Chapter 7: Costs and Cost Minimization

1. Cost concepts for decision-making
   A. **Opportunity cost**: the *opportunity cost* of an action is the value of the next best alternative to that action
      (1) opportunity cost is forward-looking
          (sunk costs are sunk/irrecoverable, so usually aren't relevant any longer)
      (2) opportunity cost can change over time
          (e.g., as current market prices and conditions change)
   B. **Explicit costs**: involve a direct $ outlay
   C. **Implicit costs**: involve no direct $ outlay
   D. **Accounting costs**: measure all *explicit* costs incurred in the past
   E. **Economic costs**: all *opportunity* costs, explicit or implicit, as of the present
   F. **Sunk costs**: can't be avoided/recovered
   G. **Unsunk costs**: can be avoided/recovered

Example: put on the flight, or not?

revenues $25,000

costs: fuel, landing fees, ground crew 20,000
      pilots' salaries, other overheads allocated to flight 10,000
2. Cost minimization in the long run
   A. Short run: some inputs (e.g., K) are fixed
      Long run: all inputs are variable
      But in both cases, cost minimization is subject to constraints – e.g., producing some specified level of output, Q
   B. Geometry here is very similar to utility maximization, but beware of some very important differences!
   C. Isocost lines: K, L combinations that have the same total cost, \( TC = wL + rK \) (\( w = \) wage rate, \( r = \) rental rate of capital) solve for K to put on same set of axes as isoquant: \( K = \frac{(TC/r)}{-(w/r)}L \)
      isocosts closer to origin entail lower costs

\[ \text{Diagram: Isocost lines} \]

D. Cost minimization (subject to constraint of producing specified level of Q) requires finding lowest possible isocost that is consistent with the specified level of Q
E. implications of cost minimization
isocost slope = isoquant slope (MRS)
or \(-\frac{w}{r} = -\frac{\text{MPL}}{\text{MPK}}\) or \(\frac{\text{MPL}}{\text{MPK}} = \frac{w}{r}\)
(another "equal bang for a buck" rule!)

e.g., at E, \(\frac{\text{MPL}}{\text{MPK}} > \frac{w}{r}\)
so \(\frac{\text{MPL}}{w} > \frac{\text{MPK}}{r}\), so raise L, cut K
(cuts costs while maintaining Q)
likewise, at F, \(\frac{\text{MPL}}{\text{MPK}} < \frac{w}{r}\)
so \(\frac{\text{MPL}}{w} < \frac{\text{MPK}}{r}\), so cut L, raise K
(cuts costs while maintaining Q)

Corner solutions: sometimes, minimizing costs can imply using no K (or no L)
(here, MRS < \(\frac{w}{r}\) even at \(L = 0\);
or MRS > \(\frac{w}{r}\) even at \(K = 0\))
3. Effects of input price changes in long run (comparative statics)
A. Effect of input price change: a pure substitution effect!
(exception: fixed-proportions production function)

B. Effect of a change in required level of Q: a pure "output" or "scale" effect!
(NB: an input can be "inferior," but both inputs can't be inferior – why?)

C. NOTE: When an input's price changes, this generally changes both (a) the cost-minimizing (K, L) combination at any given Q, and (b) the optimal (profit-maximizing) level of Q itself. We'll consider this later.
4. Long-run price-elasticity of input demand

elasticity of demand
for an input X with \( \frac{\% \Delta \text{ in demand for } X}{\% \Delta \text{ in price of } X} \)
respect to its price

e.g., elasticity of demand for L w.r.t. w,
elasticity of demand for K w.r.t. r

one key determinant of elasticity of demand for a factor: \( \sigma \), the elasticity of substitution between that factor and other factor(s)

\[
\sigma = \frac{\% \Delta K/L}{\% \Delta \text{MRS}} = \frac{\% \Delta K/L}{\% \Delta (\text{MPL}/\text{MPK})} \bigg|_{\bar{Q}}
\]

but since \((\text{MPL}/\text{MPK}) = (w/r)\)
when costs are minimized, \(\sigma = \frac{\% \Delta K/L}{\% \Delta (w/r)} \bigg|_{\bar{Q}}\)
5. Cost minimization in the short run
A. In short run (by definition), it's not possible to change some input(s) (e.g., K); so in short run, can minimize costs only w.r.t. the variable input(s) (e.g., L)
B. thus, *in the short run*, must distinguish between fixed costs and variable costs
C. *in short* run, fixed costs need not be sunk (e.g., factory could be rented out), but some fixed costs are sunk; and *all* variable costs are unsunk
D. in short run, producing a specified Q with a fixed input (e.g., K) does not (usually) entail isocost-isoquant tangency
1. **Production Function**: \[ Q = 4K^{\frac{1}{2}}L^{\frac{1}{2}} \]

   Isoquant: solve for \( K \) (to graph \( K/L \) on vertical axis):
   \[ \frac{1}{4}Q = K^{\frac{1}{2}}L^{\frac{1}{2}} \Rightarrow (\frac{1}{4}Q)^{\frac{1}{2}} = KL \]
   \[ \Rightarrow K = \frac{1}{16}L^{-1}Q^{-2} \]

   (shows \( K, L \) combinations at given \( Q \))

