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Abstract

In this paper we examine how the timing of investment affects the levels of quality chosen by firms. We show that in a model with vertical quality differentiation a game with sequential quality choice induces both firms to make smaller quality investments than they would in a game with simultaneous quality choice. Furthermore, we show that while aggregate profit is higher, both consumer and social surplus are lower under sequential quality choice.

Keywords: R&D; Endogenous quality choice; investment timing

JEL classification: D21; D43

1. Introduction

In this paper we examine how the timing of investment decisions affects the levels of quality chosen by firms. We show that in a model with vertical quality differentiation a game with sequential quality choice induces both firms to make smaller quality investments than they would in a game with simultaneous quality choice. Furthermore, we show that aggregate profit is

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higher under sequential quality choice, suggesting that sequential quality choice offers coordination benefits, an insight often overlooked in quality choice models.

We compare the subgame perfect Nash equilibrium of a game with simultaneous quality choice and a game with sequential quality choice. We consider a model with two firms who in the first stage make quality investments, either simultaneously or sequentially, and then in the second stage engage in Bertrand competition in the product market. Quality choices will be made sequentially in an industry where new products are pioneered by a single firm, such as the pharmaceutical industry. The assumption that quality choices are made simultaneously is most appropriate if the new generation of products is developed by multiple firms in the industry, such as the automobile industry.

Another interpretation of the assumption of simultaneous vs. sequential quality choice is possible if we focus on the information structure of the two games. In the extensive form of the simultaneous move game, one of the firms has a non-trivial information set. A game with the identical extensive form, except for the absence of the non-trivial information set, corresponds to a game of sequential moves. While an industry's structure might explain whether or not there is a non-trivial information set, institutional factors and government regulations that mandate informational disclosure may also determine whether the competition is one of imperfect information (i.e. a simultaneous move game) or one of perfect information (i.e. a sequential move game). For example, the Food and Drug Administration (FDA) licensing policy may provide the opportunity for credible, verifiable information disclosure (Grabowski and Vernon, 1983). Another example is the recent change in the U.S. patent system which requires early disclosure of the information in patent applications.[1] Our finding that the aggregate profit is higher in the sequential move game could explain in part why U.S. industries supported such a change.

However one chooses to interpret the model, the key insight is that the timing of quality choice not only has implications for the quality of products available but also has important consequences for measures of welfare, such as producer, consumer and social surplus. The remainder of the paper proceeds as follows. In the next section we sketch a model of vertical quality differentiation where each firm's development efforts result deterministically in a quality level. The firms' quality choices are made either simultaneously or sequentially depending on whether or not there is information disclosure.

[1] If firms chose to voluntarily publish their patent applications and the formulae underlying their FDA license requests, rivals would not view this voluntary disclosure as credible since the information could not be verified. See Tirole (1988) and Kreps (1991) for discussions on the difference between the "observability" and the notions of "credibility" and "verifiability".

In Section 3 we will characterize equilibrium under the two quality choice regimes. Consumer and social surpluses are examined in Section 4. Concluding remarks and a discussion of the robustness of our results are in Section 5.

2. The model

We consider a two-stage game with two identical firms, 1 and 2, who produce products differentiated by vertical quality. Investments in quality are made in the first stage and products are produced and sold in the second stage. In the first stage each firm makes an investment that deterministically determines the quality, q_i, $i = 1, 2$, of its product. We consider two regimes for the first stage: one where choices are made sequentially and one where they are made simultaneously. The question of how the first mover of the sequential game is determined is an important one since it critically influences the distribution of profits. However, much of our analysis is unaffected by this issue, and so it will be convenient to assume that in the sequential game firm 1 chooses its quality first, and then firm 2 makes its choice. In Section 4 where we discuss profits under the two regimes we will address the issue of how the first mover is determined.

We assume that at the end of this first stage the quality decisions of the two firms are made public, making the quality choices common knowledge at the end of the first stage regardless of the timing of the moves in the first stage. While this assumption has the benefit of eliminating informational problems that might arise if quality choices were not common knowledge in the second period, it also characterizes industries where the new product introduction can be separated into two parts: development and production. In the second stage, firms simultaneously announce prices; then consumers decide which product (if any) to purchase and sales are made.[2] Given that quality choices are revealed at the end of the first stage, it seems most natural to assume that at the second stage prices are chosen simultaneously.

The games can be solved backwards for the subgame perfect Nash equilibrium. Since the situation at the beginning of the second stage is the same in either case, we primarily focus on the first stage of the model and only briefly describe the equilibrium of the second stage of the model. The emphasis on the first stage is sensible since, as described below, the second stage equilibrium is uniquely determined by the first stage quality choices and therefore, for our purposes, the second stage may be reduced to the equilibrium payoffs as function of quality choices. Those interested in a

[2] In Section 5 we will discuss the robustness of our results in the light of our assumption of simultaneous price choice.

more detailed presentation of the second stage should consult Gabszewicz and Thisse (1979), Shaked and Sutton (1982), and Aoki (1988).

In a model with vertically differentiated quality, given two products of different qualities sold at the same price, *all* consumers unambiguously rank one quality as being better than the other. In order for both types of products to have positive sales, there must be heterogeneity in consumers' willingness to pay for quality, which we capture by assuming that there is a continuum of consumers indexed by t, uniformly distributed over the interval [0,1]. A type t consumer will get a surplus of $v_t(q, p) = tq - p$ if he buys one unit of product of quality $q \in [0, \infty)$ at price $p \in [0, \infty)$.

