Money Shocks in a Markov-Switching VAR for the U.S. Economy

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September 17, 2012

Abstract

In this brief note a two-state Markov-Switching VAR (MS-VAR) on output, prices and money is estimated for the U.S. economy. Building on the perspective of a New-Keynesian theoretical model, a set of short-run identification restrictions is used to conduct structural impulse response analysis under the two possible regimes that the economy operates. Results suggest that when the economy is under distress (regime zero), money shocks tend to be more severe and produce larger responses in both output and prices than during times of healthy growth and relative stability.

1 Introduction

Estimation of vector autoregressive (VAR) models has been subject to multiple extensions ever since their introduction to economics after the seminal work of Sims (1980). A particularly useful extension is that in which the parameters to be estimated are allowed to vary over time (TVP). In turn, several versions of such an extension have been devised, with the local-level and the regime-switching models being two of the most popular. In this short piece, the latter approach is taken and Markov-switching VAR for the U.S. economy is estimated for a system including output, prices and money. In this model, all the parameters to be estimated (intercepts, autoregressive coefficients and variances) are allowed to vary over the two regimes considered.

Results from the current exercise are in line with much of the literature on the subject; when the economy is allowed to operate under two regimes, one of them is associated with recessions and turbulent times. Under such a regime the economy is subject to more severe money shocks and the variables exhibit larger responses to them. The paper is organized as follows. Section two presents the main tests of specification and describes the main features of the estimation procedure; section three lays out the identification strategy; section four summarizes the main results and section five concludes.

2 Specification and estimation

This section presents the main tests of specification and estimation strategy. First, unit root tests for the macroeconomic aggregates: log real GDP, log of GDP deflator and log of M2 are presented in table A1. Results from these tests suggest that all the variables considered contain a unit root in the levels but not in the first differences. Accordingly, the vector of endogenous variables to include in the VAR is \( X_t = [\Delta y_t \quad \Delta m_t \quad \Delta p_t] \) where a lower case stands for natural log.

Next, a number of VAR(p) models of the form:

\[ X_t = \delta + \Phi_0 + \Phi_1 X_{t-1} + \ldots + \Phi_p X_{t-p} + \varepsilon_t \]
are estimated for \( p = 1 \ldots 8 \) and the corresponding information criteria (IC) are drawn in order to determine the appropriate lag-length of the system that needs to be estimated. Following the IC presented Table A2, a VAR(2) specification is used in what follows.

Now, in order to allow for time-varying parameters (TVP), it is assumed that, at any given \( t \), the economy under study is operating under one of two possible regimes as in Krolzig (1996).\(^1\) Let \( s_t \in \{0, 1\} \) be a random variable summarizing the state of the economy at \( t \) and assume that \( s_t \) follows a first order Markov process, that is:

\[
\Pr (s_t = j | s_{t-1}^t) = \Pr (s_t = j | s_{t-1}) \quad \text{for } j = 0, 1
\]

In particular, it is assumed that \( s_t \) is characterized by a Markov chain whose transition probabilities are unknown and therefore must be estimated. This implies estimating the transition probability matrix:

\[
P = \begin{bmatrix}
\Pr (s_t = 1 | s_{t-1} = 1) & 1 - \Pr (s_t = 1 | s_{t-1} = 1) \\
1 - \Pr (s_t = 1 | s_{t-1} = 1) & \Pr (s_t = 1 | s_{t-1} = 1)
\end{bmatrix}
\]

where \( p, q \) are given by:

\[
p = \frac{\exp(p^*)}{1 + \exp(p^*)} \\
q = \frac{\exp(q^*)}{1 + \exp(q^*)}
\]

With this switching-regime arrangement in place, the model to be estimated becomes a Markov-Switching VAR (MS-VAR):

\[
X_t = \delta_{s_t} + \Phi_{0,s_t} X_t - 1 + \Phi_{2,s_t} X_{t-2} + \varepsilon_t
\]

\[
\varepsilon_t \sim NID (0, \Sigma_{s_t})
\]

with the parameters of the model following:

\[
\Phi_{i,s_t} = (1 - s_t) \Phi_{i,s_0} + s_t \Phi_{i,s_t} \\
\Sigma_{s_t} = (1 - s_t) \Sigma_{s_0} + s_t \Sigma_{s_t}
\]

and the companion form of the system being:

\[
X_t = \delta_{s_t} + F_{s_t} X_{t-1} + \nu_t
\]

with \( X_t = [X_t', X_{t-1}']' \) and:

\[
F_{s_t} = \begin{bmatrix}
\Phi_{1,s_t} & \Phi_{2,s_t} \\
I_3 & 0
\end{bmatrix}
\]

