Bubbles and the Current Crisis

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Novelty: bubbles can be expansionary.

Main idea: plegeable income of entrepreneurs depends on the terminal value of capital, so a bubble enables them to borrow and invest more. This increases investment efficiency and can more than compensate for the fact that bubbles absorbs savings.
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Model

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- All agents work when young, consume when old
- Risk neutral
Each firm $j$ that can produce in period $t$ has access to technology

$$F(l_{jt}, k_{jt}) = k_{jt}^\alpha l_{jt}^{1-\alpha}$$

A consequence is that the wage is given by the marginal product of labor:

$$w_t = (1 - \alpha)(k_t / l_t)^\alpha$$

$$= (1 - \alpha)k_t^\alpha$$

the last equality holding because $l_t = 1$. ($k_t$ is the aggregate capital/labor ratio).
For existing firms in period $t$, the investment technology is the usual one:

$$k_{j,t+1} = Z_{jt} + (1 - \delta)k_{jt}$$

New firms in period $t$ (which can only start producing in $t + 1$) have

$$k_{j,t+1} = \pi_t Z_{jt} + (1 - \delta)k_{jt}$$

where $\pi_t > 1$ and can be random ("investment efficiency"). Note that efficiency would entail that all investment should be done by new firms.
• Each generation $t$ has unit size
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A fraction $\varepsilon$: entrepreneurs
Savings

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Savings

- Each generation $t$ has unit size
- A fraction $\varepsilon$: entrepreneurs
- Only entrepreneurs can start new firms
- Anyone can buy old firms and operate them
Any agent can borrow and lend at some stochastic interest rate $R_{t+1}$
Borrowing and Lending

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- Any agent can borrow and lend at some stochastic interest rate $R_{t+1}$.
- $E_t R_{t+1} = "the " interest rate at t$.
- Collateral constraint of agent $j$:

\[
R_{t+1} f_{jt} \leq \phi_{t+1} \left[ F(l_{j,t+1}, k_{j,t+1}) - w_{t+1} l_{j,t+1} + V_{j,t+1} \right] \\
= \phi_{t+1} \left[ r_{t+1} k_{jt+1} + V_{j,t+1} \right]
\]

where $f_{jt} =$ amount borrowed, $\phi_{t} =$ "financial friction", $V_{j,t+1} =$ price of the firm acquired by $j$, and $r_{t} = \alpha k_{t}^{\alpha-1}$.
Optimality conditions

If non-entrepreneurs are willing to lend to entrepreneurs and to buy old firms, then

\[ E_t R_{t+1} = \max \frac{E_t [r_{t+1} k_{jt+1} - R_{t+1} f_{jt+1} + V_{jt+1}]}{V_{jt} + Z_{jt} - f_{jt}} \]

where \( r_t = \alpha k_t^{\alpha-1} \) and the max is s.t.

\[ k_{j,t+1} = Z_{jt} + (1 - \delta) k_{jt} \]
Likewise, if entrepreneurs are willing to start new firms, then

\[ E_t R_{t+1} \leq \max \frac{E_t \left[ r_{t+1} k_{jt+1} - R_{t+1} f_{jt+1} + V_{jt+1} \right]}{Z_{jt} - f_{jt}} \]

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\[ k_{j,t+1} = \pi_t Z_{jt} + (1 - \delta) k_{jt} \]

and

\[ R_{t+1} f_{jt} \leq \phi_{t+1} \left[ r_{t+1} k_{jt+1} + V_{j,t+1} \right] \]
Look for equilibrium in which entrepreneurs are always constrained and nonentrepreneurs are not.
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Conjecture that

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Equilibrium

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- The conjecture assumes that there is no bubble
Entrepreneurs only invest in new firms and borrow as much as they can. The collateral constraint then becomes:

\[ f_{jt} = \frac{\pi_t E_{t+1} \phi_{t+1}}{1 - \pi_t E_t \phi_{t+1}} w_t \]
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Nonentrepreneurs have more than enough savings to buy old firms and lend to entrepreneurs:

$$(1 - \varepsilon) w_t \geq V_t + f_t^N$$
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Capital evolves according to:

\[ k_{t+1} = \left[ 1 + \frac{(\pi_t - 1) \varepsilon}{1 - \pi_t E_t \phi_{t+1}} \right] (1 - \alpha) k_t^\alpha \]
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- Capital evolves as in the typical OG model, with shocks to "technology."