We assume that there is no cost of production but that firm i incurs a cost $C(q_i)$ to develop quality q_i. We scale the quality index so $q_i = 0$ corresponds to "no improvement" or the status quo. We assume that the cost function has the particular form $C(q_i) = kq_i^2$, $k > 0$, where k is interpreted as an efficiency parameter, and where higher values of k correspond to industries that are less efficient at R&D. This cost function implies that better quality is more expensive and is increasingly so.[3]

At the second stage, given the available quality and price combinations, a consumer will buy one unit of a product if surplus is (i) positive and (ii) greater than the surplus from consuming the other product. As has been shown in the related work of Motta (1993) and Aoki (1988), the equilibrium revenue from the second stage for firm 1, $R_1(q_1,q_2)$, can be written as

Lemma 1.

$$R_1(q_1, q_2) = \begin{cases} \dfrac{4(q_1)^2(q_1 - q_2)}{(4q_1 - q_2)^2} & \text{if } q_1 \geq q_2, \\ \dfrac{q_2 q_1(q_2 - q_1)}{(4q_2 - q_1)^2} & \text{if } q_1 < q_2. \end{cases} \quad (1)$$

(Proofs are in the Appendix unless otherwise noted.)

Firm 2's revenue, $R_2(q_1,q_2)$, can be defined analogously. From (1) it follows that $R_i(q_1,q_2)$ is continuous at any (q_1,q_2) and is differentiable at all points except at $q_1 = q_2$. The non-differentiability at $q_1 = q_2$ is due to the fact that the revenue functions are derived from the two equilibrium sales revenue functions, one for the firm supplying the higher quality product and the other for the firm supplying the lower quality product (see Appendix).

A typical revenue function for firm 1 is depicted in Fig. 1. When firm 2's quality is q_2, firm 1's revenue is depicted by the solid revenue line; the dashed revenue line depicts firm 1's revenue when firm 2's quality is slightly larger, say q_2'. We summarize the relationships depicted in the figure in the following lemma.

[3] The specific functional form chosen is not necessary for the existence of pure strategy Nash equilibria but does allow us to derive analytical conclusions.

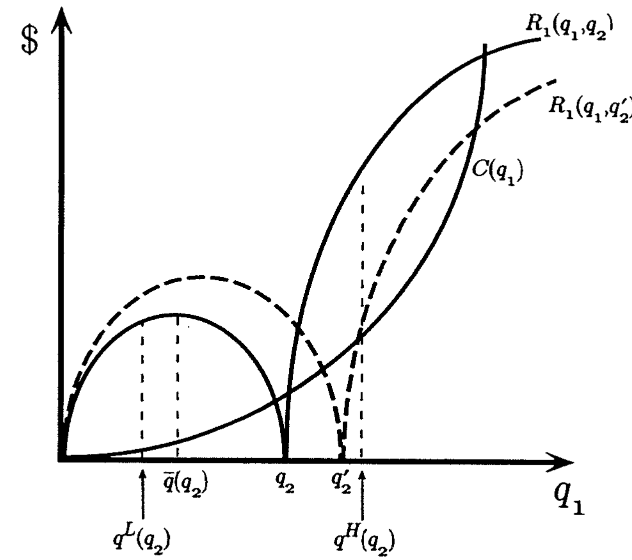

Fig. 1.

Lemma 2. When $q_i > q_j$,

$$\frac{\partial R_i(q_1, q_2)}{\partial q_i} > 0 \quad \text{and} \quad \frac{\partial R_i(q_1, q_2)}{\partial q_j} < 0, \quad (2)$$

$$\frac{\partial R_j(q_1, q_2)}{\partial q_i} > 0, \quad (3)$$

$$\frac{\partial R_j(q_1, q_2)}{\partial q_j} \gtreqless 0 \Leftrightarrow q_j \lesseqgtr \bar{q}, \quad \text{where } \bar{q} = (4/7)q_i. \quad (4)$$

Eqs. (2) and (3) imply that firms benefit from qualities being further apart. The exception to this rule is that the lower quality firm prefers not to have too low a quality in order to prevent low willingness to pay consumers from switching to the outside good (i.e. not purchase either product). Therefore, there is an optimal distance for the lower quality firm to maintain from its higher quality rival (Eq. (4)).

The second order properties of the revenue functions are characterized by

$$\frac{\partial^2 R_i(q_1, q_2)}{\partial q_i^2} < 0, \quad \text{for } q_1 \neq q_2, \ i = 1, 2, \tag{5}$$

$$\frac{\partial^2 R_i(q_1, q_2)}{\partial q_1 \partial q_2} > 0, \quad \text{for } q_1 \neq q_2, \ i = 1, 2. \tag{6}$$

Eq. (5) implies that firm i's revenue function is locally concave in q_i, except where $q_1 = q_2$. Eq. (6) implies that when $q_i > q_j$, the negative impact of an increase in rival's quality on firm i's revenue, $\partial Ri(q_1, q_2)/\partial q_j < 0$ is smaller when qualities are further apart. On the other hand, when $q_i < \bar{q} < q_j$ the positive impact of an increase in its own quality $\partial Ri(q_1, q_2)/\partial q_i > 0$ is greater when qualities are further apart.[4]

The payoff from the first stage, $\Pi_i(q_1, q_2)$, consists of revenue from the second stage less the cost of quality investment. Therefore, the payoff relevant for characterizing the subgame perfect Nash equilibrium is

$$\Pi_i(q_1, q_2) = R_i(q_1, q_2) - C(q_i). \tag{7}$$

3. Simultaneous and sequential quality choice equilibria

3.1. Characterizing the best-response correspondence

We begin by solving for the optimal quality choices assuming that the firms make their choices simultaneously. From the properties of the revenue functions discussed above, we know that an optimal choice will always exist and will be an interior solution. Therefore, firm i's best-response correspondence, $q_i = \beta_i(q_j)$, is characterized by the first-order condition

$$\frac{\partial \Pi(q_1, q_2)}{\partial q_i} = \frac{\partial R_i(q_1, q_2)}{\partial q_i} - C'(q_i) = 0.$$

As depicted in Fig. 1 there are always two local maxima, one below and one above the rival's quality level, denoted by $q^{\mathrm{L}}(q_j)$ and $q^{\mathrm{H}}(q_j)$, where $q^{\mathrm{L}}(q_j) \leq q_j \leq q^{\mathrm{H}}(q_j)$.

