End of Term Administrative Issues

1. Rewritten midterms due today
2. Recitation tomorrow
3. Next week: Recitation moved to Tuesday, classes Wed and Thu
4. Last week: Recitation on Tuesday, Dec 10th
5. Project Due: Dec 12th
6. Final Exam: Wed Dec 18, 9-12 am, Scott 206
To address the shortcomings of real business cycle models, many advocate the introduction of nominal price and wage rigidities.

- Dominant framework: the dynamic New Keynesian model (a modern IS-LM model)
- Versions of the NK model are basic to actual monetary policy analysis and formulation
- Read: Gali chapters 2,3 (also recommended: Romer, ch. 7)
A Basic Dynamic Model

- Combine elements from Galí, chapter 3
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The representative household has preferences

\[
\sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \kappa \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]
\]

and budget constraint

\[
P_t C_t + M_t + Q_t B_t \leq M_{t-1} + B_{t-1} + W_t N_t + T_t
\]
A Basic Dynamic Model

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- Money in the utility function: Sidrauski (1967)
It is convenient to define $A_t = B_{t-1} + M_{t-1}$. The budget constraint can be rewritten as:

$$P_t C_t + Q_t M_t + Q_t B_t + M_t (1 - Q_t) \leq M_{t-1} + B_{t-1} + W_t N_t + T_t$$

Or

$$P_t C_t + Q_t A_{t+1} + (1 - Q_t) M_t \leq A_t + W_t N_t + T_t$$
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- This defines $Q_t$ as the price at $t$ of a dollar at $t + 1$, so $1/Q_t = 1 + i_t$
- So $1 - Q_t = i_t / (1 + i_t)$ is the opportunity cost of holding money
Optimality conditions

FOCs include the usual Euler:

\[ C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} R_{t+1} \]

where

\[ R_{t+1} = \frac{1}{Q_t} \frac{P_t}{P_{t+1}} = \frac{(1 + i_t)}{1 + \pi_{t+1}} \]

Also, optimal labor supply:

\[ N_t^\varphi = C_t^{-\sigma} (W_t / P_t) \]

and the demand for money equation:

\[ \kappa (M_t / P_t)^{-\nu} = \left[ C_t^{-\sigma} (1 - Q_t) \right], \text{ or} \]

\[ M_t / P_t = \left[ \frac{1}{\kappa} C_t^{-\sigma} \left( \frac{i_t}{1 + i_t} \right) \right]^{-1/\nu} \]
Rewriting the Euler equation as:

\[ Q_t = E_t \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \]

it is clear that

\[ Q_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \]

is the *stochastic discount factor* for nominal payoffs between \( t \) and \( t + 1 \).
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The modern literature emphasizes that producers set prices optimally. Hence there must be some departure of perfect competition. Usual choice: *monopolistic competition*. The idea: final goods are aggregates of imperfectly substitutable varieties. In the household problem, 

$$C_t = \left[ \int_0^1 C_t(i)^{1-1/\varepsilon} di \right]^{\varepsilon/(\varepsilon-1)}$$
If a household consumes $C_t$ units of the composite good, the minimum cost of purchasing it is

$$P_t C_t = \text{Min}\{C_t(i)\} \quad \int P_t(i) C_t(i) \, di \quad \text{s.t.} \quad C_t = \left[ \int_0^1 C_t(i)^{1-1/\varepsilon} \, di \right]^{\varepsilon/(\varepsilon-1)}$$

whose solution is

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

where

$$P_t = \left[ \int P_t(i)^{1-\varepsilon} \, di \right]^{1/(1-\varepsilon)}$$
Each variety $i$ is produced by a single firm with only labor:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

- Assume, first, that each firm sets a nominal price $P_t(i)$ for its variety to maximize profits.
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- Assume, first, that each firm sets a nominal price $P_t(i)$ for its variety to maximize profits.
- The constraints are the production function and the demand for variety $i$:

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Production

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$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$

