THE REVIEW OF ECONOMICS AND STATISTICS

THE STABILITY OF MONEY DEMAND AND FORECASTING THROUGH CHANGES IN REGIMES

Bruce Mizrach and Anthony M. Santomero*

Abstract—The stability of the Goldfeld money demand equation is examined using a technique due to Brown, Durbin and Evans. We find a structural break in this equation in the early 1960s. Further tests isolate four regimes of monetary behavior over the period 1959:2 to 1983:4. Forecasts into the fourth regime, 1980:1–1983:4, from the first three regimes and two time series models are shown to perform rather poorly. Combinations of these forecasts using the procedure proposed by Granger and Ramanathan provide superior estimates, revealing that there is useful information in several of the previous regimes.

It is now more than ten years after Goldfeld’s (1973) synthesis of the money demand literature. That article contributed a consensus specification of this important function that was both consistent with the transactions demand framework, and also seemed to track the data well. While some economists continued to make cases for alternative specifications, the notion that real money demand depended on some form of income or transactions term and the related opportunity costs of holding non-interest-bearing assets was widely accepted. In a quarterly equation, this acceptance certainly attained the status of consensus.

As is well known, this consensus was short-lived. Merely three years later Goldfeld (1976) felt compelled to write that his equation began to systematically overpredict the amount of real money balances actually held by the public. A vast literature has developed since that time to suggest where the “missing money” might be found. This literature has not come close to reforming the consensus of 1973.

In this paper we offer some insight as to why this has been the case. Using formal tests for the stability of regression coefficients over time, it is shown that a structural break occurred in the Goldfeld quarterly money demand specification as early as 1962:3. In all, four regimes are isolated over the period of 1959:2 to 1983:4.

This rather disconcerting finding about the instability of money demand calls into question our ability to forecast this important time series using simple single equation behavioral models. In this regard, our results are consistent with those of Cooley and Leroy (1981). However, we attempt to address the issue of efficient estimation in such an unstable environment. A method of combining forecasts from the pre- and post-break models which offers some hope of producing more efficient forecasts, is employed, and a statistically optimal forecast of short-term real money demand is estimated. We show that this far outperforms forecasts obtained from any of the individually estimated functions used heretofore.

I. The Traditional Specification of the Demand for Money Function

As a preliminary step to this work, more than 50 of the trial specifications tested by Goldfeld in his 1973 article were estimated using data up to 1983.4. Searching for any anomalies, yet sensitive to the criticisms of data mining by Cooley and Leroy (1981), we chose to utilize Goldfeld’s “conventional” equation for all subsequent tests, with one minor exception, substituting the passbook savings’ rate for Goldfeld’s constructed time-deposit rate series.

Data names and definitions, taken from the quarterly data banks of the MPS model (in logs) are:

\[ M1 \]—Quarterly average \( M1 \), billions of current dollars.\(^2\)

\[ PGNP \]—Implicit price deflator for gross national product, 1972 = 100.

\[ m1 \]—Real money balances, \( M1/PGNP \).

\[ XGNP \]—Real gross national product, 1972$.

\[ RCP \]—Commercial paper rate (4 to 6 months).

\[ RSLP \]—Rate on savings and loan passbook deposits.

Following the stock-adjustment model, we include a lagged dependent variable in the specification:

\[ m^* = f(r, y) \] desired money holdings \hspace{1cm} (1)

\[ m_t - m_{t-1} = \gamma(m^*_t - m_{t-1}) \] adjustment to desired level \hspace{1cm} (2)

\[ m1 = \alpha_0 + \alpha_1 m1_{t-1} + \alpha_2 XGNP + \alpha_3 RCP + \alpha_4 RSLP + \mu_t \] \hspace{1cm} (3)

Point estimates are thus impact elasticities, the coefficient on the lagged money stock is one minus the speed

\(^2\) Roley (1984) suggests the use of end-of-quarter \( M1 \), rather than the quarterly average, because of a reduction in serial correlation. In our specification, though, the \( H \)-statistic jumps to 2.81 with end-of-quarter data.
of adjustment; and long-run elasticities are the point estimates over the speed of adjustment.