In addition, from (2) and (3) there exists a unique $\hat{q}$ such that $\Pi_1(q^{\mathrm{L}}(\hat{q}), \hat{q}) = \Pi_1(q^{\mathrm{H}}(\hat{q}), \hat{q})$ and $\Pi_2(\hat{q}, q^{\mathrm{L}}(\hat{q})) = \Pi_2(\hat{q}, q^{\mathrm{H}}(\hat{q}))$. In other words, if the rival's quality is $\hat{q}$ (i.e. $q_j = \hat{q}$), firm i is indifferent between the

[4] As will be shown later, the equilibrium quality choice is characterized by the first-order condition of maximization in which case $\partial R_i(\cdot)/q_i > 0$. In other words, in equilibrium the low quality choice will be strictly less than $\bar{q}$.

local maxima, and $\beta_i(q_j)$ takes two values. The best-response function, $\beta_i(q_j)$, is discontinuous at $\hat{q}$ and can be expressed as

$$\beta_i(q_j) = \begin{cases} q^{\mathrm{L}}(q_j) & \text{if } q_j \geq \hat{q}, \\ q^{\mathrm{H}}(q_j) & \text{if } q_j \leq \hat{q}. \end{cases} \tag{8}$$

Firm 1's best-response correspondence is graphed in Fig. 2.

Lemma 3. The best-response correspondences have the following properties:

(1) *$\beta_i(q_j)$ is discontinuous at $q_j = \hat{q}$*
(2) *$d\beta_i(q_j)/dq_j > 0$, $\forall q_j \neq \hat{q}$.*
(3) *$\beta_i(q_j) \neq q_j$, $\forall q_j$.*

We have also depicted some of firm 1's iso-profit ($d\Pi_1(q_1, q_2) = 0$) curves in Fig. 2. Iso-profit curves close to the 45° line correspond to negative profit. Along the upper segment of the best-response function profit increases as

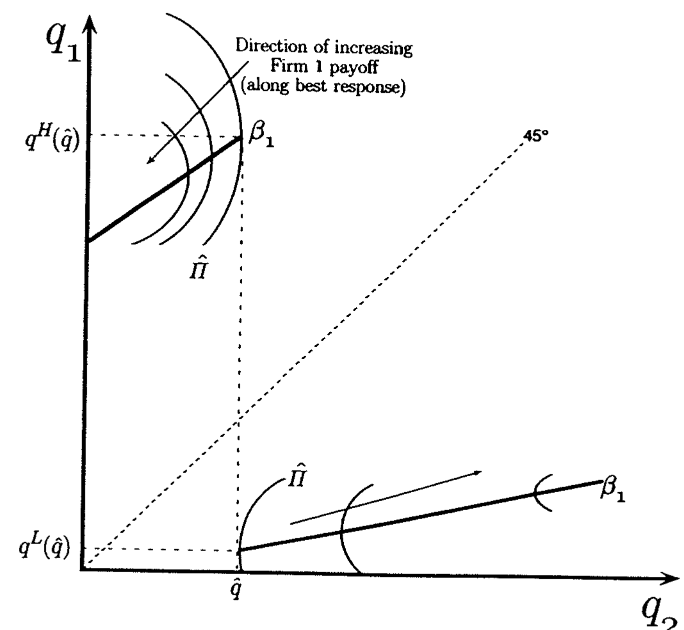

Fig. 2.

we move to the southwest. In contrast, along the lower segment of the best-response function profit increases as we move to the northeast.

3.2. Simultaneous quality choice

By graphing both firms' best-response functions $\beta_i(q_j)$, $i=1,2$ we can solve for the Nash equilibrium in the game with simultaneous quality choices (Fig. 3). A pure strategy Nash equilibrium will be the intersection of the best-response functions.

Lemma 4. In the simultaneous quality choice game, there are two pure strategy equilibria, which are identical except for the identity of the firms.

Existence is shown by proving that the best-reponses correspondences intersect.[5] We will refer to the equilibrium involving $q_i > q_j$ as E_i^{SIM}. In addition, one can show the following (proof in the Appendix),

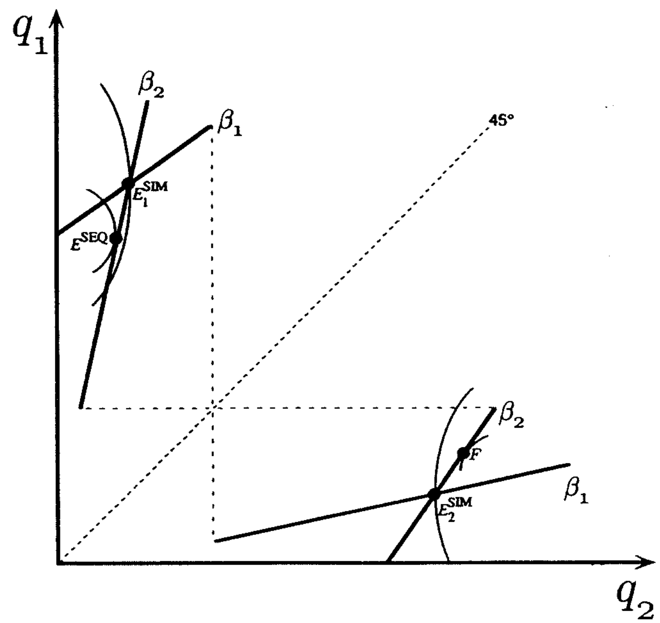

Fig. 3.

