Basic Modeling Decisions

- Large Economies versus Small Economies
- Commodity Structure: Exports, Imports, Nontraded Goods
- Relative Prices: Terms of Trade, Real Exchange Rate
- Financial Markets
- Nominal Rigidities: PCP versus LCP
- Optimal Monetary Policy
We follow Gali and Monacelli (2005) and Gali (2008).

- The world is a continuum of small economies of zero measure
- Each economy has a representative household
- Each country produces varieties of a differentiated good
Consider a single (Home) country. The Home agent’s consumes an aggregate of a Home composite good and imports:

\[ C = \left[ (1 - \alpha)^{1/\eta} C_H^{(\eta-1)/\eta} + \alpha^{1/\eta} C_F^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \]
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This implies that

\[ C_H = (1 - \alpha) \left( \frac{P_H}{P} \right)^{-\eta} C, \quad C_F = \alpha \left( \frac{P_F}{P} \right)^{-\eta} C \]

where \( P \) is the CPI:

\[ P = \left[ (1 - \alpha) P_H^{1-\eta} + \alpha P_F^{1-\eta} \right]^{1/(1-\eta)} \]
The Home composite is Dixit-Stiglitz:

\[ C_H = \left[ \int C_H(j)^{\frac{\epsilon - 1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon - 1}} \]
Commodity Structure

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- The imports composite is C.E.S. of the composite good produced in every other country \( i \):

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Here,

\[ C_i = \left[ \int C_i(j) \frac{\varepsilon - 1}{\varepsilon} \, dj \right]^{\frac{\varepsilon}{\varepsilon - 1}} \]
The previous implies that

\[ C_H(j) = \left( \frac{P_H(j)}{P_H} \right)^{-\varepsilon} C_H \]

\[ P_H = \left[ \int P_H(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)} \]

Analogous expressions are derived for \( C_i, C_i(j), P_F, \) and \( P_i \)
Assuming complete markets, the budget constraint is written as:

\[ P_t C_t + E_t [Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t \]

where \( Q_{t,t+1} \) is the stochastic discount factor for Home currency payoffs, which in equilibrium must satisfy

\[ Q_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \]
Optimal Labor Supply requires:

\[ \sigma^\varphi N^\varphi = \frac{W}{P} \]
1. Optimal Labor Supply requires:

\[ C^\sigma N^\varphi = \frac{W}{P} \]

2. The nominal interest rate is

\[ \frac{1}{1 + i_t} = E_t Q_{t,t+1} = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \]
Some Definitions I

1. The Home *terms of trade* is given by $P_F / P_H$ or, in log deviations from ss, 
   \[ s = p_F - p_H \]

2. The CPI definition now gives (prove it!):
   \[ p = p_H + \alpha s \]

3. Hence the difference between CPI inflation and "domestic" inflation is
   \[ \pi_t = \pi_{Ht} + \alpha \Delta s_t \]

4. The *Law of One Price* says that the price of an imported good at Home equals its price in its country of origin, $P^i_j$, corrected by the (nominal) exchange rate $\mathcal{E}_i$:
   \[ P_i(j) = \mathcal{E}_i P^i_j(j) \]
The bilateral *real exchange rate* between Home and country $i$ is the ratio of their CPIs, $Q_i = \mathcal{E}_i P^i / P$

See Gali or GM to see the appropriate multilateral definitions

To first order,

$$s = e + p^* - p_H$$

where $e$ is a nominal exchange rate index and $p^*$ a world CPI index

The real exchange rate is related to the terms of trade by:

$$q = (1 - \alpha) s$$
Perfect risk sharing is given by the familiar condition:

$$C_t = \vartheta_i C_t^i Q_{it}$$

To first order, and assuming symmetry,

$$c_t = c_t^* + \frac{1}{\sigma} q_t$$

$$= c_t^* + \frac{1 - \alpha}{\sigma} s_t$$
Implications for Demand I

The demand for Home variety $j$ is

$$ Y(j) = C_H(j) + \int C_H^i(j) \, di $$

$$ = \left( \frac{P_H(j)}{P_H} \right)^{-\varepsilon} C_H + \int \left( \frac{P_H^i(j)}{P_F^i} \right)^{-\varepsilon} C_F^i \, di $$

$$ = \left( \frac{P_H(j)}{P_H} \right)^{-\varepsilon} \left[ (1 - \alpha) \left( \frac{P_H}{P} \right)^{-\eta} C + \int \left( \frac{P_H}{\varepsilon_i P_F^i} \right)^{-\gamma} \alpha \left( \frac{P_F^i}{P^i} \right)^{-\eta} C_i \, di \right] $$

