, but they are no longer a contract for nominal consumption opportunities because of the liquidity-in-advance constraint. Because the goods market closes before the asset market, consumption opportunities in that period are no longer available at any interest rate.

It is no longer a straightforward process to obtain a simple money demand equation. Substituting for λ_{t+1} from eq. (9a) and ϕ_{t+1} from (9b) results in a comparable expression to eq. (5) above:

$$-I_t + \beta E[U_1(1 + \pi_{t+1})^{-1} - U_2\psi_1(1 + \pi_{t+1})^{-1} - U_2\psi_2] = 0. \quad (10)$$

Note that the Lagrangian multiplier I_t still must be eliminated. To do so requires that the nonlinear rational expectations difference equation, (9d), be solved. Linearizing by Taylor expansion and neglecting higher-order terms, this may be written as

$$I_t \cong \text{constant} + \theta_1 E[I_{t+1}] + \theta_2 E[(1 + \pi_{t+1})^{-1}] + \theta_3 R_t. \quad (11)$$

Under the assumption that an invertible Wold representation exists for R_t , a solution exists for I_t of the form [see Whiteman (1983)]

$$I_t = A(L)R_t, \quad (12)$$

when $A(L)$ is polynomial in the lag operator.

Substituting into eq. (10) a comparable portfolio-balance relationship to that contained in eq. (6) can be written as

$$-A(L)R_t + E[F(m_t, c_{t+1}, \pi_{t+1})] = 0. \quad (13)$$

This expression for money demand indicates that it depends upon nominal bond rates, future consumption, and inflation. This is a trivariate system, with risky future consumption opportunities serving as the scale variable. An expression for money demand, analogous to eq. (7) above, is derived in section 5 under some restrictions on the functional forms.

4. Measuring the independent variables

The liquidity-in-advance model developed in section 3 may be investigated empirically with reference to eq. (13) above. However, the independent variables in these functions and the underlying theory itself depend critically upon the expectations of future values of c and π . The traditional approach to obtain such estimates is either to substitute actual values or unconditional sample moments into the estimated equations. In this section we attempt to more accurately capture these expectations through the use of conditional expectations. These estimates are then used in the reduced form estimation of section 5 and in a linearized version of (13).

4.1. Modelling conditional expectations

We begin with a representation of the inflation rate. Let us write a structural equation for the inflation rate π_t as

$$\pi_t = x_t \beta + \varepsilon_{1t}, \quad \varepsilon_{1t} | \Omega_{t-1} \sim N(0, \sigma_{1t}^2), \quad (14)$$

where x_t is a matrix of exogenous variables, β is a vector of parameter estimates, and ε_{1t} is a disturbance term. The conditional mean of ε_{1t} is zero, given information Ω_{t-1} , but the conditional variance is time-varying and is assumed to be a function of a set of additional exogenous variables z ,

$$\sigma_{1t}^2 = z_t \alpha. \quad (15)$$

Engle (1982) notes that heteroscedasticity often takes an autoregressive form with large errors tending to follow other large errors of either sign,

$$\sigma_{1t}^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{1t-i}^2. \quad (16)$$

This is an autoregressive conditionally heteroscedastic (ARCH) process of order q . In an application to the price level in the U.S., Engle (1983) found that the large increases in the 1970s were not associated with large increases in the conditional variance. The variance around this higher expected price level was in fact much lower than the variability in the 1940s and 1950s.

Analogously the model for real consumption purchases can be written as

$$c_t = w_t \gamma + \varepsilon_{2t}, \quad \varepsilon_{2t} | \Omega_{t-1} \sim N(0, \sigma_{2t}^2). \quad (17)$$

with w_t a matrix of explanatory variables, γ a vector of parameters, and ε_{2t} a disturbance term. Combining these two conditional expectations models, we can then write the first-order bivariate ARCH model as

$$\text{vec}(\varepsilon_t \varepsilon_t') \equiv H_t = A_0 + A_1 \eta_{t-1}, \quad (18)$$

where $\text{vec}(\varepsilon_t \varepsilon_t') = (\varepsilon_{1t}^2, \varepsilon_{1t} \varepsilon_{2t}, \varepsilon_{2t}^2)'$, A_0 is a 3×1 vector of constant terms, A_1 is a 3×3 matrix of coefficients, and $\eta_{t-1} = (\varepsilon_{1t-1}^2, \varepsilon_{1t-1} \varepsilon_{2t-1}, \varepsilon_{2t-1}^2)'$.

