Problem Set 1

Econ 504

September 2009

1. Let $Y = AF(K, L) = AK^{\alpha}L^{1-\alpha}$, with $0 < \alpha < 1$ be an aggregate production function. Define $y = Y/L$ and $k = K/L$.

(i) Show that output per capita can be expressed as $y = f(k)$. Find f;

(ii) Show that f is strictly increasing, continuously differentiable, and strictly concave on $(0, \infty)$;

(iii) Show that f satisfies the Inada conditions ($f'(k) \to 0$ as $k \to \infty$, $f'(k) \to \infty$ as $k \to 0$).

2. In the Solow growth model, assume that $f(k) = Ak$, where f denotes again output per capita, and A is a positive constant. Find the steady states (if any) and discuss their stability assuming that (i) $sA > (n+\delta)$; (ii) $sA < (n+\delta)$. A *balanced growth path* is a path for k such that its growth rate ($\dot{k}/k$) is constant. Is there a balanced growth path? What is the growth rate of output in such a path?

3. Show that if $Y = F(K, L)$ displays constant returns to scale and is such that (assuming competitive factor markets) both factor incomes are constant shares of total income, F must be Cobb Douglas.

4. Solve exercises 1.1 to 1.13 of Romer's textbook.

5. Take the deterministic, continuous time Solow model discussed in class, and assume $s = 0.1, \delta = 0.1, n = 0, \gamma = 0, f(k) = k^{\alpha}, \alpha = 0.3$. Write a GAUSS or MATLAB program that calculates the steady state, and graphs the solution of the model (i.e. the well known Solow diagram). Starting from the steady state, what happens if s increases permanently to 0.12? If δ falls to 0.8? Use your program to discuss the answers. Draw the time path of the responses (similar to Fig. 1.5 of Romer). What is the intuition, in each case?