Problem Set 4

Econ 504

October 2009

1. Consider a stochastic growth model with *habit persistence.* The planner chooses a path of consumption and capital accumulation to maximize

$$E\sum_{t=0}^{\infty}\beta^t u(c_t, c_{t-1})$$

subject to

$$c_t + k_{t+1} = A_t k_t^a$$

given k_0 and c_{-1}.Assume that $u(c_t{,}c_{t-1})$ is twice continuously differentiable, bounded, increasing and concave in (c_t, c_{t-1}). Hence current utility is a function of current and past consumption.

Finally, A_t is an exogenous shock that follows:

$$\log A_t = \text{ constant} + \rho \log A_{t-1} + \text{ white noise}$$

(i) Find the dynamic equations that characterize the optimal plan

(ii) Discuss the (nonstochastic) steady state

(iii) Linearize the system around the steady state and discuss how you would solve it.

2. Take the stochastic growth model discussed in class, but assume that capital is costly to install. The planner maximizes $E\sum_{t=0}^{\infty}\beta^t u(c_t)$ subject to

$$c_t + i_t[1 + \Psi(\frac{i_t}{k_t})] = A_t f(k_t) \tag{1}$$

and

$$k_{t+1} = (1-\delta)k_t + i_t \tag{2}$$

where A_t is an exogenous productivity shock with the usual properties. Also, Ψ is an *adjustment cost* function assumed to be strictly increasing, strictly convex, at least twice continuously differentiable with $\Psi(0) = 0$ and $\Psi'(0) = 0$. We also assume that $\Psi(x) + x\Psi'(x)$ is strictly increasing as a function of x.

(i) Find the first order conditions associated with this problem.

(ii) Show that the optimal choice of i_t/k_t is an increasing function of q_t, where q_t is the ratio of the Lagrange multiplier associated with (2) to that associated with (1).

(iii) Show that the Euler condition can be written as

$$q_t u'(c_t) = \beta E_t \left[u'(c_{t+1})[A_{t+1} f'(k_{t+1}) + (1-\delta) q_{t+1} + (\frac{i_{t+1}}{k_{t+1}})^2 \Psi'(\frac{i_{t+1}}{k_{t+1}})] \right]$$

Interpret this condition. (*Hint:* q_t can be interpreted as the price of a newly installed unit of capital in period t, often called "Tobin's q ")

3. Solve problems 2.1, 2.2, 2.6, 2.7, 2.8, and 2.9 in Romer's textbook.