- Profits are

$$P_t(i) Y_t(i) - W_t N_t(i) = P_t(i) Y_t(i) - \Psi_t(Y_t(i))$$

where, from the production function, the (total) cost function is:

$$\Psi_t = W_t \left( \frac{Y_t(i)}{A_t} \right)^{1/(1-\alpha)}$$
The solution to:

\[
\text{Max } P_t(i) Y_t(i) - \Psi_t(Y_t(i))
\]

subject to

\[
Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t
\]

is the *markup rule*:

\[
P_t(i) = \frac{\varepsilon}{\varepsilon - 1} \psi_t(Y_t(i))
\]

where \( \psi_t(Y_t(i)) = \Psi'_t(Y_t(i)) \) is the *marginal cost* function, equal to

\[
\psi_t(Y_t(i)) = \frac{W_t}{(1 - \alpha)A_t} \left( \frac{Y_t(i)}{A_t} \right)^{\alpha/(1-\alpha)}
\]
With flexible Prices, $P_t(i) = P_t$ and $Y_t(i) = Y_t$, so that the markup condition becomes:

$$P_t = \frac{\varepsilon}{\varepsilon - 1} \psi_t(Y_t)$$

$$= \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{(1 - \alpha)A_t} \left( \frac{Y_t}{A_t} \right)^{\alpha/(1-\alpha)}$$
In equilibrium, \( Y_t(i) = Y_t = A_t N_t(i)^{1-\alpha} \), so
\( N_t = N_t(i) = (Y_t/A_t)^{1/(1-\alpha)} \).

Also, \( Y_t = C_t \), so we get:
\[
\frac{W_t}{P_t} = N_t^{\varphi} C_t^{\sigma} = (Y_t/A_t)^{\varphi/(1-\alpha)} Y_t^{\sigma}
\]

Hence, with flexible prices, \( Y_t \) solves
\[
1 = \frac{\varepsilon}{\varepsilon - 1} \frac{(Y_t/A_t)^{\varphi/(1-\alpha)} Y_t^{\sigma}}{(1 - \alpha)A_t} \left( \frac{Y_t}{A_t} \right)^{\alpha/(1-\alpha)}
\]

Main Implication: *natural output* \( Y_t = Y_t^n \) is a function only of \( A_t \)
Special case: $\alpha = 0$, then we have

$$1 = \frac{\varepsilon}{\varepsilon - 1} \frac{(Y_t^n)^{\sigma+\varphi}}{A_t^{\varphi+1}}$$

or, taking logs,

$$0 = \log \left( \frac{\varepsilon}{\varepsilon - 1} \right) + (\varphi + \sigma) y_t^n - (1 + \varphi) a_t$$

Monetary aspects do not matter
The Calvo Model

- Calvo (1983) assumed that, in each period, a firm can set a new price with probability $1 - \theta$. 

$$
\sum_{k=0}^{\infty} \theta^k E_t Q_t, t+k
$$

subject to

$$Y_{t+k} = P_t Y_t + \epsilon_t C_{t+k} + \sigma P_t Y_t + \kappa$$

and with $Q_{t+k} = \beta_k C_{t+k}$
The Calvo Model

- Calvo (1983) assumed that, in each period, a firm can set a new price with probability $1 - \theta$.
- A firm that can reset price then chooses $P_t^*$ to maximize:

$$\sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} \left[ P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|k}) \right] \}$$

subject to

$$Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k}$$

and with

$$Q_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$$
Optimal Pricing and the Price Level

The FOC is
\[ \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} \left[ P_t^* - \frac{\varepsilon}{\varepsilon - 1} \psi_{t+k|t} \right] \} = 0 \] (1)

where \( \psi_{t+k|t} = \Psi_{t+k}'(Y_{t+k|k}) \), so this is an obvious generalization of the static case.