Classical estimation of the parameters in \( \Theta = (\Phi_{s_t}, \Sigma_{s_t}, P) \) involves maximizing the log-likelihood:

\[
\mathcal{L} = \sum_{t=1}^T \log \left\{ \sum_{s_t=0}^1 f \left( X_t | s_t, \phi_{t-1} \right) \Pr (s_t | \phi_{t-1}) \right\}
\]

where \( f \left( X_t | s_t, \phi_{t-1} \right) \) is the conditional (normal) density and \( \Pr (s_t | \phi_{t-1}) \) are obtained following Kim and Nelson (1999, p.63). Thus, the problem reduces to maximizing (2) given some initial values \( \Theta_0 \). In order to

\(^1\)The notation used here, however, follows Kim and Nelson (1999) closely.
obtain such vector of initial values, an unrestricted, constant parameter VAR(2) is estimated for the entire sample via OLS and the estimated coefficients and covariance matrix are used as $\Phi_{0,s}, \Sigma_{0,s}$. Obtaining initial values for the transition probabilities is not so trivial, however, and in this application these are chosen to reflect complete ignorance; that is, $p_0^s = q_0^s \approx 0$ which in turn implies initial values for all the entries of the $P$ matrix equal 0.5.

The estimated parameters under each state are presented in Table 1. Along with these estimates, vectors, $|\lambda^*|$, of the largest three eigenvalues (in modulus) from the companion matrices $F_s$, are presented as a check for VAR stability:

### Table 1A. Estimation Results for State 0

<table>
<thead>
<tr>
<th>Delta y_{t-1}</th>
<th>Delta p_{t-1}</th>
<th>Delta m_{t-1}</th>
<th>Delta y_{t-2}</th>
<th>Delta p_{t-2}</th>
<th>Delta m_{t-2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.065</td>
<td>-0.1628</td>
<td>0.103</td>
<td>0.082</td>
<td>-0.898</td>
<td>0.277</td>
</tr>
<tr>
<td>8.77e-05</td>
<td>0.248</td>
<td>0.001</td>
<td>0.026</td>
<td>0.223</td>
<td>0.021</td>
</tr>
<tr>
<td>0.057</td>
<td>-0.397</td>
<td>0.285</td>
<td>0.028</td>
<td>0.582</td>
<td>0.231</td>
</tr>
</tbody>
</table>

$$\Sigma = \begin{bmatrix} 1.16E-02 & -0.1628 & 0.103 & 0.082 & -0.898 & 0.277 \\
-6.06E-04 & 1.10E-04 & 6.03E-04 & 5.07E-04 & 7.53E-03 \\
6.03E-04 & -5.07E-04 & 7.53E-03 & & & 
\end{bmatrix}, \quad |\lambda^*| = \begin{bmatrix} 0.726 \\
0.497 \\
0.485 
\end{bmatrix}$$

### Table 1B. Estimation Results for State 1

<table>
<thead>
<tr>
<th>Delta y_{t-1}</th>
<th>Delta p_{t-1}</th>
<th>Delta m_{t-1}</th>
<th>Delta y_{t-2}</th>
<th>Delta p_{t-2}</th>
<th>Delta m_{t-2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.006</td>
<td>-0.131</td>
<td>0.104</td>
<td>0.115</td>
<td>-0.1376</td>
<td>0.200</td>
</tr>
<tr>
<td>1.8e-04</td>
<td>0.809</td>
<td>1.18e-04</td>
<td>0.055</td>
<td>0.073</td>
<td>0.018</td>
</tr>
<tr>
<td>0.016</td>
<td>-0.309</td>
<td>0.068</td>
<td>0.019</td>
<td>-0.014</td>
<td>-0.014</td>
</tr>
</tbody>
</table>

$$\Sigma = \begin{bmatrix} 4.29E-04 & -0.131 & 0.104 & 0.115 & -0.1376 & 0.200 \\
-5.02E-04 & 6.64E-04 & 2.85E-04 & -3.68E-04 & 2.70E-04 & 
\end{bmatrix}, \quad |\lambda^*| = \begin{bmatrix} 0.833 \\
0.601 \\
0.436 
\end{bmatrix}$$

