The New Keynesian Model

Basic Issues

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January 2013

Basic Ingredients of the New Keynesian Paradigm

- Representative agent paradigm
- Nominal rigidities, price setting
- Phillips Curve, Dynamic IS
- Focus on policy rules
- Welfare based analysis of policy

Aggregate Supply

The final good is a Dixit-Stiglitz aggregate of individual varieties:

$$Y_t = \left[\int Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

hence the demand for each variety is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$

where the price level is

$$P_t = \left[\int P_t(i)^{1-\varepsilon} di \right]^{1/(1-\varepsilon)}$$

The Pricing Problem

The reason to assume monopolistic competition is to obtain a model in which prices are set by producers.
Producer i has technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

Assuming that labor is homogeneous, nominal marginal cost is then:

$$MC_t(i) = \frac{dN_t(i)}{dY_t(i)} = \frac{1}{1-\alpha} \left(Y_t(i)\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{A_t}\right)^{\frac{1}{1-\alpha}} .W_t$$

In the absence of nominal rigidities, each producer would set price as a constant markup over marginal cost:

$$P_t(i) = \mathcal{M} MC_t(i) = \mathcal{M} MC_t = P_t$$

where $\mathcal{M} = \frac{\varepsilon}{\varepsilon - 1}$

Calvo Pricing

Nominal rigidities are often introduced via the "Calvo protocol", which implies that the optimal pricing rule is

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M} MC_{t+k|t} \right) \right\} = 0$$

The Calvo protocol also implies that

$$P_t^{1-\varepsilon} = (1-\theta) P_t^{*1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}$$

Linear Approximation: The Phillips Curve

The log linear approximation to these eqs. (assuming that inflation is zero in ss) yields:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ mc_{t+k|t} + p_{t+k} \right\}$$

$$p_t = (1 - \theta) p_t^* + \theta p_{t-1}$$

One has to work a little to express $mc_{t+k|t}$ in a simple way:

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha} (p_t^* - p_{t+k})$$

where

$$mc_t = (w_t - p_t) - \frac{1}{1-\alpha} (a_t - \alpha y_t)$$

is a measure of average marginal cost (I ignore irrelevant constants)

Inflation

Combining, you get

$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t$$

where λ depends on $\theta, \alpha, \varepsilon$.

Notice that this means

$$\pi_t = \lambda E_t \sum_{k=0}^{\infty} \beta^k mc_{t+k}$$

This says that inflation goes up when marginal costs are expected to go up or, equivalently, when markups fall short of desired ones.

The Output Gap

Woodford suggested to express marginal costs in terms of the "gap " of output from its flexible price or "natural " value. To do this, one expresses marginal costs as:

$$mc_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \varphi}{1 - \alpha} a_t$$

This holds whether or not prices are flexible. But if they are, mc_t is constant, say mc. This leads to a definition of natural output as:

$$mc_t - mc = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_t - y_t^n)$$

The difference $\tilde{y}_t = y_t - y_t^n$ is the *output gap*

The NKPC

Combining everything, we finally get the NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

Note that the output gap has entered the determination of inflation because it completely summarizes the role of marginal costs in pricing. In more complicated models, marginal costs may depend on other variables, such as relative prices or the real exchange rate (in open economies).

Aggregate Demand

The representative agent's problem in Gali's chapter 3 is standard. (Note, however, that complete financial markets are implicitly assumed). In equilibrium, $Y_t = C_t$, and log linearizing the Euler equation yields

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho)$$

This can be usefully rewritten by substituting output with the output gap:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n)$$

where

$$r_t^n = \pi + \sigma E_t \Delta y_{t+1}^n$$

is a good enough definition of the *natural interest rate.*
This is the *dynamic IS* and essentially says that aggregate demand increases when the (actual) real interest rate falls below the natural interest rate.

Recap, so far

We have AS, the NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

and AD, the IS:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n)$$

These two equations will determine the behavior of inflation and the output gap once we relate the nominal interest rate to the same two variables.

Interest Rate Rules

- If one assumes that i_t follows an exogenous process, however, the price level is indeterminate.
- Leeper, Woodford: in practice, central banks adjust policy rate in response to *endogenous* variables
- Taylor: this is in fact what the Fed does
- Example: Taylor

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

- Equilibrium is then determinate if

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

- Taylor Principle

Policy Analysis: The Efficient Allocation

Gali (section 4.1) considers the efficiency problem and shows that efficiency requires:

$$
\begin{aligned}
C_t(i) &= C_t \\
N_t(i) &= N_t \\
-\frac{U_{nt}}{U_{ct}} &= MPN_t
\end{aligned}
$$

These are intuitive.
Equilibrium allocations may differ because of two distortions: imperfect competition and nominal rigidities.

