Bubbles, Asset Prices, and Financial Frictions

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Introduction: Bubbles and Crises

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- This seems to be consistent with evidence.
- In fact, one can envision a theory in which financial frictions (collateral constraints) depend on asset prices. If the latter are affected by bubbles, then there may be an important connection between bubbles and credit collapse.

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- The sad fact is that we know very little about bubbles, and are short of a satisfactory theory.
- Let us review some of the literature. (Blanchard and Fisher ch. 5 is a useful summary.)

A Simple Linear Case

Consider the expectational equation

$$y_t = aE_t y_{t+1} + cx_t$$

where x_t is an exogenous process of "fundamentals " and a, c are constants.

A stochastic process y_t is a *solution* if it satisfies the equation for all t.

The equation can be derived, for instance, from an equation for asset prices:

$$p_t = \frac{1}{1+r} E_t \left[p_{t+1} + d_t \right]$$

The Fundamental Solution

Iterating:

$$\begin{aligned} y_t &= cx_t + aE_t y_{t+1} \\ &= cx_t + aE_t \left[cx_{t+1} + aE_{t+1} y_{t+2} \right] \\ &= cx_t + caE_t x_{t+1} + a^2 E_t y_{t+2} \end{aligned}$$

so, for any T,

$$y_t = c \sum_{j=0}^{T} a^j E_t x_{t+j} + a^{T+1} E_t y_{t+T+1}$$

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Assume that $|a| < 1$.Then, assuming that the last term in the RHS converges to zero with T, this suggests that the process

$$y_t^* = c \sum_{j=0}^{\infty} a^j E_t x_{t+j}$$

is a solution.

Of course, we need the sum to converge, which requires that $|E_t x_{t+j}|$ grow asymptotically no faster than $|1/a|$. In this case, this is called the *fundamental* solution.

Bubbly Solutions

Is the fundamental solution y_t^* the *only* solution? Let y_t denote any other solution and define a *bubble* as

$$b_t = y_t - y_t^*$$

Recalling the original equation

$$y_t = aE_t y_{t+1} + cx_t$$

and noting that the fundamental solution solves it

$$y_t^* = aE_t y_{t+1}^* + cx_t$$

It follows that

$$b_t = aE_t b_{t+1}$$

Rewriting this as

$$E_t b_{t+1} = \frac{1}{a} b_t$$

emphasizes that a bubble, if it exists, must "grow at the rate of interest". In fact, $E_t b_{t+j} \longrightarrow \infty$ with j

Examples of Bubbles

1. A deterministic, ever expanding bubble:

$$b_t = b_0 a^{-t}$$

2. A stochastic, bursting bubble:

$$\begin{aligned} b_{t+1} &= \frac{1}{aq} b_t + \varepsilon_{t+1} && \text{with probability } q \\ &= \varepsilon_{t+1} && \text{with prob } (1-q) \end{aligned}$$

where ε_t is an *arbitrary* process with $E_t \varepsilon_{t+1} = 0$

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- For instance, if y_t is the price of an asset that can be freely disposed of, and $a > 0$, there cannot be *negative* bubbles. (If $b_t < 0$, then $E_t b_{t+j}$ would become arbitrarily large and negative, so $E_t y_{t+j}$ would too.)

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- In general, these kinds of side conditions suggest that it is quite hard *not to* rule out bubbles.
- The literature has focused on *general equilibrium* arguments.

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- This kind of work has become less popular with advances in finance.

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- Main finding: it is hard to have *rational* bubbles in general equilibrium
- One case: infinite number of agents and *dynamically inefficient* equilibria

Infinite Horizons

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- If short sales are allowed, a reverse argument implies that there cannot be a positive bubble either

- But the result survives even without short sales: if $p_t > p_t^*$, anyone buying the asset must be planning to sell it at some point to make a capital gain. But, because there is a finite set of agents, there must be a finite time at which nobody wants to hold the asset.

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- This indicates that, if bubbles are to exist in general equilibrium, it must be that new agents come into the economy over time (as in overlapping generations models).

Finite Horizons

Consider the Diamond OG model: without bubbles, the dynamics of capital is given by

$$\begin{aligned} k_{t+1} &= \frac{1}{1+n} s\left[w(k_t), r(k_{t+1})\right] \\ &\equiv \frac{1}{1+n} s\left[k_t, k_{t+1}\right] \end{aligned}$$

Bubbles on Useless Assets

Let there be an asset M that is intrinsically useless, and suppose that it sells at price p_t (in terms of goods).
If anyone holds the bubble, arbitrage requires

$$1 + f'(k_{t+1}) = \frac{p_{t+1}}{p_t} = \frac{B_{t+1}}{B_t}$$

where $B_t = Mp_t$ is the total size the bubble. In per capita terms,

$$b_{t+1} = \frac{1 + f'(k_{t+1})}{1+n} b_t$$

Capital accumulation is

$$k_{t+1} = \frac{1}{1+n}[s(k_t, k_{t+1}) - b_t]$$

Note that bubbles can reduce capital accumulation here, because part of savings go to pay for the bubble.

Equilibrium

Equilibrium then requires

$$b_{t+1} = \frac{1 + f'(k_{t+1})}{1+n} b_t$$

$$k_{t+1} = \frac{1}{1+n}[s(k_t, k_{t+1}) - b_t]$$

Also, $k_t \geq 0$ and (with free disposal) $b_t \geq 0$

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- This is equivalent to the requirement that the (bubbleless) Diamond economy be *dynamically inefficient*

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- Since the bubble must grow at the rate of interest, then eventually it becomes too large.
- This reasoning does not hold if there is dynamic inefficiency
- In fact, in this case bubbles can lead to a Pareto improvement!

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- Let P_t^* be its fundamental value, and B_t the bubble, so $P_t = P_t^* + B_t$
- By arbitrage,

$$P_t^* = 1 + \frac{1}{1 + f'(k_{t+1})} P_{t+1}^*$$

and

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- This implies that asymptotically the asset price is dominated by the bubble.

Final Remarks

1. Because bubbles are associated with dynamic inefficiency, they are also associated with multiple equilibria.
2. Dynamic inefficiency, and hence bubbles, can emerge from financial constraints. Recent work (e.g. the paper by Martin and Ventura in the reading list) exploit the connection.