Real Business Cycles
Econ 504

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While the use of the stochastic Ramsey model as a basis for Macroeconomics is now commonplace, the Kydland-Prescott proposal was revolutionary at the time.
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While the use of the stochastic Ramsey model as a basis for Macroeconomics is now commonplace, the Kydland-Prescott proposal was revolutionary at the time.

This program led to advances in many areas (policy analysis, computational economics, econometrics,...)
Consider the model in section 3.1 of DD: the representative agent maximizes

$$E \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to

$$y_t = z_t f(k_t, n_t) = c_t + i_t$$

$$k_{t+1} = (1 - \delta)k_t + i_t$$

$$n_t + l_t = 1$$

The main difference with respect to previous version: the choice of labor ($n_t$) versus leisure ($l_t$).
The first order conditions for the agent’s problem include (you should show this):

\[ u_1(c_t, l_t) = \beta E_t u_1(c_{t+1}, l_{t+1}) [z_{t+1} f_1(k_{t+1}, n_{t+1}) + (1 - \delta)] \]

and

\[ u_2(c_t, l_t) = u_1(c_t, l_t) w_t = u_1(c_t, l_t) z_t f_2(k_t, n_t) \]

The last equation gives labor supply (by the usual condition that the MRS between consumption and leisure equals their relative price, the inverse of the real wage).
As emphasized by King, Plosser, and Rebelo, it is desirable to choose functional forms that are consistent with balanced growth:

- A Cobb Douglas production function, \( y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \)
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- CRRA-Cobb Douglas utility:

\[
  u(c, l) = \frac{(c^\phi l^{1-\phi})^{1-\phi}}{1-\phi}
\]
Popular Assumptions on Preferences

- King, Plosser, and Rebelo emphasized that *balanced growth* demands:

\[ u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} v(l), \quad \sigma \neq 1, \sigma > 0 \]

\[ = \log c + v(l) \quad \text{for } \sigma = 1 \]
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- Why is this important? In a competitive equilibrium, the marginal rate of substitution between consumption and leisure must equal the (inverse of the) marginal product of labor. Here (with \(\sigma \neq 1\)),

\[
\frac{c^{1-\sigma}}{1-\sigma} \nu'(l) = \frac{cv'(l)}{(1-\sigma)\nu(l)} = MPL
\]

In a balanced growth path, the MPL and \(c\) grow at the same rate, so \(l\) can be constant. KPR showed the converse.
Another common assumption: Greenwood, Hercowitz, Huffman (GHH):

\[ u(c, l) = v(c - h(1 - l)) \]

With GHH preferences, the MRS between \( c \) and \( l \) is simply \( 1/h'(1 - l) \). So labor supply depends only on the wage.
DD also assume that the productivity shock follows an AR(1) process in logs:

\[
\log z_t = (1 - \rho) \log \bar{z} + \rho \log z_{t-1} + \varepsilon_t,
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\varepsilon_t \sim iid(0, \sigma^2)
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Now, to have a workable version of the model, we only need to choose model parameters (for preferences, technology, and the productivity process).

To do the latter, Kydland and Prescott argued that the parameters should be calibrated. More recent work has investigated how the parameters can be estimated in various ways.
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One principle is that, if one is interested in explaining business cycle phenomena, it is desirable to set parameters on the basis of long run considerations.
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Another: in the nonstochastic steady state, the (gross) interest rate is equal to $1/\beta$. Hence, since the average interest rate in the US is about 4-5% per year (1 to 1.25% per quarter), it is natural to set $\beta = 1/1.01$ or $\beta = 1/1.0125$.
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In a balanced growth path, the rate of growth of consumption (about 0.023 in the data) is given by (in continuous time):

$$\frac{\dot{c}}{c} = \frac{1}{\phi}(r - \rho)$$

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Lucas (2003): Since $\rho$ is assumed to be nonnegative, an upper bound for $\phi$ is $r/(\dot{c}/c) = 0.05/0.023 \approx 2.2$. In practice, $\phi$ is taken to be between 1 and 5.
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For instance, the parameters of the utility function are often chosen so as to ensure that, in the nonstochastic steady state, \( n = 1 - l = 1/3 \). This is because of empirical findings on the allocation of time (Ghez and Becker 1975, Juster and Stafford 1991).
Finally, how can we calibrate the process for the productivity shocks $z_t$?
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Popular: after calibrating $\alpha$, derive the Solow residual:

$$\log z_t = \log y_t - \alpha \log k_t - (1 - \alpha) \log n_t$$

and estimate the parameters $\rho, \bar{z}, \sigma^2$ of the AR(1) process:

$$\log z_t = (1 - \rho) \log \bar{z} + \rho \log z_{t-1} + \epsilon_t,$$

$$\epsilon_t \sim iid(0, \sigma^2)$$