Inefficiencies related to Imperfect Competition

- To isolate those, consider *flexible* price equilibria.
- In those, all varieties have the same price and are produced in the same quantities, so $C_t(i) = C_t$ and $N_t(i) = N_t$.
- On the other hand, markup pricing means

$$P_t = \mathcal{M} MC_t = \mathcal{M} \frac{W_t}{MPN_t}$$

so

$$-\frac{U_{nt}}{U_{ct}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

since $\mathcal{M} = \varepsilon/(\varepsilon - 1) > 1$

- In words, the real wage is less than the marginal product of labor, which leads to inefficiently low employment
- Accordingly, much of the literature assumes that there is a subsidy $\tau = 1/\varepsilon$ to the wage, which restores the efficiency of flex price equilibria.

Inefficiencies Related to Nominal Rigidities

In equilibrium, average markups are

$$\mathcal{M}_t = \frac{P_t}{MC_t} = \frac{P_t}{(1-\tau)W_t / MPN_t} = \frac{P_t \mathcal{M}}{W_t / MPN_t}$$

assuming a subsidy $\tau = 1/\varepsilon$ to the wage.

Then we get

$$-\frac{U_{nt}}{U_{ct}} = \frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t}$$

The factor $\frac{\mathcal{M}}{\mathcal{M}_t}$ can fluctuate and is an inefficient wedge between MRS and MPN

In addition, under Calvo Pricing, different varieties of the same good have different prices, which leads to violations of the efficiency conditions $C_t(i) = C_t$ and $N_t(i) = N_t$. These are usually referred to as costs of *price dispersion*.

The Optimality of Zero Inflation

In the basic model, zero inflation implies that equilibrium outcomes coincide with natural (flex price) outcomes, which are efficient by construction.

Not much else needs to be said, except to check that this is consistent with the way we have expressed the model:

$\pi_t = 0$ implies $\tilde{y}_t = 0$ by the NKPC

Then $\mathcal{M}_t = \mathcal{M}$ and $-\frac{U_{nt}}{U_{ct}} = MPN_t$

Finally, all goods are priced the same, so $C_t(i) = C_t$ and $N_t(i) = N_t$.

The Implementation Question

If zero inflation is optimal, how do we attain it?
One possibility: if inflation is zero, then the nominal interest rate must equal the natural interest rate r_t^n.
But a *policy rule* $i_t = r_t^n$ leads to indeterminacy. (Why?)
Instead, one can consider a rule:

$$i_t = r_t^n + \phi_\pi \pi_t$$

$==>$ If $\phi_\pi > 1$, the equilibrium is determinate, and implies $\pi_t = 0$ at all times.

Note that this rule is "complex "in that it requires observing r_t^n.

Evaluating Simple Rules

For many reasons, one may want to compare alternative *simple* rules, such as Taylor's:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

Gali (ch. 4 and Appendix) shows that, if the flex price outcome is efficient, equilibrium outcomes can be ranked according to the loss function:

$$\mathcal{W} = \frac{1}{2} E \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\tilde{y}_t) + \frac{\varepsilon}{\lambda} var(\pi_t) \right]$$

Note the interpretation of the coefficients:

$\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ captures the importance of the wedge between MRS and MPN

$\frac{\varepsilon}{\lambda}$ matters because of price dispersion (λ depends on θ)

Policy Tradeoffs

The basic model displays a *divine coincidence:* stabilizing inflation leads to flexible price equilibria which are efficient. Hence there is no policy tradeoff.

The literature has then asked: what if flexible price outcomes are not efficient? This makes the policy question much harder.

Gali (ch. 5) starts with the assumption that flexible price outcomes are efficient in the *steady state.* In this case, it is useful to define the *welfare relevant output gap,* $x_t = y_t - y_t^e$, as the (log) difference between output and its efficient level.

A second order approximation to welfare is then given by

$$E \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha_x x_t^2)$$

In turn, the NKPC can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

where $u_t = \kappa(y_t^e - y_t^n)$ is a purely exogenous process (*cost push*).

Optimal Policy: Commitment Versus Discretion

Consider the problem of maximizing:

$$E \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha_x x_t^2)$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

for $t = 0, 1, 2, \ldots$

This is a relatively simple problem (subsection 5.1.2)

However, the solution is *time inconsistent:* if the policymaker reoptimizes at some time $t \geq 1$, she will depart from the original plan (Calvo, Kydland-Prescott)

One possible solution under discretion: the policymaker at t assumes that his choice at t will not affect what happens in future periods.
Then the choice problem at t is to maximize $\pi_t^2 + \alpha_x x_t^2$ subject to

$$\pi_t = \kappa x_t + u_t + \text{constant}$$

The solution is simply $x_t = -\frac{\kappa}{\alpha_x}\pi_t$.

But other solutions may be possible!

The Inefficient Steady State Case

In the more general case of an inefficient steady state, Gali shows that the second order approximation to welfare has a linear term:

$$E \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha_x \hat{x}_t^2) - \Lambda \hat{x}_t$$

where $\hat{x}_t = x_t - x$, and x is the welfare relevant output gap in the zero inflation steady state.

This is problematic because one can no longer use a linear approximation to the equilibrium conditions to evaluate welfare up to second order.

Several solutions in the lit:

1. Assume that the steady state distortion is "small "
2. Approximate equilibrium to second order
3. Solve for optimum policy with original nonlinear specification, then linearize.