We estimate these conditional moments by specifying models for inflation and consumption behavior. The equation for the inflation rate is drawn from Engle (1983).¹ Our equation for consumption is a fourth-order autoregres-

¹Estimation by ordinary least squares, using the data outlined in appendix A.1 below, resulted in

$$\Delta \pi_t = 0.0033 - 0.3169 * \Delta \pi_{t-1} - 0.2019 * \Delta \pi_{t-2} - 0.0652 * \Delta pm_{t-1} + 0.0198 * \Delta w_{t-1},$$

(0.024) (2.988) (2.056) (0.374) (0.365)

$$SE = 1.520, \quad \text{Interval: } 1959:4-1988:1,$$

where Δpm_t is the first difference in the import price deflator and Δw_t is the first difference of nonfarm hourly earnings.

sion.² Residuals from these equations were tested for the presence of ARCH disturbances and the standard Lagrange multiplier test rejected the null of no ARCH errors for the trivariate system.³

4.2. Maximum-likelihood estimation of asset return moments

There are two problems with least-squares estimates of the ARCH process, i.e., the potential for negative lags and the inefficiency of estimates.

To counter the first problem, we estimated an ARCH(8) model with the alphas restricted to the positive orthant and with sums less than one.⁴ These weights were then used to form a two-parameter model

$$H_t = A_0 + A_1^* (L\pi, Lc, L\pi c)', \quad (19)$$

with A_1 restricted to be a diagonal matrix.

Engle and Kraft (1983) show that efficient estimation of (19) requires maximizing the conditional log-likelihood for each observation⁵

$$L = \sum_{t=1}^T \text{constant} + 0.5 * \ln |H_t| - 0.5 * \varepsilon_t' H_t^{-1} \varepsilon_t. \quad (20)$$

The estimator is asymptotically normally distributed, with the final scoring step regression computing a consistent estimate of the covariance matrix. Results from maximum-likelihood estimation for the conditional moments

²The equation estimated was

$$\Delta c_t = 6.714 + 0.0977 * \Delta c_{t-1} + 0.1088 * \Delta c_{t-2} + 0.1741 * \Delta c_{t-3} + 0.1255 * \Delta c_{t-4},$$

(2.917) (1.21) (1.113) (1.747) (1.259)

$$\text{SE} = 12.824, \quad \text{Interval: 1959:4-1988:1.}$$

³Engle, Granger, and Kraft (1983) show that, if we restrict eq. (18) to a diagonal representation, three times the sample size times the sum of the R^2 of the three regressions is distributed $\chi^2(3(p+1))$. For the sample $3T(R_1 + R_2 + R_{12}) = 104.5$, which easily rejects the null of no ARCH errors at the 99% level.

⁴The eight lagged alphas for the price equation variance were $L\pi = (0.074, 0.133, 0.283, 0.360, 0.128, 0.050, 0.001)$; for consumption, $Lc = (0.142, 0.118, 0.237, 0.146, 0.232, 0.232, 0.005, 0.091)$; for the covariance, $L\pi c$, negative weights were allowed: $(0.248, -0.061, 0.244, -0.004, 0.339, 0.100, 0.134, 0.320)$.

⁵Denote $\alpha = \text{vec}[A']$, $z_t' = (1, \eta_{t-1}')$, and P , such that $\text{vec}(\varepsilon_t \varepsilon_t') = P' \eta_t$, $Z_t = P'(I \otimes z_t')$, $Q_t^{-1} = (H_t^{-1} \otimes H_t^{-1})$, and $f_t = \text{vec}(\varepsilon_t \varepsilon_t' - H_t)$. The score and the information matrix can then be written as $\partial L / \partial \alpha' = 0.5 \sum_{t=1}^T Z_t' Q_t^{-1} f_t$, $I_{\alpha\alpha} = 0.5 \text{E}(\sum_{t=1}^T Z_t' Q_t^{-1} f_t)$. Facilitating the scoring step is that the parameter iterations $\alpha^{(i+1)} = \alpha^{(i)} + [Z' Q^{(i)-1} Z]^{-1} Z' Q^{(i)-1} f^{(i)}$ can be performed using OLS auxiliary regressions.

