1. (a) \[ \text{Var}(X) = E[(x - E(x))^2] = E[x^2 - 2xE(x) + E(x)^2] \]
\[ = E(x^2) - 2E(x)E(x) + E(x)^2 = E(x^2) - E(x)^2 \]
\[ = E(x^2) - \mu_x^2 \]
\[ \text{Var}(X + Y) = E[((x + y) - E(x + y))^2] = E[(x - E(x)) + (y - E(y))]^2 \]
\[ = E[(x - E(x))^2 + [y - E(y)]^2 + 2(x - E(x))(y - E(y))] \]
\[ = E[x - E(x)]^2 + E[y - E(y)]^2 + 2E(x - E(x))(y - E(y)) \]
\[ = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y) \]
\[ \text{Var}(c) = E(c - E(c))^2 = 0 \]

(b) \[ \text{Var}(\hat{X}) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \]
\[ \text{Cov}(\hat{X}, \hat{Y}) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \]

(c) \[ \bar{W} = \frac{1}{T} \sum_{i=1}^{T} W_i; \quad E(W_i) = \mu_W \]
- \( W_i \): sample point
- \( \bar{W} \): sample mean
- \( \mu_W \): true mean

(d) Cross section data is a sample of a number of observational units all drawn at the same point in time, such as the GDP of all countries in 1998. Time series data is a set of observations drawn on the same observational unit at a number of points in time, such as GDP of America from 1980 - 1998.

(f) An example of discrete PDF:
- \( X \): the result of casting a die
- Possible values: \( X_1, X_2, \ldots, X_6 \)
- Probability of each possible value occurring

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<thead>
<tr>
<th>( f(x) )</th>
<th>( X = 1 )</th>
<th>( X = 2 )</th>
<th>( X = 3 )</th>
<th>( X = 4 )</th>
<th>( X = 5 )</th>
<th>( X = 6 )</th>
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The distribution is a uniform distribution.

Properties of continuous PDF:

(i) \( f(x) \geq 0 \)
(ii) \( \int_{-\infty}^{\infty} f(x)dx = 1 \)
(iii) \( \int_{a}^{b} f(x)dx = P(a \leq x \leq b) \)

2. \( T = 18 \)
\[ \bar{X} = 166.1 \]
\[ Y = 189.8 \]

\[ Cov(X, Y) = \frac{1}{17}(647,573) - \frac{18}{17}(166.1)(189.8) = 4694.425 \]

\[ or \quad \simeq \frac{1}{18}(647,573) - (166.1)(189.8) = 4433.623 \]

\[ Var(X) = \frac{1}{17}[536,578 - 18(166.1)^2] = 2347.399 \]

\[ or \quad \simeq \frac{1}{18}(536,578) - (166.1)^2 = 2216.988 \]

\[ Var(Y) = \frac{1}{17}[812,250 - 18(189.8)^2] = 9600.575 \]

\[ or \quad \simeq \frac{1}{18}(812,250) - (189.8)^2 = 9067.210 \]

3. (a) \[ \rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{Cov(X, Y)}{\sqrt{Var(x)\sqrt{Var(Y)}}} \]

(b) \[ \hat{\rho} = \frac{S_{XY}}{S_X S_Y} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})}} \]

(c) (i) \(-1 \leq \rho_{XY} \leq 1\)

(ii) \[ \rho_{XY} = 0 \Rightarrow Cov(X, Y) = 0 \]

It means that \(X\) and \(Y\) are uncorrelated.

(d) \[ \hat{\rho} = \frac{4964.425}{\sqrt{2347.399\sqrt{9600.575}}} = 0.98878 \]

\[ or \quad \simeq \frac{4433.623}{\sqrt{2216.988\sqrt{9067.210}}} = 0.9887 \]

(e) There is a strong positive correlation between \(X\) and \(Y\).

4. (a) \[ Z = \frac{\bar{X} - \mu_X}{\sigma_X} = \frac{1.5 - 2}{\sqrt{0.16}} = -1.25 \]

The critical value \(Z_{\alpha} = -1.645; \quad \alpha = 0.1. \) Since \(Z_{\alpha} < Z\), we do not reject \(H_0\).

(b) The level is the probability that we reject \(H_0\) when \(H_0\) is true.

(c) \[ Z = \frac{1.5 - 2}{\sqrt{0.16}} = -1.25 \]

The critical value \(Z_{\alpha} = -1.28; \quad \alpha = 0.1. \) Since \(Z_{\alpha} < Z\), we do not reject \(H_0\).

(d) \[ Z = \frac{1.5 - 2}{\sqrt{0.16}} = -1.25 \]

The critical value \(Z_{\alpha} = 1.28; \quad \alpha = 0.1. \) Since \(Z_{\alpha} > Z\), we do not reject \(H_0\).

5. The \(t\)-distribution is an approximation of the \(Z\)-distribution. This approximation becomes more accurate as the degrees of freedom increases. In fact, as the degree of freedom approaches infinity, the \(t\)-distribution becomes \(Z\)-distribution. The \(Z\) is used when \(\mu_X\) and \(\sigma_X\) are known. The \(t\) is used when all that is known is \(\bar{X}\) and \(Var(X)\)