Professor N.R. Swanson – Economics 322 – Rutgers University

Problem Set 1

1.

(a) Prove that var$(X) = E(X^2) - \mu_X^2$. What is var$(X+Y)$? What is var(c), c is a constant?

(b) If you don't know the true population data, but are given a 'sample' of data, what estimator of the population variance could you construct? What about the population covariance?

(c) Say that the average wage of a college graduate in economics is $\overline{W} = \frac{1}{T}\sum_{i=1}^{T} W_i$, where W_i is the wage of individual i. Say that μ_W is the true average wage. What is the difference between W_i, $\overline{W}$, and μ_W? What is the difference between μ_X and $\overline{X}$? Why is this difference relevant to the examination of economic data?

(d) What is Cross Sectional Data? Time Series Data? Give 2 examples of each and discuss.

(e) Where does econometrics "fit in" in economics?

(f) Write down an example of a discrete PDF. What are three properties of a continuous PDF, say f(x)?

2.

Given the following data on state approp. and tuition and fees, calculate [X = St.App.,Y = tuition and fees]

$$\overline{X} = \frac{1}{T}\sum_{t=76-77}^{93-94} X_t,$$

where T is the total number of observations. Also, calculate $\overline{Y}, \widehat{cov}(X,Y), \widehat{var}(X), \widehat{var}(Y)$.

3.

(a) Define the correlation coefficient, ρ_{XY}, for two RVs X and Y.

(b) If we only have a "sample" rather than "all" of the population data for X and Y, how could we construct an estimator of ρ, say $\widehat{\rho}$ (or r)? Write down an expression for $\widehat{\rho}$ given a sample of N observations.

(c) What are the "boundary" values that ρ_{XY} can take? If $\rho_{XY} = 0$, what does this mean? Draw a picture illustrating your argument.

(d) Calculate $\widehat{\rho}$, the sample correlation coefficient between state appropriation and tuition and fees.

(e) What can you hypothesize concerning the 'relationship' between the variables, given the value of $\widehat{\rho}$ which you have calculated?

4.

Assume $\overline{X} \sim N\left(\mu_X, \sigma^2_{\overline{X}}\right)$

Let

$H_o : \mu^*_X = 2$

$H_1 : \mu^*_X \neq 2$

(a) Use the Z-test to check whether to reject H$_o$ or not, when $\overline{X} = 1.5, \sigma^2_{\overline{X}} = 0.16$, at a 10% level of significance.

(b) What is the "level" of the test in a)? What does this mean? Draw a picture to show the level of the test.

(c) Do the same test one-sided

$H_o : \mu^*_X = 2$ with $\alpha = 0.10$.

$H_1 : \mu^*_X < 2$ i.e., at a 10% significance level.

(d) Repeat (c), but for $H_1 : \mu^*_X > 2$. Do the results agree? Why or why not?

5.

What is the difference between the t and Z distributions? When do we use t instead of Z for tests of the type given in (4)?

Sources of Income for a University
As a Percent of the General Funds Budget (in Millions $)

	State Appropriation X		Tuition & Fees Y		Other Income		Total Income
76-77	103	54%	72	38%	13	7%	190
77-78	103	53%	77	40%	13	7%	195
78-79	109	53%	81	40%	12	6%	203
79-80	117	53%	85	39%	16	8%	218
80-81	124	52%	97	41%	17	7%	238
81-82	130	50%	114	44%	16	6%	261
82-83	137	47%	136	47%	17	6%	290
83-84	142	46%	150	48%	20	6%	313
84-85	155	45%	165	48%	21	6%	342
85-86	164	45%	178	49%	24	7%	367
86-87	174	44%	193	49%	27	7%	395
87-88	188	43%	215	49%	31	7%	434
88-89	199	41%	254	52%	32	7%	486
89-90	217	41%	277	52%	38	7%	533
90-91	230	40%	298	52%	46	8%	575
91-92	235	39%	323	53%	49	8%	608
92-93	227	36%	345	55%	53	9%	626
93-94	236	36%	358	55%	54	9%	649