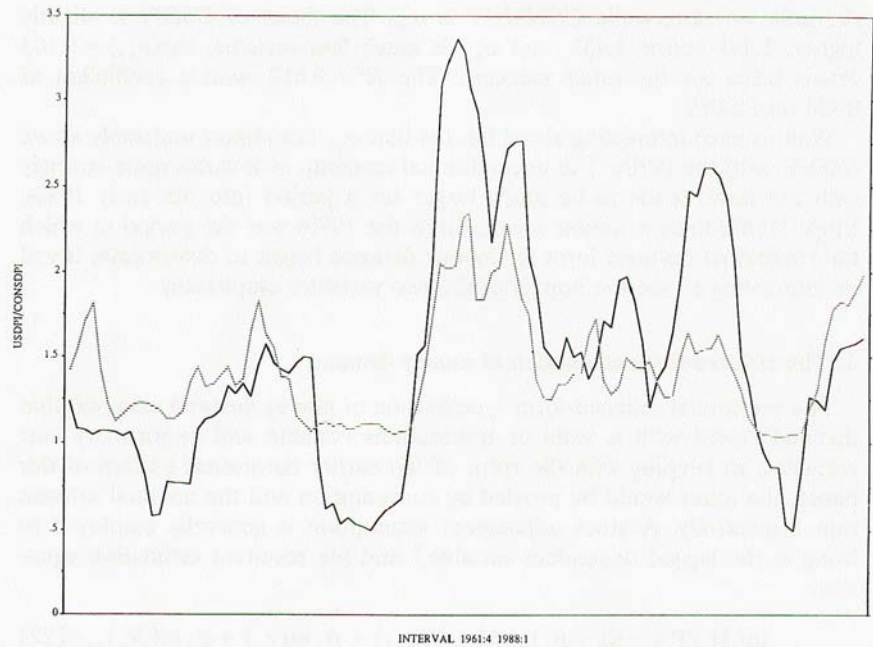


Fig. 1. Risk measures; — USDPI, ---- CONSDPI.

are

$$\sigma_{\pi}^2 = 1.0015 + 0.4931 * L\pi, \quad (21a)$$

(2.963) (2.466)

$$\sigma_c^2 = 45.8264 + 0.7834 * Lc, \quad (21b)$$

(1.564) (2.656)

$$\sigma_{\pi c} = -1.0346 + 0.5361 * L\pi c. \quad (21c)$$

(0.722) (4.031)

T-statistics are given in the parentheses; convergence was achieved in 14 iterations.

4.3. Comparison between conditional and unconditional moments

Eqs. (21) report the covariance matrix of consumption and inflation conditioned on the information set Ω_{t-1} . The ARCH structure is tantamount to assuming that all the information is contained in the lagged squared errors. Fig. 1 is an illustrative plot of the differences between conditional and unconditional moments. *USDPI* is a 12-quarter moving standard deviation of

the inflation rate, while *CONSDPI* is σ_{π_t} . The mean of *USDPI* is slightly higher, 1.481 versus 1.433, and σ_{π_t} is much less variable, $\text{var}(\sigma_{\pi_t}) = 0.103$ versus 0.506 for the other measure. The $R^2 = 0.619$, with a coefficient of 0.906 on *USDPI*.

What is most interesting about fig. 1 is that σ_{π_t} lies almost uniformly above *USDPI* until the 1970s. The unconditional moment, as it varies quite strongly with the level, tends to be much larger for a period into the early 1980s. Engle (1983) finds a similar result. Since the 1970s was the period in which the traditional reduced form for money demand began to deteriorate, it will be interesting to see the impact of the two variables empirically.

5. The reduced-form estimation of money demand

The traditional reduced-form specification of money demand assumed that demand varied with a scale or transactions variable and opportunity cost variables. In keeping with the spirit of the earlier theoretical section of this paper, the latter would be proxied by consumption and the nominal interest rate respectively. A stock adjustment assumption is generally employed to bring in the lagged dependent variable,⁶ and the resultant estimation equation:

$$\ln(M_t/P_t) = \beta_0 + \beta_1 \ln(M_{t-1}/P_{t-1}) + \beta_2 \ln(c_t) + \beta_3 \ln(R_t). \quad (22)$$

As noted earlier, Klein extended this model to include the variability of the inflation rate and found a significantly positive sign. Smirlock challenged the empirical finding and using an alternative measure of inflation variability found the opposite relationship.⁷ More recently, Sweeney (1988) has argued that the speed of adjustment and overall fit of this equation can be improved by some measure of inflation risk.