[5] The proof of Lemma 4 is not presented here since it is somewhat lengthy and does not shed light on the understanding of the model. The proof is available upon request.

Proposition 1. The firm supplying the high quality product earns higher profit. That is, firm i earns greater profit at the equilibrium E_i^{SIM} than at equilibrium E_j^{SIM}, $j \neq i$, $i, j = 1, 2$.

In other words, because the cost of quality development does not increase too quickly, each firm prefers to supply the high quality product.

3.3. Sequential quality choice

Now consider the subgame perfect Nash equilibrium when firms make quality choices sequentially. Suppose that firm 1 can make its quality choice first and that firm 2 observes firm 1's choice before it makes its own quality choice. Given what it observes, firm 2's choice should satisfy the best-response function. Firm 1 optimizes by choosing the point corresponding to the highest profit along firm 2's best-response curve. In Fig. 3, for $q_1 > q_2$, firm 1 achieves higher profit by moving southwest from E_1^{SIM} along $q_2 = \beta_2(q_1)$ to the point E^{SEQ} until firm 1's iso-profit curve is tangent to firm 2's best-response correspondence. For $q_1 < q_2$, firm 1 achieves higher profit by moving northeast from E_2^{SIM} along $q_2 = \beta_2(q_1)$ to the point F,[6] where again firm 1's iso-profit curve is tangent to firm 2's best-response correspondence. Both F and E^{SEQ} are local maxima for firm 1; however, we can show that E^{SEO} is the global maximum. This discussion can be summarized as

Proposition 2. In the equilibrium of the sequential choice game, the leader will choose to supply the high quality product.

3.4. Comparing equilibria

We begin by comparing the set of qualities offered at the sequential equilibrium, E^{SEO}, and firm 1's preferred simultaneous equilibrium, E_1^{SIM}. Since from the consumers' viewpoint, all that is relevant is the pair of qualities offered and not the identity of the firms supplying the qualities, we find it convenient to focus on the pair of qualities offered at E_1^{SIM}.

Along firm 2's best response correspondence, E^{SEO} lies to the southwest of E_1^{SIM}, implying that sequential choice leads firm 1 to choose a quality level lower than it would if choices were made simultaneously. Moreover, this lower quality induces the rival to also choose a lower quality. In other words,

[6] Of course, either F or E^{SEO} could occur at the "kink" of $\beta_2(q_1)$. Our results do not depend on the tangency of the solution.

Proposition 3. The qualities offered under sequential quality choice are pair-wise lower than the qualities offered under simultaneous quality choice.

In the sequential choice game, firm 1's quality choice directly determines firm 2's quality choice. As firm 1 increases or decreases its quality, firm 2's choice will change in the same direction. Since firm 1 benefits when firm 2 lowers its quality, it chooses to lower its own quality.

Clearly, the profit for firm 1 is greater under sequential choice than under simultaneous choice. From the iso-profit curves, it is easy to see that the profit for firm 2 is lower under sequential quality choice. We can summarize this discussion as

Proposition 4. The firm supplying the superior product earns greater profit under sequential quality choice while the firm supplying the inferior product earns greater profit under simultaneous choice.

Given that one interpretation of sequential vs. simultaneous quality choice is whether or not there is information disclosure, it is natural to ask how our results compare to results that would follow from a model with spillover (Reinganum, 1985). By spillover, we mean that a firm's R&D has externalities for other firms' R&D. For instance in a model with spillover, R&D investments by firm 1 might lower the cost of R&D for firm 2. The result that both firms make smaller quality investments in the sequential move game is consistent with the spillover effect of information disclosure. However, the distributional effects are exactly the opposite. Spillover will make the second firm more efficient at quality investment and thus increase its profit, but the resulting appropriability problem would also imply that the first firm's profit will be smaller. In contrast, we show that when information disclosure mainly serves as a coordination device, disclosure can benefit the first mover.

4. Producer, consumer, and social surplus

We now examine the welfare consequences of the two regimes. We begin with producer surplus (aggregate profit). Despite the assumptions on the functional forms, producer surplus cannot be solved analytically. However, by directly computing the equilibria we can determine the effect of the timing of quality choice on producer surplus.

In Table 1 we present the computed equilibrium qualities and profits. We note that the only parameter in the model is the measure of cost, k. Given our utility function and cost function specifications all the equilibrium values are homogenous of degree -1 in k, the solutions given hold for any k.

Table 1
Equilibrium values under simultaneous and sequential choice (firm 1 moves first)

	Simultaneous choice, E^{SIM_1}	Sequential choice, E^{SEQ}
q_1	$0.126655/k$	$0.122232/k$
q_2	$0.024119/k$	$0.023894/k$
Π_1	$0.012219/k$	$0.012235/k$
Π_2	$0.000764/k$	$0.000757/k$
$\Pi_1+\Pi_2$	$0.012983/k$	$0.012992/k$
Consumer surplus	$0.021609/k$	$0.012016/k$
Social surplus	$0.034592/k$	$0.034008/k$

A comparison reveals that aggregate profit is greater under sequential choice. This suggests that there may be circumstances where both firms will prefer sequential quality choice, even though the profit of the second mover is lower in the sequential move game. For instance, if means of side payments were available, firms would always prefer to compete in the sequential move game.