   **Marginal products**:
   \[ MPL = \frac{\partial Q}{\partial L} = \frac{1}{2} \cdot 4K^{\frac{1}{2}}L^{-\frac{1}{2}} \Rightarrow MPL = 2\, K^{\frac{1}{2}}L^{-\frac{1}{2}} \]
   \[ \Rightarrow APL = \frac{Q}{L} = 2(KL)^{\frac{1}{2}} \]

   \[ MPK = \frac{\partial Q}{\partial K} = 2K^{-\frac{1}{2}}L^{\frac{1}{2}} = 2 \left( \frac{Q}{K} \right)^{\frac{1}{2}} \]

   \[ \Rightarrow APK = \frac{Q}{K} = 4 \left( \frac{L}{K} \right)^{\frac{1}{2}} \]

   \[ MRS = -\frac{MPL}{MPK} = -\frac{2K^{\frac{1}{2}}L^{-\frac{1}{2}}}{2K^{-\frac{1}{2}}L^{\frac{1}{2}}} = -\frac{K}{L} \]

2. **Cost Minimization**: requires \( MRS = -\frac{w}{r} \) \[ \Rightarrow MPL = \frac{w}{r} \]

   \[ \Rightarrow \frac{K}{L} = \frac{w}{r} \Rightarrow \frac{K}{w} = \frac{L}{r} \quad ("equal bang for buck") \]

   solve to get optimal (cost-minimizing) \((K, L)\) for given \( Q \):
   \[ Q = 4K^{\frac{1}{2}}L^{\frac{1}{2}} \quad \text{and} \quad \frac{K}{w} = \frac{L}{r} \quad \Rightarrow K = \frac{w}{r}L \]

   so \( Q = 4 \left( \frac{w}{r}L \right)^{\frac{1}{2}} \cdot L^{\frac{1}{2}} = 4 \left( \frac{w}{r} \right)^{\frac{1}{2}} L^{\frac{3}{2}} \cdot L^{\frac{1}{2}} = 4 \left( \frac{w}{r} \right)^{\frac{1}{2}} L \quad \text{so} \quad L^* = 4Q \cdot \left( \frac{w}{r} \right)^{-\frac{1}{2}} \]

   (note that this gives demand for \( L \) at any given \( Q \) as a function of \( Q \) and \( w/r \))

   Likewise, \( K = \frac{w}{r}L \Rightarrow K^*_1 = \frac{w}{r} \cdot 4Q \cdot \left( \frac{w}{r} \right)^{-\frac{1}{2}} \]

   \[ \Rightarrow K^*_1 = 4Q \cdot \left( \frac{w}{r} \right)^{\frac{1}{2}} = 4Q \cdot \left( \frac{r}{w} \right)^{-\frac{1}{2}} \]

   Note the symmetry of \( MPL, MPK \)

   \[ K^*_1, L^*_1, \text{ etc.} \]
3) **Minimized Costs:**  
\[ TC_i = rK_i^\alpha + wL_i^\beta \]

So \[ TC_i = r \cdot 4Q_i \left( \frac{w}{r} \right)^{\frac{1}{2}} + w \cdot 4Q_i \left( \frac{w}{r} \right)^{-\frac{1}{2}} \]

\[ = 4Q_i \left( w \cdot r \right)^{\frac{1}{2}} + 4Q_i \left( w \cdot r \right)^{-\frac{1}{2}} = 8Q_i \left( w \cdot r \right)^{\frac{1}{2}} \]

Note that \[ ATC = \frac{TC}{Q} = \frac{8Q_i \left( w \cdot r \right)^{\frac{1}{2}}}{Q} = 8 \left( w \cdot r \right)^{\frac{1}{2}} \quad (\text{independent of } \frac{Q}{Q}) \]

\[ MC = \frac{dATC}{dQ} = \frac{d}{dQ} \left\{ 8Q_i \left( w \cdot r \right)^{\frac{1}{2}} \right\} = 8 \left( w \cdot r \right)^{\frac{1}{2}} = ATC ! \]

4) **Short Run:** \[ K = K \]

Cost minimization now requires \[ Q_i = 4K^{-\frac{1}{2}} \L_i^{\frac{1}{2}} \Rightarrow L_i = \frac{1}{16} Q_i K^{-1} \]

\[ TC_i = rK + wL_i = rK + \frac{1}{16} Q_i K^{-1} w \]

\[ ATC_i = \frac{rK}{Q_i} + w \frac{L_i}{Q_i} = r \frac{K}{Q} + w \cdot \frac{1}{16} \frac{Q_i}{K} \]

\[ MC = \frac{dATC}{dQ} = \frac{d}{dQ} \left\{ 8Q_i \left( w \cdot r \right)^{\frac{1}{2}} \right\} = 8 \left( w \cdot r \right)^{\frac{1}{2}} \]

Remember that in short run, \[ ATC = \frac{rK}{Q_i} + w \frac{L_i}{Q_i} = AFC + w \frac{1}{APL} \]

\[ MC = \frac{d}{dQ} \left\{ rK + wL \right\} = w \cdot \frac{dL_i}{dQ} = w \cdot \frac{1}{\left( \frac{dQ_i}{dK} \right)} \]

5) **Assignment**

A. What are returns-to-scale properties for \( Q = 4K^{\frac{1}{2}}L^{\frac{1}{2}} \)?

B. Repeat 1-4 above for general Cobb-Douglas production, \( Q = aK^{\alpha}L^{\beta} \quad (a, \alpha, \beta \text{ are }> 0) \)