We begin an investigation of the money demand function by examining whether using conditional moments can resolve this paradox. Using M1 money balances and the implicit price deflator for consumption for the dependent variable, real consumption purchases, 90-day commercial paper rate, and the two risk measures obtained above as the independent variables, we find that both the unconditional and conditional second moments enter with a significant negative sign for the sample period 1962:1–1988:1. These results are reported in table 1. See the data appendix A.2 for further details.

⁶Goodfriend (1985) has offered an alternative interpretation of the lagged dependent variable. Assuming the 'true' income and interest rate variables follow a first-order autoregressive process and are measured with error, he derives the restrictions $0 < \beta_1 < 1$, even though money demand does not depend on lagged money balances.

⁷Klein's measure is similar though perhaps more artificial. He uses a 5-year standard deviation around a 10-year mean. This measure at a quarterly frequency is closely approximated by Smirlock's 8-lag variable.

Table 1
Reduced-form estimation of money demand.^a

Interval quarterly	Constant	Lagged money	Consumption	Interest rate	RISK1	RISK2	SE	$\hat{\rho}$	DW
1962:1-1988:1	-0.226 (4.70)	0.953 (41.23)	0.048 (4.87)	-0.026 (5.54)	-0.016 (3.11)		0.0083	0.3715 (3.76)	2.038
1962:1-1988:1	-0.235 (4.58)	0.940 (37.47)	0.051 (4.81)	-0.024 (4.82)		-0.007 (2.31)	0.0085	0.394 (3.96)	2.053
1961:4-1969:4	-1.041 (5.83)	0.839 (7.39)	0.192 (4.50)	-0.052 (5.15)	-0.007 (1.29)		0.0041		1.803
1961:4-1969:4	-1.002 (5.54)	0.815 (7.21)	0.190 (4.34)	-0.046 (4.79)		-0.003 (0.78)	0.0042		1.644
1970:4-1979:4	0.012 (0.12)	0.951 (30.21)	0.014 (1.37)	-0.016 (5.41)	-0.016 (5.11)		0.0049		2.362
1970:4-1979:4	-0.118 (1.09)	0.973 (30.84)	0.026 (2.37)	-0.015 (4.95)		-0.007 (4.72)	0.0050		2.426
1980:1-1988:1	-2.498 (2.47)	0.661 (6.43)	0.406 (2.73)	-0.038 (2.85)	0.027 (1.44)		0.0102	0.091 (0.41)	1.634
1980:1-1988:1	-1.680 (2.34)	0.760 (13.97)	0.281 (2.81)	-0.042 (3.16)		0.006 (0.87)	0.0103		1.858

^aThe dependent variable is the log-difference of real M1 money balances; for data sources and description, see appendix A.2. RISK1 is the square root of the conditional variate as given in eq. (21a). RISK2 is a 12-quarter moving standard deviation of inflation rates. T-statistics are in parentheses.

Further analysis revealed this to be solely a product of the 1970s inflation. Breaking the interval into subperiods by decade, we find that the risk terms are most strongly negative in the high-inflation subsample. More striking is that the conditional variate is positive in the 1980s.

It would therefore appear that the choice of risk proxy cannot account for the differences between Klein and Smirlock's results. The change in sign in the 1980s from negative (Smirlock) to positive (Klein) suggests that we need to get closer to the structural model. Section 6 pursues this goal as well as some additional econometric issues.

6. Asset demand equations under rational expectations

6.1. Issues concerning generated regressors

Section 4 showed the importance of measuring the variance around the conditional mean. In this section, we will now show that the use of risk proxies is likely to generate inconsistent estimates.

