The Case Against JIVE

Davidson and McKinnon (2004) JAE 21: 827-833

PhD. student Freddy Rojas Cama
Econometrics Theory II

Rutgers University

November 14th, 2011

Instrumental variables performance with weak instruments

- Davidson and McKinnon (2004). The case Against JIVE. Journal of Applied Econometrics 21: 827-833.
- Blomquist and Dahlberg (1999). Small sample properties of LIML and Jaccknife IV estimators: Experiments with Weak Instruments. Journal of Applied Econometrics 14: 69-88.
- Angrist, J., W. Imbens and A. Krueger (1999). Jaccknife Instrumental Variables Estimation. Journal of Applied Econometrics 14: 57-67.

Summarizing

Finite-sample properties Evaluation
(According to each study)

	UJIVE	LIML	Comment
DM		✓	LIML the best in reducing bias
BD	?	?	Hard to find a winner!
AIK	✓		A reduced space of parameters

The System of Equations

We have the following system of equations (in matricial terms);

$$\begin{aligned} Y &= X\beta + \varepsilon && (1) \\ X &= Z\pi + \eta && (2) \end{aligned}$$

where X, Z and η are matrices of dimension $n \times L$, $n \times k$ and $n \times L$ respectively. The number of overidentified restricctions can be calculated as $r = k - L$. Also, there are M common elements in X and Z, then M columns of $n \times L$ matrix η are zero. The endogeneity comes from the following expression

$$E\left[\varepsilon_i \eta_i' \middle| Z\right] = \sigma_{\varepsilon\eta} \tag{3}$$

UJIVE 1

JIVE removes the dependence of of the constructed instrument $Z_i\widehat{\pi}$ on the endogenous regressor for observation i by using the following estimator

$$\widetilde{\pi}(i) = \left(\mathbf{Z}(i)'\mathbf{Z}(i)\right)^{-1}\mathbf{Z}(i)'\mathbf{X}(i) \tag{4}$$

the estimate of the optimal instrument is $Z_i\widetilde{\pi}(i)$; then, because ε_i is independent of X_j if $j \neq i$ we claim that

$$E\left[\varepsilon_i Z_i \widetilde{\pi}(i)\right] = 0$$

this is easily verifiable

$$\begin{aligned} E\left[E\left[\varepsilon_i Z_i' \widetilde{\pi}(i)\right] \middle| Z\right] &\equiv E\left[Z_i\left(\mathbf{Z}(i)'\mathbf{Z}(i)\right)^{-1}\mathbf{Z}(i)' E\left[\varepsilon_i \mathbf{X}(i)\right] \middle| Z\right] \\ &= 0 \end{aligned}$$

See Phillips and Hale (1977) for details.

UJIVE 1

Thus, $\widehat{X}_{i,UJIVE} = Z_i \widetilde{\pi}(i)$; then the estimator of β is

$$\widehat{\beta}_{UJIVE} = \left(\widehat{\mathbf{X}}'_{UJIVE}\mathbf{X}\right)^{-1}\widehat{\mathbf{X}}'_{UJIVE}\mathbf{Y} \qquad \text{(UJIVE 1)}$$

We require to perform the estimator in (4) by each observation i. [AIK] show a *sort of* shortcut

$$Z_i \widetilde{\pi}(i) = \frac{Z_i \widehat{\pi} - h_i X_i}{1 - h_i}$$

where $h_i = Z_i \left(Z'Z\right)^{-1} Z'_i$

Limited Information Maximum Likelihood

We can estimate the parameters in linear regressions with endogenous regressors by using the limited-information-maximum-likelihood estimator. The likelihood is based on normality for the reduced form errors and with covariance matrix, although consistency and asymptotic normality of the estimator do not rely on this assumption. The log likelihood function is

$$
\begin{aligned}
L &= \sum_{i=1}^{N} -\ln(2\pi) - \frac{1}{2}\ln|\Omega| && \text{(LIML)} \\
&\quad -\frac{1}{2}\begin{pmatrix} Yi - (Z_i\pi)'\beta \\ X_i - Z_i\pi \end{pmatrix}' \Omega^{-1} \begin{pmatrix} Yi - (Z_i\pi)'\beta \\ X_i - Z_i\pi \end{pmatrix} && (5)
\end{aligned}
$$

Experiments

DM have the following system of equations (*in matricial terms*) in order to do the simualtions;

$$\begin{aligned} Y &= \iota\beta_1 + x\beta_2 + \varepsilon && \text{(Structural eq.)} \\ x &= \sigma_\eta\left(Z\pi + \eta\right) && \text{(Reduced eq.)} \end{aligned}$$

where $X = [\iota\ x]$, Z and η are matrices of dimension $n \times 2$, $n \times l$ and $n \times 1$ respectively. The number of overidentified restricctions can be calculated as $r = l - 2$. The elements of ε and η have variances σ_ε^2 and 1 respectively, and correlation ρ.
In order to start with the simulations we need to impose values for parameters. For this, an important guide is the size of the ratio $\|\pi\|^2$ to σ_η^2.