However, even if side payments were not available, both firms may still prefer the sequential move game.[7] For example, suppose that firm 1 is the first mover with probability p. This might be the case, for instance, if firms discover new product niches at random. In this scenario what is relevant for each firm are the expected profits under sequential and simultaneous quality choices.[8] Since aggregate profit is higher under sequential quality choice it will clearly be the case at $p=0.5$ that both firms prefer the sequential move game. More generally, there exists an interval of probabilities such that both firms prefer the sequential move game. Let Π^{SEQ}_{High} (Π^{SEQ}_{Low}) denote the profit earned by the firm selling the high (low) quality product in the sequential move game; let Π^{SIM}_{High} and Π^{SIM}_{Low} be defined analogously for the simultaneous move game. Firm 1 will prefer the sequential move game if

$$p\Pi^{SEQ}_{High}+(1-p)\Pi^{SEQ}_{Low}>p\Pi^{SIM}_{High}+(1-p)\Pi^{SIM}_{Low},$$

and firm 2 will prefer the sequential move game if

$$(1-p)\Pi^{SEQ}_{High}+p\Pi^{SEQ}_{Low}>(1-p)\Pi^{SIM}_{High}+p\Pi^{SIM}_{Low}.$$

Using the values from Table 1 these conditions imply that for $0.304\leqslant p<0.696$ both firms will prefer the sequential move game.

From the demand system, we can calculate the consumer surplus for any pair of qualities q^H and q^L, where $q^H\geqslant q^L$. Total consumer surplus is the

[7] We thank an anonymous referee for suggesting the following example.

[8] To keep the argument as simple as possible, assume that firm 1 is the high quality firm in the simultaneous move game with probability p.

sum of the surplus from those who buy the high quality (CS^{H}) and from those who buy the lower quality (CS^{L}),

$$CS^{\mathrm{H}}=\frac{2(q^{\mathrm{H}}+q^{\mathrm{L}})(q^{\mathrm{H}})^2}{(4q^{\mathrm{H}}-q^{\mathrm{L}})^2},\quad CS^{\mathrm{L}}=\frac{q^{\mathrm{L}}(q^{\mathrm{H}})^2}{2(4q^{\mathrm{H}}-q^{\mathrm{L}})^2}.$$

The total consumer surplus (CS) is,

$$CS=\frac{(q^{\mathrm{H}})^2(4q^{\mathrm{H}}+5q^{\mathrm{L}})}{2(4q^{\mathrm{H}}-q^{\mathrm{L}})^2}. \tag{9}$$

Using the computed equilibria from Table 1, we can evaluate (9) for the two regimes. As seen, consumer surplus is higher under simultaneous choice. We also present an analytic proof of this calculation result to highlight the tradeoffs between quality and price competition determining consumer surplus. The analysis proceeds by deriving the iso-consumer surplus curves.

The slope of the iso-consumer surplus curves in the q^{H}–q^{L} space is given by

$$\left.\frac{dq^{\mathrm{H}}}{dq^{\mathrm{L}}}\right|_{dCS=0}=-\frac{q^{\mathrm{H}}(28q^{\mathrm{H}}+5q^{\mathrm{L}})}{2(2q^{\mathrm{H}}+q^{\mathrm{L}})(4q^{\mathrm{H}}-5q^{\mathrm{L}})}\gtreqless 0 \Leftrightarrow q^{\mathrm{H}}\lesseqgtr\frac{5}{4}q^{\mathrm{L}}. \tag{10}$$

This expression follows from partially differentiating (9) with respect to q^{L} and q^{H},

$$\frac{\partial CS}{\partial q^{\mathrm{L}}}=\frac{(q^{\mathrm{H}})^2(28q^{\mathrm{H}}+5q^{\mathrm{L}})}{2(4q^{\mathrm{H}}-q^{\mathrm{L}})^3}>0, \tag{11}$$

$$\frac{\partial CS}{\partial q^{\mathrm{H}}}=\frac{q^{\mathrm{H}}(2q^{\mathrm{H}}+q^{\mathrm{L}})(4q^{\mathrm{H}}-5q^{\mathrm{L}})}{(4q^{\mathrm{H}}-q^{\mathrm{L}})^3}\gtreqless 0 \Leftrightarrow q^{\mathrm{H}}\gtreqless\frac{5}{4}q^{\mathrm{L}}. \tag{12}$$

Eq. (11) implies that an increase in the quality of the inferior product (i.e. distance between qualities is reduced) increases consumer surplus. This is not only because consumers benefit from a better product but also because there is greater competition between the two products. The effect on consumer surplus from an increase in the level of the higher quality product (i.e. greater distance) is a bit more intricate (Eq. (12)). First, if the two qualities are initially sufficiently close ($q_1<5q_2/4$), there is significant competition to begin with and the loss in consumer surplus from the reduction in competition outweighs any gain from the availability of a better product. Second, if the two qualities are so far apart ($q_1>5q_2/4$) that there is little competition to begin with, the gain from a better product offsets the

negative effect from further reduction in competition. In Fig. 4 we depict some of the iso-consumer surplus curves.

In order to compare consumer surplus in the two regimes we need to determine in which region the equilibria are located (i.e. above or below $q_1=5q_2/4$). Straightforward substitution yields $\partial\Pi_2(q_1,q_2)/\partial q_2<0$ along $q_1=5q_2/4$, for all k. This implies that firm 2's best response function, β_2, must lie to the left of this line. In this region, consumer surplus decreases as one moves southwest, and therefore that consumer surplus is greater under simultaneous choice than under sequential choice.

Finally, we can calculate the overall effect of the timing of quality choice. The social surplus, defined as the sum of aggregate profit and consumer surplus, is shown in Table 1. The loss to consumers outweighs the potential gain to producers, which means that social surplus is greater under simultaneous choice than under sequential choice.