Consider a simple version of the uncertainty-enhanced money demand equation from eq. (22),

$$m_t = x_t \beta + \sigma_{\pi_t}^2 \delta + e_t, \quad (23a)$$

where x_t is a matrix of independent variables, excluding the portfolio moments, and $\sigma_{\pi_t}^2$ is some theoretically appropriate measure of inflationary risk. Smirlock and Klein use an n -period distributed lag on π as a measure of the conditional mean

$$\pi_t^* = \frac{1}{n} \sum_{j=0}^n \pi_{t-j},$$

generating

$$\Phi_t^2 = \frac{1}{n} \left[\sum_{j=0}^n (\pi_{t-j} - \pi_t^*)^2 \right].$$

Replacing $\sigma_{\pi_t}^2$ with Φ_t^2 will produce inconsistent estimates of δ . As Pagan (1984) notes, this is a standard errors in variables problem that arises in using a proxy for an unobserved regressor. To see this, rewrite (23a) as

$$m_t = x_t \beta + \Phi_t^2 \delta + (\sigma_{\pi_t}^2 - \Phi_t^2) \delta + e_t. \quad (23b)$$

The regressor will then be correlated with the composite disturbance term $\mu_t = (\sigma_{\pi_t}^2 - \Phi_t^2) \delta + e_t$: $E[\Phi_t^2 \mu_t] \neq 0$ and $\text{plim}(\hat{\delta}) \neq \delta$.⁸ Most damaging to existing empirical studies of money demand is that the direction of the errors in variables bias cannot generally be signed.

⁸See Pagan (1984, theorem 12, p. 241).

Using an ARCH proxy for risk, i.e., $\Phi_t^2 = E[(\pi_t - \pi_t | \Omega_{t-1})^2 | \Omega_{t-1}]$ does not escape this problem as the 'true' variable will still be measured with error. A consistent estimator requires the use of instrumental variables. To make the estimator more robust, Pagan and Ullah (1988) suggest using the fitted conditional variances (21a)–(21c) as instruments for the original residuals for the conditional mean equations. Appendix A.3 has details.

6.2. *The liquidity-in-advance specification*

To estimate eq. (13), we take a two-term Taylor expansion about the conditional means of the two variables dated $t + 1$ and a single term for the others. Assuming rational expectations, only the conditional second moments of inflation and consumption will remain.⁹ We also include the lagged dependent variables as an initial condition to the agent's problem,

$$m_t \approx \beta_0 + \beta_1 m_{t-1} + \beta_4 R_t + \beta_5 \text{var}(c_{t+1}^e | \Omega_t) + \beta_6 \text{var}(\pi_{t+1}^e | \Omega_t) + \beta_7 \text{cov}(c_{t+1}^e, \pi_{t+1}^e | \Omega_t) + v_t. \quad (24)$$

Ω_t denotes information available at time t , and $v_t = \theta_1 E[(c_{t+1} - c_{t+1} | \Omega_t)] | \Omega_t + \theta_2 E[(\pi_{t+1} - \pi_{t+1} | \Omega_t)] | \Omega_t$ is a white noise disturbance. This does not differ greatly from the reduced form of either sections 3 or 5. The conditional moments, however, are over period $t + 1$ variates.

The coefficients on the consumption and inflation variance terms, β_5 and β_6 , depend upon two competing effects, shopping time considerations and the cost of deferred consumption. On the margin, if leisure is highly valued and shopping time increases quickly with consumption, shopping time effects will be important. This effect raises the ratio of money balances to expected consumption. If the marginal utility of consumption diminishes rapidly, the agent will be highly averse to risky consumption opportunities and will have a strong incentive to defer consumption to periods of lower risk. The overall effect depends upon which of the two effects predominates.¹⁰

6.3. *Regression results using instrumental variables and rational expectations*

Table 2 reports the empirical results of estimating eq. (24). For the sample as a whole, the conditional variance of inflation enters with a significantly negative sign. This contribution comes from both 1970s inflation and 1980s deflation. Using the appropriate instrumental variables technique explains the sign reversal on inflationary risk in the 1980s. Specification errors aside, this clearly can account for the divergent results of Klein and Smirlock.

⁹Including the conditional mean does not effect the results concerning conditional variances.

¹⁰If the utility and shopping time functions are log-linear and separable, we can show $\beta_2, \beta_3 > 0$ and $\beta_5, \beta_6 < 0$. $\beta_4 < 0$ follows for a positive first coefficient on the lag polynomial for interest rates.

