Solution: Econ 322 Problem Set 2

1. (a) (i) $\hat{\beta}_1 = \dfrac{Cov(\widehat{C_t, Y_t})}{V\widehat{ar(Y_t)}} = \dfrac{40}{200} = 0.2$ $\hat{\beta}_0 = \bar{C} - \hat{\beta}_1 \bar{Y} = 30,000 - (0.2)40,000 = 22,000$
$\widehat{\eta_{CY}} = \hat{\beta}_1(\bar{Y}/\bar{C}) = (0.2)\frac{40,000}{30,000} = \frac{4}{15}$

   (ii) MPC $= \hat{\beta}_1$

   (iii) Yes, $\hat{\beta}_0 > 0$, it means that if $Y_t = 0$, then $C_t = 22{,}000$. That is, we still have to consume even if we have no income.

   (iv) $\hat{\beta}_1 = \frac{60}{200} = 0.3$
$\hat{\beta}_0 = 30,000 - (0.3)40,000 = 18,000$

   (b) $R^2 = \hat{\rho}_{cy}^2 = (\dfrac{Cov(\widehat{C_t, Y_t})}{\sqrt{V\widehat{ar(C_t)}}\sqrt{V\widehat{ar(Y_t)}}})^2 = \dfrac{40^2}{(60)(200)} = \dfrac{2}{15}$

   (c) R^2 is a measure of goodness of fit, which tells how well the sample regression line fits the data.

   (d) $\hat{\sigma}^2$=20, $\hat{\sigma}^2_{\hat{\beta}_1} = \dfrac{\hat{\sigma}^2}{(T-1)\hat{\sigma}^2_Y} = \dfrac{20}{(50)(200)} = \dfrac{20}{10000}$=0.002. *Important Note: In the solution, I have used the correct formula for* $\sigma^2_{\hat{\beta}_1}$. *In the problem set, an incorrect formula was given. For purposes of grading the assignment, whichever one you use will be assumed fine, although in future, and as written in class, the true estimator is* $\sigma^2_{\hat{\beta}_1} = \frac{\sigma^2}{(T-1)\hat{\sigma}^2_Y}$ or alternatively you can use $\sigma^2_{\hat{\beta}_1} = \frac{\sigma^2}{T\widehat{\sigma^2_Y}}$.
$H_0 : \beta_1 = 0.5$, $H_1 : \beta_1 \neq 0.5$,
$Z = \dfrac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} = \dfrac{0.2 - 0.5}{\sqrt{0.002}}$ = -6.711 < -1.96 $= -Z_{\frac{\alpha}{2}}$, Hence, we reject H_0.

   (e) $H_0 : \beta_1 = -0.1$, $H_1 : \beta_1 \neq -0.1$,
$Z = \dfrac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} = \dfrac{0.2 - (-0.1)}{\sqrt{0.002}} = 6.711 < 1.96 = Z_{\frac{\alpha}{2}}$, Hence, we reject H_0.

   (f) $H_0 : \beta_1 = 0$, $H_1 : \beta_1 > 0$,
At a 5% significant level, $Z = \dfrac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} = \dfrac{0.2 - 0}{\sqrt{0.002}} = 4.474 < 1.64 = Z_\alpha$, Hence, we reject H_0.

   (g) $C_t = \beta_0 + \beta_1 Y_t + u_t$,
$\overline{C} = \beta_0 + \beta_1 \overline{Y}$,

$C_t - \overline{C} = \beta_1(Y_t - \overline{Y}) + u_t$

$\hat{\beta}_1 = \dfrac{\sum(C_t - \overline{C})(Y_t - \overline{Y})}{\sum(Y_t - \overline{Y})^2} = \dfrac{\sum(\beta_1(Y_t - \overline{Y}) + u_t)(Y_t - \overline{Y})}{\sum(Y_t - \overline{Y})^2}$

$= \dfrac{\beta_1 \sum(Y_t - \overline{Y})^2}{\sum(Y_t - \overline{Y})^2} + \dfrac{\sum(Y_t - \overline{Y})u_t}{\sum(Y_t - \overline{Y})^2} = \beta_1 + \dfrac{\sum(Y_t - \overline{Y})u_t}{\sum(Y_t - \overline{Y})^2}.$

Bias of $\hat{\beta}_1 = E(\hat{\beta}_1) - \beta_1 = E(\beta_1 + \dfrac{\sum(Y_t - \overline{Y})u_t}{\sum(Y_t - \overline{Y})^2}) - \beta_1 = \beta_1 + E(\dfrac{\sum(Y_t - \overline{Y})u_t}{\sum(Y_t - \overline{Y})^2}) - \beta_1$

$= \dfrac{\sum(Y_t - \overline{Y})E(u_t)}{\sum(Y_t - \overline{Y})^2} = 0.$

(h) $P[-t^*_{0.05}(50) \le t \le t^*_{0.05}(50)] = 90\%$

$t = \dfrac{\hat{\beta}_1 - \beta 1}{\hat{\sigma_{\beta_1}}} = \dfrac{0.2 - \beta_1}{\sqrt{0.25}} = \dfrac{0.2 - \beta_1}{0.5}$

$-1.68 \le \dfrac{0.2 - \beta_1}{0.5} \le 1.68$

therefore, by solving β_1, we can get $0.2 - (0.5)(1.68) \le \beta_1 \le 0.2 + (0.5)(1.68)$