Using the perfect risk sharing condition $C_t = C_t^i Q_{it}$ and loglinearizing, this can be rewritten as:

$$ y_t = c_t + \frac{\alpha \omega}{\sigma} s_t $$

where

$$ \omega = \sigma \gamma + (1 - \alpha)(\sigma \eta - 1) $$
Recalling that, from perfect risk sharing, \( c_t = c_t^* + \frac{1-\alpha}{\sigma} s_t \), one gets

\[
y_t = c_t^* + \frac{1}{\sigma_\alpha} s_t
\]

where

\[
\sigma_\alpha = \frac{\sigma}{1 - \alpha + \alpha \omega}
\]
Implications for the Dynamic IS

Linearizing the Euler equation gives

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \]

Since \( c_t = y_t - \frac{\alpha \omega}{\sigma} s_t \),

\[ y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) - \frac{\alpha \omega}{\sigma} E_t \Delta s_{t+1} \]

This can be rewritten as

\[ y_t = E_t y_{t+1} - \frac{1}{\sigma_\alpha} (i_t - E_t \pi_{H,t+1} - \rho) + \alpha (\omega - 1) E_t \Delta y^*_t \]
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- The risk sharing condition plays a very important role here.
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Calvo pricing
Aside: The Currency Pricing Issue

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Traditional assumption is producer currency pricing (PCP): firm $j$ sets price for both domestic sales and exports in terms of its own currency.

Alternative: local currency pricing (LCP), under which firms set prices in terms of the currency where output will be sold.
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- The assumption means that the Law of One Price holds, and that a nominal depreciation makes Home output cheaper, leading to \textit{expenditure switching}.

- Under LCP, a nominal depreciation does not change the relative prices faced by consumers, so there is no expenditure switching.

- Instead, a depreciation raises the Home currency value of exports and export profits (Betts and Devereux).
The Calvo assumption results in the same relation between domestic inflation and marginal costs as in the closed economy:

\[ \pi_{Ht} = \beta E_t \pi_{Ht+1} + \lambda mc_t \]

But here

\[ mc_t = w_t - p_{Ht} - a_t \]
\[ = w_t - p_t + p_t - p_{Ht} - a_t \]
\[ = \sigma c_t + \phi n_t + \alpha s_t - a_t \]
\[ mc_t = \sigma c_t^* + \phi y_t + s_t + (1 - \phi) a_t \]
\[ mc_t = \sigma c^*_t + \varphi y_t + s_t + (1 - \varphi) a_t \]

Recalling \( y_t = c^*_t + \frac{1}{\sigma} s_t \),

\[ mc_t = (\varphi + \sigma) y_t + (\sigma - \sigma_a) c^*_t - (1 + \varphi) a_t \]
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- This tells us how marginal cost is affected by foreign variables, here summarized by \( c^*_t \)
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\[ mc_t = (\varphi + \sigma_\alpha) y_t + (\sigma - \sigma_a) c_t^* - (1 + \varphi) a_t \]

- This tells us how marginal cost is affected by foreign variables, here summarized by \( c_t^* \)
- Under flexible prices, real (producer) marginal cost is constant, so the above gives natural output:

\[ y_t^n = \Gamma_a a_t + \Gamma_* c_t^* \]

with \( \Gamma_a = (1 + \varphi) / (\sigma_a + \varphi) \) and \( \Gamma_* = -\alpha (\omega - 1) \sigma_\alpha / (\sigma_\alpha + \varphi) \)
The New Keynesian Phillips Curve

One can now derive

\[ \pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa \alpha (y_t - y_t^n) \]

where

\[ \kappa \alpha = \lambda (\sigma \alpha + \varphi) \]
One can now derive

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The New Keynesian Phillips Curve

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where

$$\kappa_\alpha = \lambda (\sigma_\alpha + \varphi)$$

1. Openness affects dynamics by affecting the slope of the Phillips Curve
2. Foreign shocks (to consumption) affect the natural rate of output
Three equation system:

\[ \pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa \alpha \tilde{y}_t \]

\[ \tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma_\alpha} (i_t - E_t \pi_{H,t+1} - r^n_t) \]

\[ i_t = \phi_\pi \pi_{Ht} + \phi_y \tilde{y}_t + \nu_t \]
Three equation system:

\[
\begin{align*}
\pi_{Ht} &= \beta E_t \pi_{Ht+1} + \kappa \alpha \tilde{\gamma}_t \\
\tilde{\gamma}_t &= E_t \tilde{\gamma}_{t+1} - \frac{1}{\sigma_\alpha} (i_t - E_t \pi_{H,t+1} - r^n_t) \\
i_t &= \phi_\pi \pi_{Ht} + \phi_y \tilde{\gamma}_t + \nu_t
\end{align*}
\]

- As exercise, derive \( r^n_t \)
Dynamics Under Interest Rate Rule

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- Analysis of dynamics in Gali
Three equation system:

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- As exercise, derive \( r_t^n \)
- Analysis of dynamics in Gali
- In practice, interest rate rules can depend on \( \pi_t \), not \( \pi_{Ht} \)
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Home country policy affects the relative price of its exports.
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In an open economy, there is another distortion: the so-called *terms of trade externality* (Corsetti and Pesenti).

Home country policy affects the relative price of its exports.