Additionally, the estimated transition probability matrix is:

$$P = \begin{bmatrix} 0.507 & 0.493 \\
0.479 & 0.521 
\end{bmatrix}$$

### 3 Identification strategy

In this section the main identification strategy (i.e., the mapping from the VAR estimates of the last section to the structural parameters) is laid out. Towards this end, consider the simplest New-Keynesian model like
the one found in e.g. Clarida, Gali, Gertler (1999):

\[(\text{dynamic IS or AD}): \quad \ddot{y}_t = E_t \dot{y}_{t+1} - \varphi \{ i_t - E_t \pi_{t+1} \} + g_t \quad (3a)\]

\[(\text{NKPC or AS}): \quad \pi_t = \lambda \dot{y}_t + \beta E_t \pi_{t+1} + u_t \quad (3b)\]

where \( \ddot{y}_t = y_t - y_t^c \), and \( g_t, u_t \) are government consumption and cost-push disturbances, respectively. Then the equations for \( \pi_t \) and \( \Delta y_t \) in the VAR can be thought of as approximating the behavior of such an economy. Accordingly, shocks to the inflation equation would be interpreted as aggregate supply shocks while shocks to the output growth equation can be interpreted as aggregate demand shocks. The association of shocks to the money equation with a theoretical model is not trivial, however. This is because money is an equilibrium variable and both supply and demand can be thought as reacting to output and prices. Thus shocks to the money equation are simply labeled 'money shocks'. In order to see how the model in (3a)-(3b) can be useful to impose identification restrictions in the VAR, write the structural representations for \( X_t \) as:

\[
\begin{align*}
A_{s_t} X_t &= \Upsilon_{0,s_t} + \Upsilon_{1,s_t} X_{t-1} + \Upsilon_{2,s_t} X_{t-2} + \eta_t \\
\eta_t &\sim NID(0, \Omega_{s_t})
\end{align*}
\]

so that \( \Upsilon_{i,s_t} = A\Phi_{i,s_t} \) and the "structural" shocks are now:\(^2\)

\[
\eta_t = A_{s_t} \varepsilon_t \quad \text{with} \quad \Omega_{s_t} = A_{s_t} \Sigma A'_{s_t}
\]

Assuming that the structural shocks in the vector \( \eta_t \) are orthogonal and normalizing their variances to unity (i.e., \( \Omega_{s_t} = I \)), restrictions on the matrix \( A_{s_t}^{-1} \) are required in order to ensure that \( \Omega_{s_t} = I = A_{s_t} \Sigma A'_{s_t} \).\(^3\)

Towards this end, introduce the following set of short-run identification restrictions in the spirit of Leeper, Sims and Zha (1996): supply shocks have no contemporaneous impact on output while shocks to the money equation (\( i_t \)-shocks) have no contemporaneous impact on output and prices. This last assumption is not too unrealistic; most central banks acknowledge the existence of non-trivial lags in the transmission of monetary policy. However, one needs to exert caution here since, for the reasons outlined above, the interpretation of shocks to the money equation is not straightforward. Having said that, the above recursive structure establishes a complete Wold causal chain. In the VAR terminology, output growth is ordered first, inflation is ordered second and money growth is ordered last. It should be noted that this identification strategy leaves unrestricted the contemporaneous effects that demand and supply shocks may have on money; that is, if the last equation in the system was indeed a money supply rule, it would be one in which the monetary authority sets money growth so as to respond systematically to changes in current output and to current inflation. The causal order suggested above implies that:

\[
A_{s_t} = \begin{bmatrix}
a_{11} & 0 & 0 \\
a_{21} & a_{22} & 0 \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]

giving rise to \( 3 = K(K-1)/2 \) linearly independent restrictions which suffices for just-identification of the matrix \( A_{s_t} \). In the following section it is shown exactly how the MA coefficients from the SVAR can be pinned down from the above restrictions and the estimated reduced-form VAR.