Given all this, the evolution of the price level is (Gali p 62):

\[ P_t = \left[ \theta P_{t-1}^{1-\varepsilon} + (1 - \theta) P_t^{*1-\varepsilon} \right]^{1/(1-\varepsilon)} \] (2)
From a log-linear approximation of (1)-(2) (assuming $\alpha = 0$ from now on, ugly derivation in Appendix) we get that the dynamics of inflation, $\pi_t = \log P_t - \log P_{t-1} = p_t - p_{t-1}$, are given by

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \hat{mc}_t$$

where

$$\lambda = \frac{1 - \theta}{\theta} (1 - \beta \theta)$$

and $\hat{mc}_t = mc_t - mc$ is the log deviation of real marginal cost from its steady state value $mc$. 
Marginal Costs, In Equilibrium I

Recall

\[ mc_t = (w_t - p_t) - a_t \]

and

\[ N_t^\varphi = C_t^{-\sigma}(W_t / P_t) \]

In equilibrium, \( C_t = Y_t \). Also, \( Y_t = A_t N_t \). Using this, taking logs in the last equation, and combining,

\[ mc_t = (\sigma + \varphi) y_t - (1 + \varphi) a_t \]

To express this in a different way, observe that under flexible prices real marginal costs are constant (why?) and equal to \( mc \). Hence,

\[ mc = (\sigma + \varphi) y_t^n - (1 + \varphi) a_t \]

where \( y_t^n \) is the natural or flexible price output.
It follows that

$$\hat{mc}_t = (\sigma + \varphi)(y_t - y^n_t)$$

where $y_t - y^n_t$ is the output gap.
The New Keynesian Phillips Curve

Then, combining

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda \hat{mc}_t \]
\[ \hat{mc}_t = (\sigma + \varphi)(y_t - y^n_t) \]

we arrive at

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - y^n_t) \]

with \( \kappa = \lambda(\sigma + \varphi) \). This is the basic New Keynesian Phillips Curve.
Log-linearizing the Euler equation

\[ C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}} \]

with \( Y_t = C_t \), you get

\[ -\sigma y_t = \log \beta + E_t \{ -\sigma y_{t+1} + i_t - \pi_{t+1} \} \]

Letting the output gap be denoted by \( \tilde{y}_t = y_t - y^n_t \), so \( y_t = \tilde{y}_t + y^n_t \), and replacing in the above, you get

\[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n_t) + E_t \tilde{y}_{t+1} \]

where

\[ r^n_t = -\log \beta + \sigma E_t \{ y^n_{t+1} - y^n_t \} \]

is the *natural real rate of interest*. 
Note that $r_t = i_t - E_t \pi_{t+1}$ is the real rate of interest.

Iterating forward, for any $k \geq 1$:

$$\tilde{y}_t = -\frac{1}{\sigma} E_t \left[ (r_t - r_t^n) + (r_{t+1} - r_{t+1}^n) + \ldots + (r_{t+k} - r_{t+k}^n) \right] + E_t \tilde{y}_{t+k}$$
Summarizing, we now have:

1. The NK Phillips Curve:

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The Basic New Keynesian Model

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The model is completed with a specification of monetary policy. For example, an interest rate rule (Gali, 3.4):

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \]
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- For example, an interest rate rule (Gali, 3.4):

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \]

- Note that this is equivalent to a real interest rule:

\[ r_t = i_t - E_t \pi_{t+1} = \rho + \phi_\pi \pi_t - E_t \pi_{t+1} + \phi_y \tilde{y}_t + \nu_t \]
Gali (p.50) shows that a unique equilibrium exists with a rule
\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \]
if and only if:
\[ \kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0 \]
Gali (p.50) shows that a unique equilibrium exists with a rule
\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \ddot{y}_t + v_t \] if and only if:
\[ \kappa (\phi_\pi - 1) + (1 - \beta) \phi_y > 0 \]

This is the called *Taylor Principle*
Equilibrium Under an Interest Rate Rule

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if and only if:

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- To understand it, suppose $\phi_y = 0$. Then the Principle is that $\phi_\pi > 1$, which ensures that a permanent increase in inflation is met with a higher real rate of interest
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\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \] if and only if:
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- To understand it, suppose \( \phi_y = 0 \). Then the Principle is that \( \phi_\pi > 1 \), which ensures that a permanent increase in inflation is met with a higher real rate of interest
- When \( \phi_y \), the same idea applies.
See Gali, 3.4 Some noteworthy aspects:

1. *Liquidity Effect*: the nominal interest rate *increases* and the money supply *falls* in response to a contractionary monetary shock.