Table 2
Money demand estimation under rational expectations.^a

Interval quarterly	Constant	Lagged money	Interest rates	<i>INST1</i>	<i>INST2</i>	<i>INST3</i>	SE
1962:4–1987:4	-0.038 (1.64)	1.030 (72.27)	-0.011 (3.37)	-0.004 (2.53)	0.003 (1.16)	0.000 (1.38)	0.0107
1962:4–1969:4	-0.245 (2.38)	1.208 (15.25)	-0.040 (3.23)	0.002 (0.90)	-0.001 (0.19)	0.000 (0.09)	0.0054
1970:1–1979:4	0.148 (2.87)	0.930 (29.36)	-0.016 (5.57)	-0.003 (2.10)	-0.005 (1.89)	0.000 (0.18)	0.0053
1980:1–1987:4	0.276 (4.04)	0.906 (30.43)	-0.065 (5.41)	-0.007 (2.02)	0.006 (1.28)	0.001 (1.70)	0.0104

^aThe independent variable is the log difference of real M1 money balances; for data sources and description, see appendices A.1 and A.2. *INST1* is the instrumented conditional variance of inflation. *INST2* is the instrumented conditional variance of consumption. *INST3* is the instrumented conditional variance between consumption and inflation. Instruments were 5 lags of fitted conditional moments; see appendix A.3. *T*-statistics are in parentheses.

The extended model does contribute some additional explanation to the 'missing money' as well. Apart from inflationary risk, the variability in aggregate consumption contributes to a decrease in real money balances held. While we don't see a secular decline in consumption during this period, we do see more active cash management, with a smaller transactable money stock supporting a given level of consumption.

As for the Klein-Smirlock paradox, theoretically, both can be viewed as special cases of this more general model. While the 'quality' of the money stock may have deteriorated, Klein's reduced-form estimation cannot account for the jointness of the consumption and portfolio decision. While our empirical results are supportive of Smirlock's paper, his results must be interpreted carefully because of the errors in variables problem. Repeating Smirlock's exercise in the 1980s would have reversed his conclusions.

7. Summary and conclusions

Money plays a unique role in society because of its function as a transactions medium. To include the precautionary motive, the market sequence is critical. By restricting consumption to liquidity obtained in the previous period, the agent can no longer hedge against inflationary risk through the bond market.

From the Euler equations of a liquidity-in-advance model, we obtained a specification for money demand by linearizing around the conditional moments, the appropriate measures of uncertainty. In this equation, consumption and inflationary risk and their covariance enter as arguments in the money demand relation.

In the empirical section, ARCH processes were estimated for these conditional moments, and the errors-in-variables problem was carefully considered. The results of this effort substantially support our underlying theory. We find that inflation and consumption risk reduce the demand for money in the 1970s. Inflation risk is important throughout. We trace the empirical contradictions in the literature to either misspecification or the errors-in-variables bias.

Data appendix

All the data were constructed from Citibase variables. Their mnemonics have a star.

A.1. ARCH variables

$$\pi_t = [(GDC_t^* - GDC_{t-1}) / GDC_{t-1}] * 400,$$

where GDC^* is the implicit price deflator for personal consumption expenditures.

$$pm_t = [(GDIM_t^* - GDIM_{t-1}) / GDIM_{t-1}] * 400,$$

where $GDIM^*$ is the implicit deflator for imported goods and services.

$$w_t = [(LBPUR_t^* - LBPUR_{t-1}) / LBPUR_{t-1}] * 400,$$

where $LBPUR^*$ is the index of nonform hourly earnings.

$$c_t = GC82^*,$$

where $GC82^*$ is personal consumption in billions of 1982 dollars.

$$USDPI = \left(\frac{1}{12} \left[\sum_{j=0}^{11} \pi_j - \left(\frac{1}{12} \sum_{j=0}^{11} \pi_j \right) \right]^2 \right)^{0.5}.$$

A.2. Money demand variables

The dependent variable is $(FMIQ/GDC^*)$, where $FMIQ$ is quarterly average FMI^* monthly money balances. The consumption variable is $GC82^*$. The interest rate is the logged quarterly average of $FYCP90^*$, the 90-day commercial paper rate. All these data are logged.

A.3. Instruments

From the conditional mean equations (14) and (17), we obtain the OLS residuals $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$, and $\hat{\varepsilon}_{12t}$. These residuals are used in obtaining the models (21a)–(21c) for the conditional second moments. These fitted conditional variances σ_{π}^2 , σ_c^2 , and $\sigma_{\pi c}$ are then used as instruments for the OLS residuals from the conditional mean equations. Given our specification (24), these are period $t + 1$ variates.

Instrument regressions are available from the authors.

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