1. (40 points) For the model
\[ Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + u_t \]
(a) Suppose it is known that \( \beta_2 = \beta_3 = \beta_4 \). Describe how you would obtain the best estimates for \( \beta_1 \), \( \beta_2 \), \( \beta_3 \), and \( \beta_4 \).
(b) Write down the formula for the least squares estimator of \( \beta_2 \) in (1), given that \( \beta_2 = \beta_3 = \beta_4 \).
(c) For the model
\[ Y_t = \alpha + \beta X_t + u_t \]
Consider the two estimators \( \hat{\beta} = \frac{s_Y^2}{s^2} \) and \( \hat{\beta} = \frac{\sum X_t Y_t}{\sum X_t^2} \). a: When are they the same?
(d) Assume that:
\[ Y_t = \alpha_1 + \alpha_2 X_{3t} + \text{error} \]
Derive the elasticity of \( Y \) with respect to \( X \).

2. (60 points) Using quarterly data for ten years (making the number of observations 40), the following model of demand for new cars was estimated.
\[ NUMCARS = \hat{\beta}_1 + \hat{\beta}_2 PRICE + \hat{\beta}_3 INCOME + \hat{\beta}_4 INRAT \text{E} + \hat{\beta}_5 UNEMP + \hat{u}_t \]

where \text{NUMCARS} is the number of new car sales per thousand population, \text{PRICE} is the new car price index, \text{INCOME} is per capita real disposable income (in actual dollars), \text{INRAT} \text{E} is the interest rate, and \text{UNEMP} is the unemployment rate. The following table has the estimates of the \( \beta \)'s for three alternative models.

(Values in parentheses are standard errors)

<table>
<thead>
<tr>
<th></th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>-0.071391 (0.034730)</td>
<td>-0.079392 (0.011022)</td>
<td>-0.024883 (0.007366)</td>
</tr>
<tr>
<td>income</td>
<td>0.003159 (0.001763)</td>
<td>0.00356 (0.0006266)</td>
<td></td>
</tr>
<tr>
<td>intrate</td>
<td>-0.153699 (0.049190)</td>
<td>-0.146651 (0.039229)</td>
<td></td>
</tr>
<tr>
<td>unemp</td>
<td>-0.072547 (0.098195)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ESS = \left( \sum \hat{u}_t^2 \right)$</td>
<td>23.510464</td>
<td>23.550222</td>
<td>44.65914</td>
</tr>
<tr>
<td>$\text{ADJ R}^2$</td>
<td>0.758</td>
<td>0.764</td>
<td>0.565</td>
</tr>
<tr>
<td>$\text{SGMASQ} = \left( \hat{\sigma}_u^2 \right)$</td>
<td>0.671728</td>
<td>0.654173</td>
<td>1.207004</td>
</tr>
<tr>
<td>AIC</td>
<td>0.754701</td>
<td>0.719108</td>
<td>1.29716</td>
</tr>
<tr>
<td>SCHWARZ</td>
<td>0.932092</td>
<td>0.851414</td>
<td>1.472329</td>
</tr>
</tbody>
</table>
In Model A, test the joint hypothesis that $\beta_3 = \beta_4 = \beta_5 = 0$ by carrying out the following steps.

(a) Write down the formula for the test statistic and compute it.

(b) State its distribution and d.f.

(c) State the decision rule (for the 20% level) and your conclusion.

(d) Redo steps (a), (b), (c) above, but this time test Model B for overall significance.

Next separately test each of the $\beta$'s in Model B to determine whether or not it is significantly different from zero by carrying out the following steps:

(e) Write down the d.f. for the test statistic.

(f) Write down the critical value for a 5% level of significance test.

(g) Use the information in (f) to test whether each coefficient is significant or not.

Clearly show your work and state your conclusion.

FOR PRICE ($\hat{\beta}_2$)
FOR INCOME ($\hat{\beta}_3$)
FOR INTRATE ($\hat{\beta}_4$)

(h) Based on all of your above findings, would you recommend any of the variables in Model B be omitted? If yes, why? If no, why not?

(i) For each variable in Model B, explain very briefly whether the sign of the estimated regression coefficients is as you would expect. Justify your answers carefully.

FOR PRICE ($\hat{\beta}_2$)
FOR INCOME ($\hat{\beta}_3$)
FOR INTRATE ($\hat{\beta}_4$)

(j) Which of the three models is the "best"? Briefly explain which criteria you used and why you chose the model you did.