Hints (Xtra)

2. Expectations and Linear Relationships. Assume that $X$ is independent of $\varepsilon$ with:

\[ E(X) = 0, \text{Var}(X) = 4; \ E(\varepsilon) = 0, \text{Var}(\varepsilon) = 2. \]

Assume:

\[ Y = 3X + 2 + \varepsilon. \]

Then, using the properties of expectations and variances,

a) Find $E(Y)$ and $\text{Var}(Y)$.

\textbf{Hint1} : \quad E(Y) = E(3X + 2 + \varepsilon) = E(3X + 2) + E(\varepsilon) \text{ from properties of } E \\
\textbf{Hint2} : \quad \text{Var}(Y) = \text{Var}(3X + 2 + \varepsilon) = \text{Var}(3X + 2) + \text{Var}(\varepsilon) \text{ from independence}

Each of these Hints reduces the problem to one where you need to apply the rules given in class.

b) Find the covariance between $X$ and $Y$: $\text{Cov}(X,Y) = E(XY) - E(X)E(Y)$.

The expression for the covariance given in class can be reduced to this form. You are not asked to show this, but can take it is given. It is actually not hard to prove though. If $E(X) = 0$, notice that the covariance between $X$ and $Y$ simplifies to: $\text{Cov}(X,Y) = E(XY)$. Therefore, if we had an expression for $XY$, we could find its expectation. Now, refer to the hints below.

\textbf{Hint1} : \quad E(X) = 0 \text{ and } XY = 3X^2 + 2X + X\varepsilon. \\
\textbf{Hint2} : \quad \text{Find } E(XY) \text{ using } \text{Var}(X) \text{ and independence between } X \text{ and } \varepsilon.

To develop a third hint, note that a covariance always has something to do with a cross product (e.g. $XY$ above). Similarly, a variance is related to the square of a variable. This brings up the third Hint:

\textbf{Hint3} : \quad \text{Var}(X) = E \left\{ [(X - E(X))^2] \right\} = E(X^2) \text{ if } E(X) = 0

You may not see how to do the problem immediately, but follow the hints and at some point you will realize what is happening.