\(^2\)This representation is what Lütkepohl (2005) calls the A-model.

\(^3\)It is important to stress the fact that the matrix \( A_{s_t} \) is state-dependent. This follows from the normalization of the structural shocks' variances under both regimes. Alternatively, one could define \( \Omega_{s_t} \) to be diagonal and state-dependent in which case \( A \) would be constant across regimes.
4 Impulse response analysis

4.1 Constructing structural impulse responses

As a first step towards understanding identification in the current model, write the corresponding VMA representation of (4):

\[ X_t = \mu_{s_t} + \Gamma_{s_t} (L) \eta_t \]

and of (1):

\[ X_t = \mu_{s_t} + \Psi_{s_t} (L) \varepsilon_t \]

Next, notice that since \( \Omega_{s_t} = I \), then:

\[ \Sigma = A_{s_t}^{-1} A_{s_t}^{-1\prime} \]

implying that \( A_{s_t}^{-1} \) is lower triangular and may therefore be obtained as the Choleski factor of \( \Sigma \). Now, from the VMA representations one has that:

\[ X_t = \mu_{s_t} + \Psi_{s_t} (L) \varepsilon_t = \mu_{s_t} + \Psi_{s_t} (L) A_{s_t}^{-1} \eta_t \]

so that \( \Gamma_{s_t} (L) = \Psi_{s_t} (L) A_{s_t}^{-1} \). Naturally, the elements in \( \Psi_{s_t} (L) \) can be obtained from the results in section 2 as:

\[ \Psi_{j,s_t} = F^{(j)}_{11,s_t} \]

where \( F^{(j)}_{11,s_t} \) corresponds to the upper-left block of the matrix \( F^j \) (i.e. \( F \) raised to the \( j \)-th power; see Hamilton (1994, p.260) for details). Finally, with \( \Psi_{j,s_t} \) and \( A_{s_t}^{-1} \) at hand, the structural MA coefficients required to construct structural IRF (SIRF) can be trivially recovered as \( \Gamma_{j,s_t} = \Psi_{j,s_t} A_{s_t}^{-1} \). It is possible now to introduce individual shocks and study how they reverberate through the system, which is done in the next section.

4.2 Regime-dependent impulse response functions

Using the results from the last section, SIRFs are constructed for the output-prices-money system and presented in figure 1. The first thing to notice is that, the signs of the SIRFs under both regimes are the same for every variable considered. This suggest that rather than asymmetric responses, the two regimes differ mainly in the size of the variable response to shocks. Mreover, results are satisfactory on a number of dimensions. For instance, in response to a negative supply shock (i.e. a positive cost-push shock), prices rise and output is depressed for a number of periods. Following the substantial acceleration of inflation, money growth falls too, reflecting a high responsiveness of the money supply rule to any deviation in prices. Furhtermore, a positive money shock produces a considerable but short lived increase in output growth and a substantial and persistent acceleration in inflation. However, it should be noticed that the responses of inflation and money growth to an aggregate demand shock are puzzling.
As can be inferred from smoothed transition probabilities shown in figure A2, the economy spends most of the time at regime one; more precisely, the probability of being at state one is close to one in much of the period. However, figure 2 points to an interesting association; it is around the time (one-to-two quarters before or after) the economy experiences a recession that the probability of operating under regime zero (being in state zero) increases substantially.