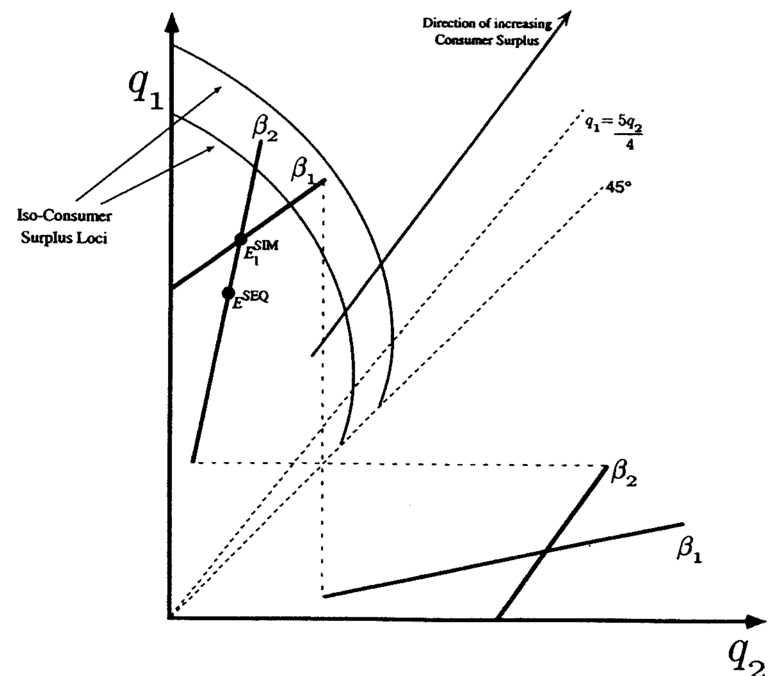

Fig. 4.

5. Concluding comments

With respect to the generality of our results, although our model assumed a particular demand system, the key result–that the first mover prefers to reduce its quality–depends on only a few properties of the payoff functions. The first key property is that, in equilibrium, firms prefer to sell the superior product (Propositions 1 and 2); the second key condition concerns the upward sloping best response correspondences.

The first condition is a very general property and seems likely to hold for a wide class of demand and cost specifications. A stronger requirement, that the profit of the higher quality firm is greater than that of the lower quality firm for any pair of qualities, not only would imply our results but will hold for a broad class of demand specifications, i.e. as long as cost of quality does not rise too quickly.

How would our results change if this condition did not hold? For instance, suppose the cost of marginal quality investment is very high. In this case, the first mover might prefer to precommit to be the low quality firm, i.e. the equilibrium would be the point F in Fig. 3. How the qualities at this equilibrium compare with those at the simultaneous choice equilibrium depends on which simultaneous choice equilibrium is preferred. If E_1^{SIM} is preferred, then even if the first mover chooses F, the first mover still makes smaller quality investment in the sequential game. However, the first firm's quality investment is so small that the follower becomes the firm with higher quality. If E_2^{SIM} is preferred, then in the sequential game both firms choose higher qualities. Note that the factor that would lead the firm to prefer E_2^{SIM} is the same factor that leads the firm to prefer F: the higher cost of marginal quality. Thus, for industries where cost of quality improvement is high relative to revenue, sequential quality choice may lead to higher quality investments. We would like to stress, however, that the situation that we have modeled (i.e. where cost does not increase too quickly) is in fact the interesting case since in the alternative scenario, the effect of the timing of quality choice is minimal since firms choose to make very small R&D investments in either case.

The second key condition, the upward sloping best-response correspondence, is influenced by the nature of the second-stage competition: price or quantity. Our comparison of the sequential and simultaneous choice games is primarily determined by the *strategic* substitutability or complementarity of qualities, which depends on the nature of competition in the second stage. In our model qualities are strategic complements because there is Bertrand competition in the sales stage. On the other hand, if the sales stage is characterized by Cournot competition, it can be shown that qualities are strategic complements for the high quality firm but strategic substitutes for the low quality firm (Aoki, 1988). Thus, in this game the comparison of the sequential and simultaneous choice games depends on whether the first mover prefers to sell the high quality product. Aoki (1988) has shown that the first mover will choose to sell the high quality product. Since the first mover chooses along the second mover's best response correspondence (which exhibits strategic substitutability) the first mover will increase its quality in order to reduce the quality of the second mover. That is with Cournot competition, in the simultaneous choice game, the equilibrium quality of the superior product is higher and the equilibrium quality of the inferior product is lower relative to the equilibrium in the simultaneous choice game. Thus as this discussion suggests, the relationship between quality choice and the sequencing of moves is sensitive to the nature of competition at the sales stage.

The common approach in models of vertical quality differentiation has been to assume that the marginal cost of production is zero. We have followed this approach also. Obviously, our results will hold when the marginal cost of production is a positive but small constant. Another possibility not considered here is when the marginal cost of production varies with quality. In our model the marginal cost of quality itself varies but marginal cost of production is independent of quality, namely zero. If the marginal production cost depends on quality, one would expect it to be increasing in quality. In other words, a higher quality product costs more per unit to produce, due to the necessity of higher quality materials or higher skilled workers. However, as long as the marginal production cost does not increase too much relative to product improvement, our results should still hold. If the marginal production cost rises too much relative to product improvement, such innovations will not be undertaken.[9]

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[9] Arrow (1962) distinguishes between product and process innovations. In our model R&D results in a better product–a product innovation. If the marginal production cost depends on quality, R&D would also have the characteristic of a process innovation. In fact if better product is associated with higher production cost, R&D would result in a product innovation and a process de-innovation.

Appendix

A.1. Proof of Lemmas 1–3

At the second stage it is convenient to characterize the equilibrium in terms of high and low quality; to avoid confusion, we use superscripts to denote the level of relative quality, i.e. high or low, and subscripts to denote the identity of the firm, i.e. firm 1 or 2.

