Problem-solving in models of demand: some suggestions

(1) See Mathematical Appendix in text (pp. 664-683) to review calculus and other mathematical techniques

(2) How-To summaries in the Study Guide:
  ▪ computing demands and demand curves (p. 117)
  ▪ computing income and substitution effects (p. 122)
  ▪ computing CV and EV (p. 128)

(3) Draw a diagram to make it easier to see what's going on!

(4) Ask: how many unknowns are there? how many equations are there?

An extended example appears in the following pages.
Chapter 5

Study Guide:

- Obtain D curves: p. 117
- Calculate income + substitution effects: p. 122
- Calculate EV, CV: p. 128

it helps to draw a picture!

Mathematical Appendix - in text, p. 664

**BASELINE:** \[ U = 10x - \frac{x^2}{2} + y \]

\[ I = 20 \quad P_x = 1 \quad P_y = 1 \quad (\text{later, } P_x \rightarrow P_x' = 2) \]

1. **CALCULATE DEMANDS**
   
   Use \( I = P_x x + P_y y \) and \( \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \)
   
   So \[ 20 = 1 \cdot x + 1 \cdot y \]
   
   or \[ 20 = x + y \]
   
   Since \( x = 9 \) (see below)
   
   we have:
   
   \[ 20 = x + y \]
   
   \[ 20 = 9 + y \] \[ \Rightarrow y = 11 \]
   
   So we can now calculate \( U \) at the optimum:
   
   \[ U = 10x - \frac{x^2}{2} + y \]
   
   \[ = 10 \cdot 9 - \frac{81}{2} + 11 \]
   
   \[ = 90 - 40.5 + 11 \] \[ \Rightarrow U = 60.5 \]
If \( p_x \) rises to 2, use same methods:

\[ I = p_x x + p_y y \quad \text{and} \quad \frac{MU_x}{p_x} = \frac{MU_y}{p_y} \Rightarrow \frac{10-x}{2} = \frac{1}{1} \]

So \( 20 = 2 \cdot x + 1 \cdot y \)

Since \( x = 8 \), solve for \( y \):

\[ 20 = 2 \cdot x + 1 \cdot y \]
\[ 20 = 16 + y \quad \Rightarrow \quad y = 4 \]

To get new level of utility at new optimum, use \( U \) function:

\[ U = 10 \cdot x - \frac{x^2}{2} + y \Rightarrow U = 10 \cdot 8 - \frac{64}{2} \]

So \( U = 52 \)

2) TO CALCULATE INCOME + SUBSTITUTION EFFECTS

* * * draw a picture !!! * * *

<table>
<thead>
<tr>
<th>Location</th>
<th>( U )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>old A</td>
<td>60.5</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>new B</td>
<td>52</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>&quot;decomposition&quot;</td>
<td>B</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

**SUBSTITUTION EFFECT:**
Keep \( U \) same, move to new \( \frac{P_y}{P_x} \)

**INCOME EFFECT:**
Keep \( \frac{P_y}{P_x} \) same, move to new \( U \)
DECOMPOSITION BASKET—used to measure substitution effect, to decompose total effect into income and substitution effects.

Keep \( U \) constant at 60.5, but change to new \( \frac{P_x}{P_y} \) (how much I is necessary to do this?)

decomposition basket must satisfy both \( \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \) AND \( U = 10x - \frac{x^2}{2} + y \)

so \( \frac{10-x}{2} = 1 \) and \( 60.5 = 10x - \frac{x^2}{2} + y \) \( \text{old level of } U \)

\[ 10 - x = 2 \]

\[ 60.5 = 10 \cdot 8 - \frac{64}{2} + y \]

\[ 60.5 = 80 - 32 + y \quad \Rightarrow \quad y = 12.5 \]

Use budget constraint to calculate how much I \( \text{old} \) be necessary to achieve this level of \( U \):

\[ I = p_x \cdot x + p_y \cdot y = 2(8) + 1(12.5) = 28.5 \]

So we can now calculate income and substitution effects as follows:

<table>
<thead>
<tr>
<th></th>
<th>Location</th>
<th>( U )</th>
<th>( I )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>old</td>
<td>A</td>
<td>60.5</td>
<td>20</td>
<td>9</td>
<td>11</td>
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<tr>
<td>&quot;decomposition&quot;</td>
<td>B</td>
<td>60.5</td>
<td>28.5</td>
<td>8</td>
<td>12.5</td>
</tr>
<tr>
<td>new</td>
<td>C</td>
<td>52</td>
<td>20</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>income effect (C-B)</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>-8.5</td>
</tr>
<tr>
<td>substitution effect (B-A)</td>
<td></td>
<td></td>
<td></td>
<td>-1</td>
<td>+1.5</td>
</tr>
</tbody>
</table>
COMPENSATING/EQUIVALENT VARIATION

\[ CV = \text{change in } \$ \text{ necessary to keep } \overline{\text{old } U} \text{ at new prices} \]

(\text{starting from new basket of goods})

\[ CV = \text{distance } \overline{KJ} \text{ (measured in units of } y) \]

\[ = \text{change in } I \text{ necessary to keep } U = 60.5 \]

\[ = \text{change in } I \text{ necessary to buy basket } B \]

\[ = 28.5 - 20 = 8.5 \]

(see previous page)
$EV = \text{change in \$ necessary to keep new Utility at old prices (starting from old basket)}$

$EV = \text{distance } KL \text{ (measured in units of y)}$

$= \text{change in income necessary to keep } U = 52 \text{ (its new level) at old set of relative prices}$

$= \text{change in income necessary to buy basket } D$

To buy D, we need

\[ \frac{10-x}{1} = \frac{1}{1} \quad \text{(when } p_x = 1, \quad p_y = 1) \]

and we need $U = 52 = 10x - \frac{x^2}{2} + y$

\[ \frac{10-x}{1} = \frac{1}{1} \Rightarrow \begin{cases} x = 9 \\ \text{so } U = 52 = 10 \cdot 9 - \frac{81}{2} + y \\ \text{so } 52 = 90 - 40.5 + y \\ \text{so } y = 2.5 \end{cases} \]

Income needed to buy $x = 9, y = 2.5$

When $p_x = 1, p_y = 1$ is:

$\begin{align*}
I &= 1 \cdot 9 + 1 \cdot 2.5 = 11.5 \\
\text{So change in } I &= EV \\
&= 11.5 - 20 = -8.5
\end{align*}$

(Rise in $p_x$ was equivalent to $\Delta I = -8.5$)