Notice how every time there is a protracted fall in output growth, one finds a peak in the probability of being in state zero at the quarter preceding or following the bottom of the recession. This is particularly true of the double dip recession of the late seventies and early eighties, but is also visible during the more recent 2001 recession. In the next section a method for uncovering year-specific IRF is presented along with the main results of this note; the effect of money shock in the years
4.3 Year-specific responses to money shocks

In order to construct year-specific IRF, a (much) simplified version of the methodology proposed by Krolzig (2006) is followed. In particular, define the \( h \)-step impulse response for year \( t \) as:

\[
\hat{\gamma}_{t,h} = \Pr (s_t = 0 | \phi_T) \times \Gamma_{h,s_0} + \Pr (s_t = 1 | \phi_T) \times \Gamma_{h,s_1}
\]

where \( \Pr (s_t = j | \phi_T) \) is the smoothed probability of being in state \( j \) given the information available for the whole time series \( \phi_T \) as was introduced in section 2. Using this method, one can study the response of all the variables in the system to a money shock in each one of the years 1965, 1975, 1985 and 1995. These results are plotted in figure 3:

![Figure 3. Year-specific Impulse response functions](image)

The results in figure 3 are interesting on a number of counts. For one thing, it is evident that during periods of growth and relative stability like 1985 and 1995, money shocks are relatively small and have a corresponding small and short-lived effect on both output and prices. On the other hand, in years like 1975, when the economy was more likely to be operating under regime zero given that it reached the bottom of the first oil-crisis recession, money shocks were much more severe; they produce a short-run acceleration in money growth of about 1 percentage point, which in turn results in output growth and inflation both eventually accelerating by about the same magnitude. These results are not too surprising, since they point to a commonplace in monetary history, namely, that often times, recessions can be associated with sharp contractions in the money supply as was originally suggested by Friedman and Schwartz (1963) for the case of the Great Depression.

5 Final remarks

In this short note, a case has been made for allowing the parameters of an output-prices-money VAR. In particular, the approach taken here has been one in which the economy is allowed to operate under two alternative regimes and, accordingly, the responses of variables in the system to exogenous shocks are allowed to vary with such regimes. The results summarized above are not too surprising in that they associate one regime with times of economic distress along the lines suggested before by e.g. Hamilton (1990). Moreover, the results point to the familiar conclusion that during recessions money shocks tend to be more severe and have larger effects compared to their counterparts during buoyant times.

A number of caveats to the current exercise apply, however. First, then number of regimes under which the economy is allowed to vary is chosen somewhat ad-hoc and results may not be robust to allowing for multiple regimes. Furthermore, they may not be robust to allowing for serial correlation in the transition probabilities.
References


Appendix: Additional Figures and Tables

Table A1. Unit root tests

<table>
<thead>
<tr>
<th></th>
<th>KPSS</th>
<th>5% CV</th>
<th>Phillips-Perron</th>
<th>5% CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^*$</td>
<td>0.181</td>
<td>0.146</td>
<td>-2.732</td>
<td>-3.433</td>
</tr>
<tr>
<td>$m^*$</td>
<td>0.374</td>
<td>0.146</td>
<td>-0.415</td>
<td>-3.433</td>
</tr>
<tr>
<td>$p^*$</td>
<td>0.322</td>
<td>0.146</td>
<td>-0.397</td>
<td>-3.433</td>
</tr>
<tr>
<td>$\Delta y^{**}$</td>
<td>0.153</td>
<td>0.463</td>
<td>-11.01</td>
<td>-2.876</td>
</tr>
<tr>
<td>$\Delta m^{**}$</td>
<td>0.516</td>
<td>0.463</td>
<td>-6.572</td>
<td>-2.876</td>
</tr>
<tr>
<td>$\Delta p^{**}$</td>
<td>0.399</td>
<td>0.463</td>
<td>-3.160</td>
<td>-2.876</td>
</tr>
</tbody>
</table>

* regression includes constant and trend; ** regression includes a constant

Table A3. Information criteria for lag-length selection

<table>
<thead>
<tr>
<th></th>
<th>Lags</th>
<th></th>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
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<tr>
<td>SIC</td>
<td>-22.6</td>
<td>-22.5</td>
<td>-22.4</td>
<td>-22.2</td>
<td>-22.1</td>
<td>-21.8</td>
<td>-21.7</td>
<td>-21.5</td>
</tr>
<tr>
<td>AIC</td>
<td>-22.8</td>
<td>-22.9</td>
<td>-22.9</td>
<td>-22.9</td>
<td>-22.9</td>
<td>-22.8</td>
<td>-22.8</td>
<td>-22.7</td>
</tr>
</tbody>
</table>

Figure A2. Smoothed probabilities of the two states

- State 0
- State 1