Problem Set 3

Econ 504

October 2009

1. Consider the following version of the optimal growth model:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} = C_t + I_t$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

There is no uncertainty, and the representative household maximizes:

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

(i) Assume $L_t = 1$.Suppose that $u(C) = \log C$, and that there is complete depreciation of capital ($\delta = 1$). Find the value function associated with this problem, and show that the optimal policy function is to set consumption as a fixed fraction of income.

(ii) Now assume $0 < \delta < 1$ and that $u(C) = C^{1-\sigma}/(1-\sigma), \sigma > 0, \sigma \neq 1$. Still under the assumption $L_t = 1$, find the first order conditions for an optimum, and the transversality condition. Find the steady state. Linearize the first order conditions, and discuss how you would solve for the dynamic paths of output, capital, and consumption.

(iii) Finally, suppose L_t is a choice variable (hours worked), and that household preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{L_t^{1+\chi}}{1+\chi} \right]$$

with $\chi > 0$. Solve the model as in (ii).

2. (Sargent 1987). This exercise allows for consumption goods to be different from capital goods. A planner maximizes $\sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$, where C_t denotes consumption of a good produced in a sector (labeled 1) with the technology

$$C_t \leq F_1(K_{1t}, L_{1t})$$

Capital is produced in sector 2 with the technology

$$K_{t+1} \leq F_2(K_{2t}, L_{2t})$$

For $i = 1, 2$, F_i, K_{it}, and L_{it} denote sector $i's$ production function, capital input, and labor input respectively, so that

$$L_{1t} + L_{2t} \leq L_t$$

$$K_{1t} + K_{2t} \leq K_t$$

Discuss the solution of this problem as completely as you can. In particular, find the first order conditions and interpret them.

3. Solve exercises 3 and 4 of chapter 3 of Farmer's book (pages 60-61).

You will need to use a computer program, such as GAUSS or MATLAB, to solve exercise 4. Also, as discussed in class, you will need a procedure to obtain the stable solution of the linearized dynamic system. One procedure that works well is solab.m, written by Paul Klein and available from his web page (http://www.ssc.uwo.ca/economics/faculty/klein/) Take a system of the form

$$AE_tX_{t+1} = BX_t$$

where

$$X_t = \begin{bmatrix} k_t \\ a_t \\ c_t \end{bmatrix}$$

is a vector of predetermined variables (k_t), exogenous shocks (a_t) and jumping variables (c_t). Note that k_t, a_t, c_t can be vectors.

Let

$$s_t = \begin{bmatrix} k_t \\ a_t \end{bmatrix}$$

denote the vector of states. You will be looking for a (saddlepoint stable) solution of the form

$$c_t = Fs_t$$

$$E_t s_{t+1} = \begin{bmatrix} k_{t+1} \\ E_t a_{t+1} \end{bmatrix} = Ps_t$$

To solve for F and P using Klein's programs, simply define A, B, and $ns =$ the number of state variables (the dimension of s_t), and then call solab.m:

$$[F\ P] = solab(A, B, ns)$$

There are other packages available in the Internet to do the same thing. Feel free to explore them.