Setting up parameters

- DM fix values of the π_j to be equal excepting $\pi_1 = 0$.
- The parameter which does varies is denoted by $R^2_\infty = \frac{\|\pi\|^2}{\|\pi\|^2 + \sigma^2_\eta}$ (This is the asymptotic R^2)
- R^2_∞ is monotonically increasing function of the the ratio $\|\pi\|^2$ to σ^2_η.
- A small value of R^2_∞ implies that the instruments are weak.
- In the experiments, DM vary the sample size n, the number of overidentifying restrictions (r), the correlation between errors (ρ) and R^2_∞

Performance evaluation

- Because LIML and JIVE estimators have no moments (see davison and Mackinnon; 2007). DM reports the median bias, this is

$$\text{median bias} = \beta_{d0.5} - \beta$$

- As measure of dispersion DM reports the nine decile range, this is

$$9\text{d} - \text{range} = \beta_{d0.95} - \beta_{d0.05}$$

- Ackberg and Devereux (2006) suggest to consider

$$\text{Trimmed mean bias} = \frac{1}{n}\sum_j \beta_j - \beta$$

where $\beta_j \in [\beta_{d0.99}\ \beta_{d0.01}]$.

Results (I): Median bias evaluation

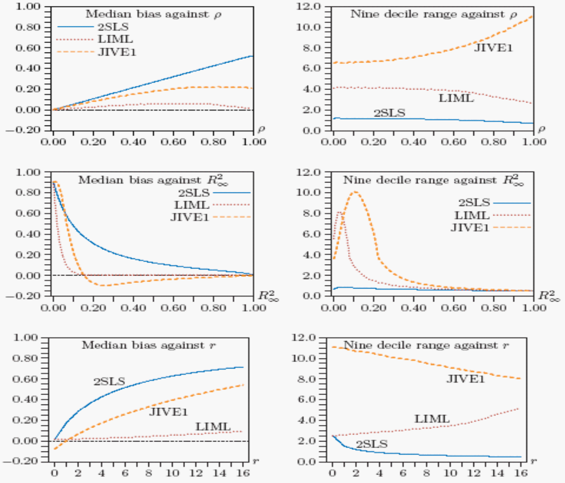

Figure A. Median bias and nine decile range when $n = 50$

Results (II): Median bias evaluation

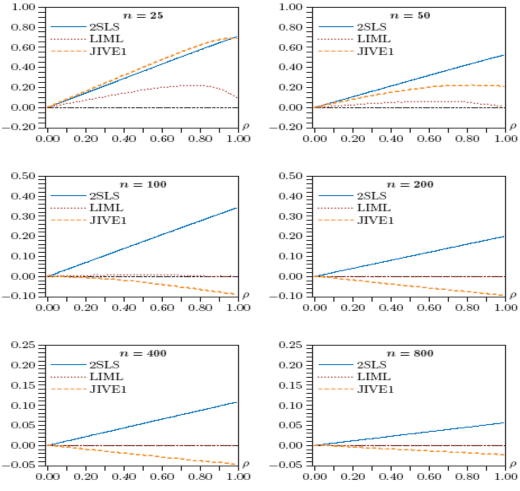

Figure 1. Median bias of three estimators, $r = 5$, $R^2_\infty = 0.1$

Results (III): Median bias evaluation

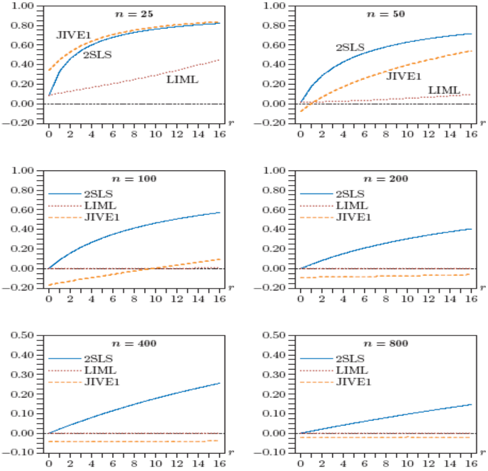

Figure 3. Median bias of three estimators, $R^2_\infty = 0.1$, $\rho = 0.9$

Results (V): Dispersion

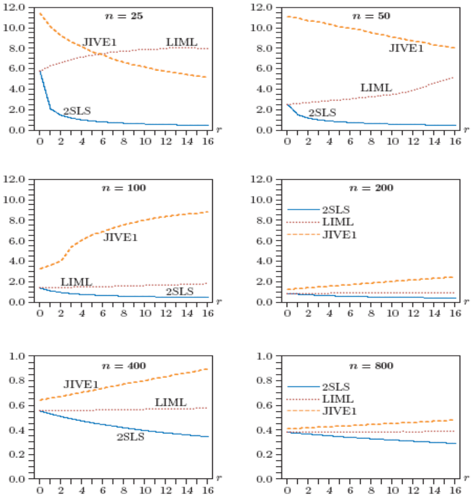

Figure 6. Nine decile range of three estimators, $R_\infty^2 = 0.1$, $r = 0.9$

Conclusions

1. AIK and BD give divergent views of JIVE performance. DM's paper re-examinate this issue.
2. DM conclude that in most regions of the parameters space they have studied, JIVE is inferior to LIML regarding to median bias, dispersion and reliability of inference.
3. LIML should be preferred whenever you need to deal with estimators which have no moments.
4. DM points out that, however, montecarlo simulations does no support unambiguoulsy the usage of LIML; when the instruments are weak the dispersion is significant in this estimator.

References

- Angrist, J., W. Imbens and A. Krueger (1999). Jaccknife Instrumental Variables Estimation. Journal of Applied Econometrics 14: 57-67.
- Blomquist and Dahlberg (1999). Small sample properties of LIML and Jaccknife IV estimators: Experiments with Weak Instruments. Journal of Applied Econometrics 14: 69-88.
- Davidson, R., and J. McKinnon (2004). The case Against JIVE. Journal of Applied Econometrics 21: 827-833.
- Davidson, R., and J. McKinnon (2006). Reply to Ackerberg and Devereux and Blomquist and Dhalberg on "The case Against JIVE". Journal of Applied Econometrics 21: 843-844.