Answer Key to Homework 1

Part I

1) E
2) C
3) D
4) A
5) B
6) C
7) C
8) A
9) C
10) B
11) C
12) E
13) C
14) D
15) C
16) D
17) A
18) C
19) C
20) A
21) (a) \( Y = C + I + G \)
   \[ Y = 350 \]
   (b) \( C = 40 + 0.8Y = 320 \)
   \[ I = 20 \]
   \[ G = 10 \]
   (c) \( Y = 269.1667 \)
22) This figure shows that since the 1980s, United States foreign assets and liabilities have both been increased rapidly. However, liabilities have risen faster than assets, resulting in a substantial net foreign debt.
23) In this case, the United States makes a two billion dollars capital transfer to Argentina, which should appear as a negative two billions entry in the capital account. The associated credit is in the financial account, in the form of a two billion dollars reduction in U.S. assets held abroad, i.e., a net asset export, and therefore a positive balance of payments entry.
Part II

For questions 1-4, recall that the crucial questions are:

(i) What consumption possibilities are available to residents of the economy?
(ii) Which one of those will the typical resident choose?

If you can answer (i) and (ii) the other questions are easy, as you will see.

The answer to (i) is given by the budget constraints, which can be written in present value form (see your class notes and Schmitt Grohe-Uribe):

\[ C_1 + \frac{C_2}{1 + r^*} = Q_1 + \frac{Q_2}{1 + r^*} + (1 + r^*)B_0^* \]

The left hand side is the present value of consumption. The right hand side is the present value of income plus initial wealth \((1 + r^*)B_0^*\).

With the values given in the problem set, the preceding equation becomes:

\[ C_1 + \frac{C_2}{1.1} = 5 + \frac{10.2}{1.1} + 1 = 6 + \frac{10.2}{1.1} \]

The answer to (ii) is the optimality condition that, for an optimum, the slope of the indifference curve at the optimum (the marginal rate of substitution, or MRS) must equal the ratio of relative prices. We have that

\[ MRS = \frac{\partial U}{\partial C_1} \frac{\partial U}{\partial C_2} = \frac{1}{\beta} \sqrt{\frac{C_2}{C_1}} \]

and, in this case, the relevant ratio of relative prices are \(1/(1/(1 + r^*)) = 1 + r^*\) (see the PV budget constraint). So optimal consumption requires:

\[ \frac{1}{\beta} \sqrt{\frac{C_2}{C_1}} = 1 + r^* \]

With the values given (in particular, \(1/\beta = 1.1 = (1 + r^*)\)) this simplifies to

\[ C_1 = C_2 \]

Using this and (1) you get \(C_1 = C_2 = 8\). This answers questions 1 and 2.

For question 3, note that the trade balance in period 1 is simply \(Q_1 - C_1 = 5 - 8 = -3\).

For question 4, note that the current account balance in period 1 is \(r^*B_0^* + Q_1 - C_1\). Since \(Q_1 - C_1 = -3\) and \(r^*B_0^*\) is positive but smaller than 3 in absolute value, the answer is (c).

Question 5: at the world interest rate \(r^* = 0.1\), we have seen that the typical household would have wanted to borrow in the world market. With capital controls this is forbidden, so the household will consume all of its available income and wealth in period 1. Hence \(C_1 = Q + (1 + r^*)B_0^* = 6\).

Question 6: for the same reason, \(C_2 = Q_2 = 10.2\).
Question 7: With capital controls, the domestic interest rate must be such that the previous values for $C_1$ and $C_2$ induce domestic residents neither to borrow nor to lend (why?). But this requires that

\[
MRS = \frac{\partial U/\partial C_1}{\partial U/\partial C_2} = \frac{1}{\beta} \sqrt{\frac{C_2}{C_1}} = 1 + r
\]

where $r$ is the domestic interest rate. With $\beta = 1/1.1$, $C_1 = 6$, and $C_2 = 10.2$, we get $r = 0.43$ approx.

Q. 8. The answer is (c). Just plug the values of $C_1$ and $C_2$ in the definition of $U(...)$ in the cases with and without capital controls.

Question 9. Redo the analysis for questions 1-2 with the new assumption that $Q_1 = 9.2$. This leads to (d).

Question 10. The answer is (b). Think about it.

**Part III**

Question 1: (c) The firm equates the marginal product of capital to the marginal cost $(1 + r^*)$

Q. 2. The MPK is $\partial Q_2/\partial K_2 = F'(K_2) = 1/2\sqrt{K_2}$. Since the interest rate is $r^* = 0.1$,

\[
1/2\sqrt{K_2} = 1.1 \implies K_2 = 0.21 = I_1
\]

Q. 3. Plug the previous value of $K_2 = I_1$ into the expression for $\Pi_2$ to get (a).

Q. 4-5. Refer to the discussion of questions 1-4 of part II: The PV budget constraint of the typical household is

\[
C_1 + \frac{C_2}{1 + r^*} = Q_1 + \frac{\Pi_2}{1 + r^*}
\]

This leads to

\[
C_1 + \frac{C_2}{1.1} = 10 + \frac{0.23}{1.1}
\]

Now, optimal consumption requires:

\[
MRS = \frac{\partial U/\partial C_1}{\partial U/\partial C_2} = \frac{1/C_1}{1/C_2} = 1 + r^*
\]

i.e.

\[
C_2 = (1.1)C_1
\]

Plug into the budget constraint to get $C_1 = 5.1, C_2 = 5.6$

Q. 6. The current account (which is also equal to the trade account) is $Q_1 - I_1 = 4.69$ approximately.

Q. 7. Just redo the analysis. It leads to $K_2 = 0.84$, etc. In particular, you should find that the current account deteriorates in period 1. Why? Investment increases due to higher productivity of capital. This also increases the household’s second period income, which induces more consumption and less saving in the first period.