Prob2: Simultaneous Equations: Part I

In part I of this problem set, we will explore estimation of simultaneous equations models. To this end, we will first generate data from a demand-supply model. As the true model generates the data, you will know the true parameter value. It will then be possible to compare estimates with true parameter values in order to evaluate OLS and 2SLS estimates. Throughout this problem set, the true model is given as:

\[
\text{Demand} : \quad Q = B1 + B2 \times P + e \\
\text{Supply} : \quad P = a1 + a2 \times Q + a3 \times W + u.
\]

1. Generating Data. First, specify parameter values by typing:

\[
\begin{align*}
B1 &= 5; \\
B2 &= -1; \\
a1 &= 1; \\
a2 &= 1; \\
a3 &= 1;
\end{align*}
\]

Generate 1000 values for the exogenous variable \(W\) by typing:

\[W = \text{rndn}(1000, 1) + 5\]

Generate values for the (standard normal) errors by typing:

\[
\begin{align*}
e &= \text{rndn}(1000, 1); \\
u &= \text{rndn}(1000, 1);
\end{align*}
\]

If you substitute for \(Q\) in the supply equation and then solve for \(P\), you can
generate (reduced form) values for P by typing:

\[ P = (a_1 + a_2 b_1 + a_2 e + a_3 W + u)/(1 - a_2 b_2); \]

Here, * denotes multiplication and / denotes division. Using values for the demand parameters, P, and e, generate data on Q by typing:

\[ Q = B_1 + \ldots + e; \]

2. OLS Estimation, Demand. There are several ways to estimate a linear model in Gauss using OLS. To obtain the estimates, standard errors, and all of the other output with which you may be familiar, estimate the demand equation by OLS by typing:

\[
\text{call OLS}(0,Q,P);
\]

Comparing the parameter estimates with the true values, are you surprised? Please explain. Here, the estimates are given next to the variable names, followed by their standard errors.

3. 2SLS Estimation, Demand. There is another command in Gauss that will provide OLS estimates. It is useful in that it saves the estimates under a name that you provide, and can therefore be employed easily in subsequent calculations. As this other command does not automatically estimate a constant term, we need to employ a variable that is always 1. Type:

\[
X = W \sim ones(1000,1);
\]

The "variable" X (an Nx2 matrix) now has two columns, the first of which contains data on W. The second column contains 1000 1’s, and will be useful for estimating a constant term. You can now run a regression of P on W and a constant term by typing:

\[
\text{cp} = p/X;
\]

Here, cp holds the values for the estimated coefficients. If you type cp, you will
see the results. As the first stage in 2SLS, you can now estimate expected price by typing:

\[ Z = X \times cp; \]

In the second stage of 2SLS, estimate B1 and B2 by running a linear regression of Q on Z (and a constant term). To obtain results for this second stage, type:

\textit{call ols(0,Q,Z)}. \\
Comparing your results with the true parameter values, are your results as expected? \textbf{Please explain.}

4. **2SLS Estimating, Supply.** To attempt to obtain consistent estimates of the supply equation, in the first stage run a linear regression of Q on W (the reduced form for Q). As in 3, estimate the coefficients by typing:

\[ cq = Q/X; \]

Then, construct and estimate of expected Q by typing:

\[ Z = X\times cq. \]

You can now attempt to obtain second stage results from an OLS regression of P on Eqant and W by typing:

\textit{call ols(0,P,Z~W)};

Are your results as expected? \textbf{Please explain.}

\textbf{Hint1: If standard errors reported, and are their magnitudes reasonable?}

5. **Optional.** Return to the demand model above and let \( \hat{v} \) be the reduced form residual from the price equation: \( \hat{v} = P - X \times cp \). Include \( \hat{v} \) in the demand model to obtain:

\[ Q = B1 + B2 \times P + c \times \hat{v} + e^*, \]

\[ c = \text{cov}(e, v) / \text{var}(v); \quad e^* = e - c \times \hat{v} \]
a) Show that the first order conditions for the control-estimator are approximately satisfied at the true parameter values when the sample size is large. In obtaining this result, you may act as if $\hat{v} = v$ because $\hat{v}$ is based in consistent parameter estimates.

b) Suppose that you observed $e$ and $v$ and ran a linear regression of $e$ on $v$. Explaining you answer very carefully, how would the estimated coefficient on $v$ compare with $c$ above when the sample size is large? In answering this question, you may simplify the problem by leaving out the constant term;

c) For the estimates of $B1$ and $B2$, show that the Control method is equivalent to IV in the linear model. In examining conditions (equations) that the estimates must satisfy, you may assume that the solution is unique.