2. The impact of a positive technology shock on output and labor employment can be *negative*.
Some Policy Implications

Suppose that there is a social loss function is of the form $E \sum \beta^t L_t$, where the current loss is:

$$L_t = v \tilde{y}_t^2 + (1 - v) \pi_t^2$$
Some Policy Implications

Suppose that there is a social loss function is of the form $E \sum \beta^t L_t$, where the current loss is:

$$L_t = \nu \tilde{y}_t^2 + (1 - \nu) \pi_t^2$$

- Justifying this loss function: Romer 11.3, Woodford (2003), Gali 4
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- Justifying this loss function: Romer 11.3, Woodford (2003), Gali 4
- If AS is given by the basic New Keynesian Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

then it is feasible for the central bank to keep $\pi_t = \tilde{y}_t = 0$
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- But this minimizes the loss function!
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- But this minimizes the loss function!
- Justification for inflation targeting
Appendix: Loglinearizing AS Equations I

Rewrite the pricing equation (1) as:

\[
\sum_{k=0}^{\infty} \theta^k E_t Q_{t+k} \frac{P_t^*}{P_{t-1}} = \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} \theta^k E_t Q_{t+k} \frac{\psi_{t+k|t}}{P_{t-1}} \\
= \mathcal{M} \sum_{k=0}^{\infty} \theta^k E_t Q_{t+k} \frac{\psi_{t+k|t}}{P_{t+k}} \frac{P_{t+k}}{P_{t-1}} \\
= \mathcal{M} \sum_{k=0}^{\infty} \theta^k E_t Q_{t+k} MC_{t+k|t} \Pi_{t-1,t+k}
\]

where \( \mathcal{M} = \frac{\varepsilon}{\varepsilon - 1} \), \( MC_{t+k|t} = \frac{\psi_{t+k|t}}{P_{t+k}} \), \( \Pi_{t-1,t+k} = \frac{P_{t+k}}{P_{t-1}} \)

Loglinearizing around a zero inflation steady state:

\[
\sum_{k=0}^{\infty} (\beta \theta)^k (P_t^* - p_{t-1}) = \sum_{k=0}^{\infty} (\beta \theta)^k E_t [\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})]
\]
or:

\[
\frac{p_t^* - p_{t-1}}{1 - \beta \theta} = \sum_{k=0}^{\infty} (\beta \theta)^k E_t[\hat{mc}_{t+k}|t + (p_{t+k} - p_{t-1})] \tag{3}
\]

where \( \hat{mc}_{t+k}|t = mc_{t+k}|t - mc \). (Note also \( mc = -\log M \).)

From now on, assume \( \alpha = 0 \), which implies that

\[
mc_{t+k}|t = mc_{t+k} = (w_{t+k} - p_{t+k}) - a_{t+k}. \] (The general case is in Gali).

Then the pricing equation (3) becomes:

\[
p_t^* - p_{t-1} = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t[\hat{mc}_{t+k} + (p_{t+k} - p_{t-1})]
\]
which can be rewritten as:

\[ p_t^* - p_{t-1} = \beta \theta E_t(p_{t+1}^* - p_t) + (1 - \beta \theta) \hat{m}c_t + \pi_t \]

In turn, the price level equation (2) gives:

\[ \pi_t = p_t - p_{t-1} = (1 - \theta)(p_t^* - p_{t-1}) \]

Combining the last two equations, we get

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda \hat{m}c_t \]

where

\[ \lambda = \frac{1 - \theta}{\theta} (1 - \beta \theta) \]