In ordinary least squares (OLS) estimation of (3), Durbin's $h$-statistic confirmed the presence of serial correlation. While Goldfeld corrected for autocorrelation using the Cochrane-Orcutt procedure, it has since been shown by Betancourt and Kelejian (1981) that this iterative technique can generate inconsistent estimates. A maximum-likelihood estimator due to Beach and McKinnon (1978) is instead employed. Results for the "conventional" equation estimated through the end of Goldfeld's sample, 1972:4, and 1983:4 are reported as equations 1 and 2 of table 1. As a check for robustness, Hildreth-Liu grid search regressions are also given for both sample periods, table 1, equations 3 and 4.

An initial source of concern is the passbook savings variable which becomes insignificant in the most recent sample period. This fate, however, befall all the other feasible rates paired with the commercial paper rate. In all these cases, the commercial paper rate retained its significance.

The other source of concern is perhaps more serious. The coefficient on the lagged money stock increases over time, which suggests a slowing of the speed of adjustment of actual money holdings to the desired quantity. This slowing appears to be counterintuitive in light of the recent advances in financial trading. However, it should be noted that estimated interest-rate coefficients trend in the same direction of the speed of adjustment, resulting in a somewhat smaller change in its long-run elasticity.

II. Tests for Stability

A. Analyzing the Residual Pattern

When examining the stability of the structural relationships in our money demand equation, the analysis cannot center upon the characteristics of the OLS residuals because, in general, they are not independent and distributed $N(0, \sigma^2)$. A highly powerful method for transforming the residuals into a form in which departures from structural constancy could be assessed was first proposed by Brown, Durbin, and Evans (1975), hereafter BDE, using what is known as the recursive residuals.

The recursive residuals are generated by successive estimation of the model, extending the end of the sample by one period each time. BDE propose two tests of these residuals. The first, called the CUSUM, sums the sequence of standardized one-period ahead forecast errors. It is designed to capture systematic under- or overprediction. It is susceptible to large but cancelling errors though. For this reason, the sum of the squared residuals, the CUSUMSS, is also examined. Monte Carlo evidence on the power of the tests may be found in Garbade (1977).

There is an additional complication in our application of this technique of having a non-scalar covariance matrix:

$$\mu \sim N(0, \sigma^2 \Sigma)$$

where $\Sigma$ is a $T \times T$ positive-definite matrix, and if

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant</th>
<th>Lagged Money $m_{t-1}$</th>
<th>$GNP$</th>
<th>$XGNP$</th>
<th>Commercial Paper $RCP$</th>
<th>Saving Rate $RSLP$</th>
<th>$\rho$</th>
<th>$\bar{R}^2$</th>
<th>OLS $H$-statistic</th>
<th>Estimated Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m_{t}$</td>
<td>1.033</td>
<td>0.573</td>
<td>0.238</td>
<td>-0.013</td>
<td>-0.155</td>
<td>0.316</td>
<td>0.990</td>
<td>3.21</td>
<td>1959:2-</td>
<td>1972:4</td>
</tr>
<tr>
<td></td>
<td>(5.34)</td>
<td>(5.41)</td>
<td>(5.09)</td>
<td>(3.19)</td>
<td>(3.94)</td>
<td>(2.34)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. $m_{t}$</td>
<td>0.243</td>
<td>0.950</td>
<td>0.055</td>
<td>-0.019</td>
<td>-0.043</td>
<td>0.201</td>
<td>0.956</td>
<td>1.95</td>
<td>1959:2-</td>
<td>1983:4</td>
</tr>
<tr>
<td></td>
<td>(2.99)</td>
<td>(32.56)</td>
<td>(2.79)</td>
<td>(4.73)</td>
<td>(1.10)</td>
<td>(2.00)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. $m_{t}$</td>
<td>-1.098</td>
<td>0.566</td>
<td>0.256</td>
<td>-0.013</td>
<td>-0.192</td>
<td>0.300</td>
<td>0.988</td>
<td>4.01</td>
<td>1959:2-</td>
<td>1972:4</td>
</tr>
<tr>
<td></td>
<td>(5.54)</td>
<td>(5.39)</td>
<td>(5.28)</td>
<td>(3.23)</td>
<td>(2.31)</td>
<td>(2.31)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. $m_{t}$</td>
<td>-0.259</td>
<td>0.950</td>
<td>0.061</td>
<td>-0.019</td>
<td>-0.057</td>
<td>0.200</td>
<td>0.955</td>
<td>4.01</td>
<td>1959:2-</td>
<td>1983:4</td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
<td>(32.48)</td>
<td>(2.77)</td>
<td>(4.67)</td>
<td>(1.24)</td>
<td>(2.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. $m_{t}$</td>
<td>-0.340</td>
<td>0.647</td>
<td>0.119</td>
<td>-0.024</td>
<td>-0.117</td>
<td>-0.705</td>
<td>0.999</td>
<td>-3.61</td>
<td>1959:2-</td>
<td>1962:3</td>
</tr>
<tr>
<td></td>
<td>(1.51)</td>
<td>(3.89)</td>
<td>(2.07)</td>
<td>(6.15)</td>
<td>(2.98)</td>
<td>(2.93)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. $m_{t}$</td>
<td>-1.095</td>
<td>0.527</td>
<td>0.251</td>
<td>-0.013</td>
<td>-0.151</td>
<td>0.293</td>
<td>0.984</td>
<td>4.73</td>
<td>1962:4-</td>
<td>1973:4</td>
</tr>
<tr>
<td></td>
<td>(3.89)</td>
<td>(3.78)</td>
<td>(3.83)</td>
<td>(2.66)</td>
<td>(2.75)</td>
<td>(1.88)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. $m_{t}$</td>
<td>-0.614</td>
<td>0.999</td>
<td>0.065</td>
<td>-0.022</td>
<td>0.115</td>
<td>-0.179</td>
<td>0.962</td>
<td>-0.44</td>
<td>1974:1-</td>
<td>1979:4</td>
</tr>
<tr>
<td></td>
<td>(2.83)</td>
<td>(12.55)</td>
<td>(4.04)</td>
<td>(3.66)</td>
<td>(1.10)</td>
<td>(0.75)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. $m_{t}$</td>
<td>-2.222</td>
<td>0.605</td>
<td>0.368</td>
<td>-0.061</td>
<td>0.000</td>
<td>-0.467</td>
<td>0.990</td>
<td>-1.94</td>
<td>1980:1-</td>
<td>1983:4</td>
</tr>
<tr>
<td></td>
<td>(2.06)</td>
<td>(4.58)</td>
<td>(2.34)</td>
<td>(5.06)</td>
<td>(0.00)</td>
<td>(1.74)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
known, we could make the transformation, $P^*P = I$, back to the standard model. In general $P$ is unknown, and, as noted in section I, we make an estimate of $\rho$ using a maximum likelihood estimator. However, there is no assurance that residuals from the true $\rho$ and $\hat{\rho}$ will have the same asymptotic distributions. Recently, Dufour (1982) suggested considering a grid of values of $\rho$, possibly inside some neighborhood of $\hat{\rho}$, and checking the robustness of the main conclusions of any investigation involving a non-scalar matrix. Hence, the CUSUM procedures are all repeated for a range of values from $\rho = -0.9$ to 0.9 by intervals of 0.1.

B. Determining the Point of Discontinuity

While the recursive residuals serve as good guides to patterns in regression relationships, the confidence limits of the CUSUM tests provide us only arbitrary points at which to reject certain models. Because time must pass for forecast errors to accumulate, they fail to capture exact points of discontinuity. The CUSUM and residual plots, then, indicate in what region the estimated equation shows signs of instability, but further tests are required to identify accurately points of discontinuity.

The most important of these tests, which enables us to consider whether a regression may have switched regimes, is due to Quandt (1960). Using this method the sample is divided into two segments (1...r) and (r + 1...T). The null hypothesis, $H_0$, is that both $A$ and $\sigma$ are constant over the two segments, while the competing hypothesis, $H_1$, is that they come from two separate models. To investigate this we compute the log maximum likelihood ratio:

$$
\lambda_c = \log \left( \frac{ML/H_0}{ML/H_1} \right) = \frac{1}{2} \log \hat{\sigma}_1^2 + \frac{1}{2}(T - r) \log \hat{\sigma}_2^2 - \frac{1}{2} T \log \hat{\sigma}^2. \tag{5}
$$

This likelihood-ratio statistic is computed moving throughout the entire sample. The point of discontinuity is diagnosed as the value of $r$ at which $\lambda$ attains its minimum. Because several regimes may exist, interest will also center on local minima.

The shortcoming of this approach is that the distribution of $\lambda$ under $H_0$ is unknown. Therefore, it does not lend itself to exact significance testing. For this reason, one must fall back upon the Chow test to give us an $F$-statistic for whether the subsamples are drawn from different populations. A high $F$ rejects the null hypothesis of continued stability.