Let p^{L} and p^{H} denote the prices for the low and high quality products and let $\rho^{\mathrm{H}}(p^{\mathrm{H}},p^{\mathrm{L}})$ and $\rho^{\mathrm{L}}(p^{\mathrm{H}},p^{\mathrm{L}})$ denote the second-stage revenue function (as a function of prices) for the high and low quality firm. Using the definition of $v_i(\cdot)$ and the uniform distribution of consumers, these functions can be expressed as

$$\rho^{\mathrm{H}}(p^{\mathrm{H}},p^{\mathrm{L}})=\begin{cases} p^{\mathrm{H}}\left(1-\dfrac{p^{\mathrm{H}}}{q^{\mathrm{H}}}\right) & \text{if } p^{\mathrm{H}}\leqslant\min\left\{q^{\mathrm{H}},\dfrac{q^{\mathrm{H}}p^{\mathrm{L}}}{q^{\mathrm{L}}}\right\},\\ p^{\mathrm{H}}\left(1-\dfrac{p^{\mathrm{H}}-p^{\mathrm{L}}}{q^{\mathrm{H}}-p^{\mathrm{H}}}\right) & \text{if } \min\left\{q^{\mathrm{H}},\dfrac{q^{\mathrm{H}}p^{\mathrm{L}}}{q^{\mathrm{L}}}\right\}<p^{\mathrm{H}}<\min\{q^{\mathrm{H}},q^{\mathrm{H}}-q^{\mathrm{L}}+p^{\mathrm{L}}\},\\ 0 & \text{if } p^{\mathrm{H}}\geqslant\min\{q^{\mathrm{H}},q^{\mathrm{H}}-q^{\mathrm{L}}+p^{\mathrm{L}}\}. \end{cases} \tag{A1}$$

and

$$\rho^{\mathrm{L}}(p^{\mathrm{H}},p^{\mathrm{L}})=\begin{cases} p^{\mathrm{L}}\left(1-\dfrac{p^{\mathrm{L}}}{q^{\mathrm{L}}}\right) & \text{if } p^{\mathrm{L}}\leqslant\min\{q^{\mathrm{L}},q^{\mathrm{L}}-q^{\mathrm{H}}+p^{\mathrm{H}}\},\\ p^{\mathrm{H}}\left(\dfrac{p^{\mathrm{H}}-p^{\mathrm{L}}}{q^{\mathrm{H}}-p^{\mathrm{H}}}-\dfrac{p^{\mathrm{H}}}{q^{\mathrm{H}}}\right) & \text{if } \min\{q^{\mathrm{L}},q^{\mathrm{L}}-q^{\mathrm{H}}+p^{\mathrm{H}}\}<p^{\mathrm{L}}<\min\left\{q^{\mathrm{L}},\dfrac{q^{\mathrm{L}}p^{\mathrm{H}}}{q^{\mathrm{H}}}\right\},\\ 0 & \text{if } p^{\mathrm{L}}\geqslant\min\left\{q^{\mathrm{L}},\dfrac{q^{\mathrm{L}}p^{\mathrm{H}}}{q^{\mathrm{H}}}\right\}. \end{cases} \tag{A2}$$

Note that the revenue functions include the possibility that only the high- or low-type firm supplies the market. Differentiating Eqs. (A1) and (A2) and solving gives the equilibrium demand, x^{H} and x^{L}, and prices, p^{H} and p^{L},

$$x^{\mathrm{H}}(q^{\mathrm{H}},q^{\mathrm{L}})=\frac{2q^{\mathrm{H}}}{4q^{\mathrm{H}}-q^{\mathrm{L}}},\quad p^{\mathrm{H}}(q^{\mathrm{H}},q^{\mathrm{L}})=\frac{2q^{\mathrm{H}}(q^{\mathrm{H}}-q^{\mathrm{L}})}{4q^{\mathrm{H}}-q^{\mathrm{L}}},$$

$$x^{\mathrm{L}}(q^{\mathrm{H}},q^{\mathrm{L}})=\frac{q^{\mathrm{H}}}{4q^{\mathrm{H}}-q^{\mathrm{L}}},\quad p^{\mathrm{L}}(q^{\mathrm{H}},q^{\mathrm{L}})=\frac{q^{\mathrm{L}}(q^{\mathrm{H}}-q^{\mathrm{L}})}{4q^{\mathrm{H}}-q^{\mathrm{L}}}.$$

The equilibrium sales revenue (for the second stage) for the high and low quality firms can be expressed as functions of quality choices alone,

$$r^{\mathrm{H}}(q^{\mathrm{H}},q^{\mathrm{L}})=\frac{4(q^{\mathrm{H}})^2(q^{\mathrm{H}}-q^{\mathrm{L}})}{(4q^{\mathrm{H}}-q^{\mathrm{L}})^2}, \tag{A3}$$

$$r^{\mathrm{L}}(q^{\mathrm{H}},q^{\mathrm{L}})=\frac{q^{\mathrm{H}}q^{\mathrm{L}}(q^{\mathrm{H}}-q^{\mathrm{L}})}{(4q^{\mathrm{H}}-q^{\mathrm{L}})^2}. \tag{A4}$$

Lemma 1 follows directly from (A3) and (A4).

Lemma 2 and Eqs. (5) and (6) follow directly from differentiating (A3) and (A4).

