(10) 1. (Omitted Variables): Suppose that we regress
\[ y_i = \beta_1 + \beta_2 x_{i2} + u_i \]  (1)
and obtain the OLS estimators of \( \beta_1 \) and of \( \beta_2 \). However, the true model is
\[ y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + u_i \]  (2)
Assume that the assumptions stated on pp.156–157 of the text hold.
Show that the OLS estimators, \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \) are biased.

(30) 2. (Irrelevant Variables): Suppose that the true model is equation (1) but we regress (2):
\[ y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + u_i \]
and obtain the OLS estimators, \( \hat{\beta}_1 \), \( \hat{\beta}_2 \) and \( \hat{\beta}_3 \). Show that
\[ E\hat{\beta}_1 = \beta_1 \]
\[ E\hat{\beta}_2 = \beta_2 \]
\[ E\hat{\beta}_3 = 0 \]
This shows that the expected value of the OLS estimator of the coefficient attached to the irrelevant variable is zero.

(20) 3. In the second midterm examination we estimated the Cobb-Douglas production function with constant returns to scale:
\[ \ln \left( \frac{Q}{L} \right) = \beta_1 + \beta_2 \ln \left( \frac{K}{L} \right) + u \]
Instead of constraining the coefficients:
\[ \beta_2 + \beta_3 = 1 \]
Let us test the hypothesis
\[ H_0 : \beta_2 + \beta_3 = 1 \quad \text{versus} \quad H_1 : \beta_2 + \beta_3 > 1. \]
To carry out this test we run the regression:

\[ \ln Q = \beta_1 + \beta_2 \ln L + \beta_3 \ln K + u \]

Test the hypotheses

\[ H_0 : \beta_2 + \beta_3 = 1 \quad \text{versus} \quad H_1 : \beta_2 + \beta_3 > 1. \]

(i) The data set in **Eviews** format will be sent to you. There are three variables:

- \( x \) = data on output
- \( l \) = data on labor ("l" is "ell" it is not 1 (one))
- \( k \) = data on capital

(ii) run the OLS by the following command:

\[ \text{LS log}(X) c \text{ log}(L) \text{ log}(K) \]

and obtain the estimated covariance matrix by clicking "view" and then "covariance matrix".

(iii) Discuss the test result.