This implies the existence of an "optimal tariff" motive.
Let us consider a simpler specification of demand for home goods:

\[ Y_t = (1 - \alpha) \left( \frac{P_{ht}}{P_t} \right)^{-\eta} C_t + \alpha \left( \frac{P^*_h}{P_t} \right)^{-\eta} C^*_t \]

where \( P^*_t \) is the world price level and

\[ P^*_h = P_{ht} / \mathcal{E}_t \]

This can be rewritten as:

\[ Y_t = (1 - \alpha) p_{ht}^{-\eta} C_t + \alpha p_{ht}^{-\eta} Q_t^\eta C^*_t \]

where \( p_{ht} = \frac{P_{ht}}{P_t} \) and

\[ Q_t = \frac{\mathcal{E}_t P^*_t}{P_t} \]

is the real exchange rate.
Recalling:

\[ P = \left[ (1 - \alpha)P_H^{1-\eta} + \alpha P_F^{1-\eta} \right]^{1/(1-\eta)} \]

and \( P_F = \mathcal{E}\mathcal{P}^* \), it follows that

\[ 1 = \left[ (1 - \alpha)p_h^{1-\eta} + \alpha Q^{1-\eta} \right]^{1/(1-\eta)} \]

so we can write

\[ p_h = g(Q) = \left[ \frac{1 - \alpha Q^{1-\eta}}{(1 - \alpha)} \right]^{1/(1-\eta)} \]
Finally, recall the risk sharing condition:

\[ C = C^* Q^{1/\sigma} \]

The *Ramsey problem* is to maximize \( u(C) - v(N) \) subject to risk sharing, \( p_h = g(Q) \), and

\[
AN = (1 - \alpha)p_h^{-\eta} C + \alpha p_h^{-\eta} Q^\eta C^*
\]
You should show that

\[ \frac{1}{\sigma} Cu'(C) = Nv'(N)e_Q^D \]

where \( e_Q^D \) is the total elasticity of demand wrt the real exchange rate

\[ D = (1 - \alpha)g(Q)^{-\eta} C^* Q^{1/\sigma} + \alpha g(Q)^{-\eta} Q^\eta C^* \]
Under flexible prices,

\[ P_h = \mu MC = \mu (1 - \nu)(W/A) \]

This leads to (supply the details):

\[ Cu'(C) = Nv'(N) \frac{\mu(1 - \nu)C}{g(X)AN} \]
Flexible Price Outcome

Under flexible prices,

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This leads to (supply the details):

\[ Cu'(C) = N\nu'(N)\frac{\mu(1 - \nu)C}{g(X)AN} \]

\[ \implies \] The flex price outcome will be optimal only if \[ \frac{\mu(1-\nu)C}{g(X)AN} = \epsilon^D_Q \]
In order to make progress, one can derive a second order approximation to utility of the form

\[ -E \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} v_t' L_v v_t + v_t' L_a \hat{a}_t + \frac{1}{2} l_\pi \pi_{ht}^2 \right] \]

where \( v_t = (\hat{y}_t, \hat{c}_t, \hat{p}_{ht}, \hat{x}_t)' \), \( L_v \) and \( L_a \) are matrices, and \( l_\pi \) is a scalar (Benigno and Benigno 2005).
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where \( v_{t} = (\hat{y}_{t}, \hat{c}_{t}, \hat{p}_{ht}, \hat{x}_{t})' \), \( L_{v} \) and \( L_{a} \) are matrices, and \( l_{\pi} \) is a scalar (Benigno and Benigno 2005).

- This contains no linear terms. Then one can use the *first order* approximation to the model equations to find optimal policy.
Actually, in this model one can express all the variables in $v_t$ as a function of output:

$$v_t = N\hat{y}_t + N_e a_t$$

Replacing, the objective function can be rewritten as

$$-E \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} l_y (\hat{y}_t - \tilde{a}_t)^2 + \frac{1}{2} l_\pi \pi_{ht}^2 \right]$$

for suitable scalars $l_y$ and $l_\pi$.

Here, $\tilde{a}_t$ is a linear transformation of $a_t$ (or of other shocks, if there were). Using the same representation for $v_t$ one can write the Phillips curve as

$$\pi_{ht} = \lambda_y \hat{y}_t + \lambda_a a_t + \beta E_t \pi_{ht+1}$$
Optimal Policy, finally

The FOC for optimality can be written as

\[ \pi_{ht} + \frac{l_y}{\lambda_y l_\pi} (\Delta \hat{y}_t - \Delta \tilde{a}_t) = 0 \]

This can be seen as a "flexible inflation targeting" prescription.
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R. Chang (Rutgers)
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- Hence optimal policy "looks like" the closed economy one
- However, it must be stressed that open economy aspects of the model show up in the weight $\frac{l_y}{\lambda_y l_\pi}$ and the properties of $\tilde{a}_t$.
- This is not the only way to express the optimal policy. One could express it in terms of e.g. inflation and a real exchange rate target.