Lemma 3 follows from the following characterization of the two local maxima,

$$q^{\mathrm{L}}(q_j)=\{q_i\in\mathbb{R}\,|\,q_j^2(4q_j-7q_i)-2kq_i(4q_j-q_i)^3=0,\ q_i<q_j\}, \tag{A5}$$

$$q^{\mathrm{H}}(q_j)=\{q_i\in\mathbb{R}\,|\,16q_i^2-12q_iq_j+8q_j^2-2k(4q_i-q_j)^3=0,\ q_i\geqslant q_j\}. \tag{A6}$$

□

A.2. Proof of Proposition 1

Let $q^{\mathrm{H}*}$ denote the high quality firm's quality choice (i.e. firm 1's quality at E_1^{SIM} and firm 2's quality at E_2^{SIM}). Let $q^{\mathrm{L}*}$ denote the low quality firm's quality choice (i.e. firm 1's quality at E_2^{SIM} and firm 2's quality at E_1^{SIM}).

We show that

$$\Pi_1(q^{\mathrm{H}*},q^{\mathrm{L}*})>\Pi_1(q^{\mathrm{L}*},q^{\mathrm{H}*}). \tag{A7}$$

Given the explicit functional forms of the revenue and cost functions, it is easy to show that

$$\frac{1}{8k}<q^{\mathrm{H}*}<\frac{7}{48k},\quad \frac{1}{64k}<q^{\mathrm{L}*}<\frac{1}{32k}.$$

From the relation (2) and optimality of $q^{\mathrm{H}}(\cdot)$, we have the following inequalities,

$$\Pi_1(q^{\mathrm{H}*},q^{\mathrm{L}*})=\Pi_1(q^{\mathrm{H}}(q^{\mathrm{L}*}),q^{\mathrm{L}*})>\Pi_1\left(q^{\mathrm{H}}\left(\frac{1}{32k}\right),q^{\mathrm{L}*}\right) \tag{A8}$$

$$>\Pi_1\left(q^{\mathrm{H}}\left(\frac{1}{32k}\right),\frac{1}{32k}\right)\geqslant\Pi_1\left(\frac{1}{8k},\frac{1}{32k}\right)=\frac{53}{4800k}. \tag{A9}$$

Using optimality of $q^{L}(\cdot)$, (3), and (4), we find the upper bound of $\Pi_1(q^{L*},q^{H*})$.

$$\Pi_1(q^{L*},q^{H*}) < \Pi_1\left(q^{L*},\frac{7}{48k}\right) < \Pi_1\left(q^{L}\left(\frac{7}{48k}\right),\frac{7}{48k}\right) < r^{L}\left(\frac{7}{48k},q^{L}\left(\frac{7}{48k}\right)\right) < r^{L}\left(\frac{7}{48k},\frac{1}{32k}\right) = \frac{77}{44944k}. \tag{A10}$$

The last inequality follows from (4) and $1/32k<(4/7)(7/48k)$. From $53/4800k-77/44944k>0$ we have inequality (A7). □

A.3. Proof of Proposition 2

We need to show that the payoff for firm 1 at the local optimum $E^{SEO}=(q_1^{SEO},q_2^{SEO})$ is greater than at the local optimum $F=(q_1^F,q_2^F)$. At E^{SEO}, firm 1 is the higher quality firm while at F it is the lower quality firm, i.e. $q_1^{SEQ}>q_2^{SEQ}$ and $q_1^F<q_2^F$. We will show that

$$\Pi_1(q_1^{SEO},q_2^{SEO}) > \Pi_1(q_1^F,q_2^F). \tag{A11}$$

By definition of E^{SEQ},

$$\Pi_1(q_1^{SEO},q_2^{SEO}) > \Pi_1(q^{H*},q^{L*}).$$

Thus the lower bound we attained in (A9) is also the lower bound for $\Pi_1(q_1^{SEO},q_2^{SEO})$.

We will now find the upper bound of $\Pi_1(q_1^F,q_2^F)$. First we note that because F is on firm 2's best-response function, $q_2^F<7/48k$. Profit at F must be positive for firm 1 and thus the point must lie below the zero iso-profit curve which implies $q_1^F<1/16k$. We are now able to show the following inequalities,

$$\Pi_1(q_1^F,q_2^F) < \Pi_1\left(q_1^F,\frac{7}{48k}\right) < r^{L}\left(\frac{7}{48k},q_1^F\right) < r^{L}\left(\frac{7}{48k},\frac{1}{16k}\right) = \frac{7}{2500k}.$$

The first inequality follows from (3). The third inequality follows from the fact that $r^{L}(\cdot)$ is increasing in q^{L} for $q_1^F<(4/7)(7/48k)$ from (4). Since $53/4800k-7/2500k=989/12000k>0$, we have shown the inequality (A11). □

References

Aoki, Reiko, 1988, Cournot and Bertrand competition with vertical quality differentiation, Mimeo.

Arrow, K.J., 1962, Economic welfare and the allocation of resources for invention, in: R. Nelson, ed., The rate and direction of inventive activity (Princeton University Press, Princeton, NJ).

Gabszewicz, J. and J.-F. Thisse, 1979, Price competition, Quality and income distribution, Journal of Economic Theory 20, 340–359.

Grabowski, H.G. and J.M. Vernon, 1983, The regulation of pharmaceuticals: Balancing the benefits and risks, American Institute for Public Policy Research, Washington, DC.

Kreps, David M., 1991, A course in microeconomic theory (Princeton University Press, Princeton, NJ).

Motta, Massimo, 1993, Endogenous quality choice: Price vs. quantity competition, Journal of Industrial Economics 41, 113–132.

Reinganum, J.F., 1985, A two-stage model of research and development with endogenous second-mover advantages, International Journal of Industrial Organization 3, 275–292.

Shaked, A. and J. Sutton, 1982, Relaxing price competition through product differentiation, Review of Economic Studies 49, 3–13.

Tirole, Jean, 1988, The theory of industrial organization (MIT Press, Cambridge, MA).

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