This model seems insufficient to understand the crisis. What was the particular shock that caused such a severe downturn?
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- Capital evolves as in the typical OG model, with shocks to "technology ".
- MV: This model seems insufficient to understand the crisis.
- Main argument: what was the particular shock that caused such a severe downturn?
Now let us conjecture that there are bubbles in equilibrium:

\[
E_t R_{t+1} = r_{t+1} + (1 - \delta) = \alpha k_{t+1}^{\alpha - 1} + (1 - \delta)
\]

\[
V_{jt} = (1 - \delta) k_{jt} + b_{jt}
\]
The entrepreneurs’ collateral constraint becomes

\[ f_{jt} = \frac{\pi_t E_t \phi_{t+1}}{1 - \pi_t E_t \phi_{t+1}} w_t + \frac{E_t \phi_{t+1} b_{jt+1}^N}{(1 - \pi_t E_t \phi_{t+1}) E_t R_{t+1}} \]

This is the same as before except for the bubble component.
Also, for any existing firm,

\[ E_t R_{t+1} = \frac{E_t b_{jt+1}}{b_{jt}} \]

i.e. the expected growth rate of bubbles must equal the interest rate.
The capital accumulation equation becomes

\[ k_{t+1} = \left[ 1 + \frac{(\pi_t - 1) \varepsilon}{1 - \pi_t E_t \phi_{t+1}} \right] (1 - \alpha) k_t^\alpha \]

\[ + \frac{(\pi_t - 1)}{1 - \pi_t E_t \phi_{t+1}} \frac{E_t \phi_{t+1} b_{t+1}^N}{\alpha k_{t+1}^{\alpha - 1} + (1 - \delta)} - (b_t + b_t^N) \]

Note that now bubbles can have two opposite effects on the accumulation of capital:

- Usual crowding out effect
The capital accumulation equation becomes

\[ k_{t+1} = \left[ 1 + \frac{(\pi_t - 1) \varepsilon}{1 - \pi_t E_t \phi_{t+1}} \right] (1 - \alpha) k^\alpha_t + \frac{(\pi_t - 1)}{1 - \pi_t E_t \phi_{t+1}} \frac{E_t \phi_{t+1} b^N_{t+1}}{\alpha k^\alpha_{t+1} + (1 - \delta)} - (b_t + b^N_t) \]

Note that now bubbles can have two opposite effects on the accumulation of capital:

1. Usual crowding out effect
2. They may relax financial constraints, leading to faster capital accumulation.
The bubbles growth condition becomes

\[ E_t b_{t+1} = \left[ \alpha k_{t+1}^{\frac{\alpha - 1}{\alpha}} + (1 - \delta) \right] (b_t + b_t^N) \]

The aggregate bubble grows faster than the interest rate because of the creation of new (bubbly) firms.
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Assume \( p = \operatorname{Prob}\{ z_{t+1} = F | z_t = B \} \)

Start with some period \( t \) in which \( b_t = 0, b_t^N > 0 \)
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Afterwards, \( b_t^N = nb_t \) if \( z_t = B \)
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Assume $p = \text{Prob}\{z_{t+1} = F | z_t = B\}$.

Start with some period $t$ in which $b_t = 0$, $b_t^N > 0$.

Afterwards, $b_t^N = nb_t$ if $z_t = B$.

Assume $\text{Prob}\{z_{t+1} = B | z_t = F\}$ is negligible.
More assumptions:

- $\pi_t = \pi, \phi_t = \phi$
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More assumptions:

- $\pi_t = \pi, \phi_t = \phi$
- $\delta \approx 1$
- Define $x_t = b_t / [(1 - \alpha)k_t^\alpha] = \text{bubble as share of savings}$
- Then

$$x_{t+1} = \frac{[\alpha(1 + n) / (1 - \alpha)(1 - p)]x_t}{1 + \frac{(\pi - 1)\varepsilon}{1 - \phi \pi} \left(\frac{(\pi - 1)\phi n}{1 - \phi \pi} - 1\right) (1 + n)x_t}$$
We also need

\[ x_t \leq \frac{1 - \phi \pi - \varepsilon}{1 - \phi (\pi - n)} \frac{1}{1 + n} \equiv \bar{x} \]

Given path for \( x_t \), capital accumulation is given by

\[ k_{t+1} = \left( 1 + \frac{(\pi - 1) \varepsilon}{1 - \pi \phi} + \frac{\phi (\pi - 1)n}{1 - \phi \pi} \right) (1 + n)x_t \left( 1 - \alpha \right) k_t^\alpha \]