The Risk and Term Structure of Interest Rates
Money and Banking

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2. When short-term interest rates are low, yield curves are more likely to have an upward slope; when short-term rates are high, yield curves are more likely to slope downward and be inverted.

3. Yield curves almost always slope upward.
Term structure of interest rates: What may explain this?

- Expectations theory (ET):
  
  Assumes that bond holders consider bonds with different maturities to be perfect substitutes. That is, buyers of bonds do not prefer bonds of one maturity over another; they will not hold any quantity of a bond if its expected return is less than that of another bond with a different maturity.

  Statement of the ET: The interest rate on a long-term bond will equal an average of the short-term interest rates that people expect to occur over the life of the long-term bond.

  Example: Let the current rate on one-year bond be 6%. You expect the interest rate on a one-year bond to be 8% next year. Then the expected return for buying two one-year bonds averages \((6\% + 8\%)/2 = 7\%\). The interest rate on a two-year bond must be 7% for you to be willing to purchase it.
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Let us be more general. Suppose that we compare two strategies to invest a $1 as follows:

1. Buy a one period bond, hold it and when it matures, buy another one (roll-over strategy).
2. Buy a two period bond and hold it until maturity (buy and hold).

So we have:

\[ i_t = \text{today's interest rate on one period bonds} \]
\[ i_{E_t+1} = \text{next period expected interest rate on one period bonds} \]
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Expected return from the "buy and hold" strategy:

\[
(1 + i_{2t})(1 + i_{2t}) - 1 = (1 + i_{2t})^2 - 1 \\
= 2i_{2t} + (i_{2t})^2 \\
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Expected return from the "roll-over" strategy:

\[(1 + i_t)(1 + i^e_{t+1}) - 1 = 1 + i_t + i^e_{t+1} + i_t (i^e_{t+1}) - 1 \approx i_t + i^e_{t+1}\]
Term structure of interest rates: Expectations Theory

- The ET of the term structure tells us that both bonds will be held only if their returns are equal we equate these two results:

\[ 2i_{2t} = i_t + i_{t+1} \Rightarrow i_{2t} = \frac{i_t + i_{t+1}}{2} \]
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Or more generally:

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- So the ET explains fact # 1, namely that interest rates of different maturities move together.

- It also explains fact # 2: if the short term interest rate, \( i_t \), is "abnormally" low, then people expect it to go back to a normal level in the future; that is, they expect \( i_{t+1}^e \) to be higher than \( i_t \). Therefore, the average of \( i_t \) and \( i_{t+1}^e \) would be higher than \( i_t \) and the two period bond will have a higher interest rate.
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However, the ET fails to explain fact # 3.
Term structure of interest rates: Segmented markets theory

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- Interest rates for each maturity are determined by D&S in each market.

Investors have preferences for bonds of one maturity over another. If investors generally prefer bonds with shorter maturities that have less interest-rate risk, then this explains why yield curves usually slope upward (fact 3).
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So we can add to our break-down of interest rates. If we are comparing two bonds with the same risk structure:

![Diagram of nominal interest rate components]

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![Diagram of interest rate components](image)

- **Nominal Interest rate**
- **Expected inflation**
- **Value of money over time**
- **Liquidity premium**

Note: recall that if we are comparing bonds with different default risk, we would add a risk premium.
Thus, we can adjust the results from the ET framework to account for the liquidity premium:

\[ i_{nt} = \frac{i_t + i_{t+1}^e + i_{t+2}^e + \ldots + i_{t+n-1}^e}{2} + l_{nt} \]

Note the subindex in \( l_{nt} \); it has two components, \( n \) and \( \dot{t} \).

Usually the longer the maturity, the higher the liquidity premium. Also, the liquidity premium may change over time; in good times it may be lower and in uncertain times it may be much higher.

A similar approach is that of the preferred habitat theory: Investors have a preference for bonds of one maturity over another. They will be willing to buy bonds of different maturities only if they earn a somewhat higher expected return.
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- However, the preferred habitat theory may allow for different investors to prefer different "habitats".
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Since rising interest rates are associated with economic booms and falling interest rates are associated with recessions, the yield curve may have predictive power over the business cycle.
Term structure of interest rates: interpreting the yield curve

(a) Future short-term interest rates expected to rise
(b) Future short-term interest rates expected to stay the same
(c) Future short-term interest rates expected to fall moderately
(d) Future short-term interest rates expected to fall sharply
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