III. Empirical Results

Doing forward, one-period-ahead recursive estimation with the Beach-McKinnon method, for the sample period 1959:2 to 1961:2 up to 1983:4, we obtain recursive residuals. The residuals are predominantly positive from 1961:3 to 1971:2, then negative for all but 1973:2 in the period from 1971:3 to 1978:1. This seems to accord with the episode of the missing money. They are positive again, this time more severely, from 1978:2 to 1982:2, before returning negative from 1982:3 to 1983:3.

The early period of underprediction leads the CUSUM to encroach upon its upper boundary by 1970:3. The CUSUM of squares, though, provides conclusive evidence of an early structural break, as it crosses the lower 95% confidence boundary in 1965:2. The "missing money" period brings back in the CUSUM, making it negative by 1974:4. The CUSUM falls to its minimum in 1978:1, before the second period of positive residuals makes the CUSUM positive again in 1980:2. The high sum is in 1982:2, but the last five quarters of residuals are negative.

The CUSUM of squares is only worsened by the period of overprediction that restores the CUSUM. It lies safely below the lower boundary before a series of dramatic jumps between 1980:1–1980:2 and 1982:2–1982:3, which culminate in the CUSUM of squares rising above the lower boundary in 1982:3. They stay within the boundary until 1983:4, though they do seem to be trending up.

It was hoped by the authors that a stable relationship would be found in the period after the first break. Forward recursive estimations for 1966:2–1967:4 to the end of the sample, however, dashed these hopes. The CUSUM of squares crosses the lower boundary, nonetheless, this time in 1972:1.

To ensure that the first break was not simply a beginning of sample problem, the CUSUM procedure was done backwards. The CUSUM of squares again violated the confidence boundaries in 1966:4.

The impact of allowing $\rho$ to vary over the range of $-0.9$ to 0.9, to test Dufour's conjecture about the covariance matrix, was minimal. The CUSUM continued to reach an early maxima, between 1970:1 and 1971:3 ($\rho = 0.6$), fall to a minimum in 1978:2 or 1978:3, and reach another maxima in mid-to-late 1982. The exception to this pattern is for $\rho$ values above 0.5, where the CUSUM stays negative to the end of the sample period. The CUSUMSS violated the lower boundary by 1966:3 for all but two parameter values ($\rho = 0.8$ and 0.9). Return came as early as 1980:3 ($\rho = 0.3$ to 0.7). In all, the evidence supports an early break and return in the 1980s for the entire range of covariance estimates.

The four subgroups show the rise in the speed-of-adjustment coefficient, peaking in III, that at least indirectly leads us to reject the null. The drop in period IV suggests the presence of multiple discontinuity and defeats the hypothesis of a simple time trend.

Chow tests confirm breakpoints between I-II/III-IV; I-II-III/IV; but not I/II-III-IV. The return of the CUSUM of squares in the latter stages of the sample led us to find the interesting result that I and IV cannot be rejected as being from the same model.

IV. Optimal Forecast Techniques

Section III has left the policy economist in a quandary as to how reasonable forecasts can be generated with changes in regime. In particular, once tests for structural stability lead to rejection of a certain model, can we still exploit prior sample information.

Granger and Ramanathan (1984) propose a method for combining forecasts that "gives the smallest mean squared error and has an unbiased combined forecast even if individual forecasts are biased." Individual forecasts can also be assessed by t-tests on the coefficient being equal to zero.

Let $f_1$ be the forecast from one model and $f_2$ be the forecast from a rival model. Regress the dependent variable on the constituent forecasts and a constant term which acts as the conditional mean:

$$ m_t = B_0 + B_1 f_1 + B_2 f_2. $$

The least squares then produces a combined forecast from the fitted value of the regression with zero average error, even if $f_1$ and $f_2$ are biased.

In implementing this technique, we consider first forecasts from the three subsamples into the fourth subsample, 1980:1 to 1983:4 (table 2). Two ARIMA models were also estimated. Lower (L95) and upper (U95) 95% confidence boundary forecasts were also examined.

From this group, the forecast of a model that excluded data from the second subsample provided the best forecast. The time series models produce the third and fourth best forecasts, indicating they are less hindered by structural breaks, but they cannot match the best structural forecasts.

The OLS combinations of estimated money demand functions outperformed any single-regime forecast. These results are reported in table 2 for the forecast interval of 1980:1 to 1983:4. The best forecast is provided by:

$$ m_t - m_{t-4} = -0.0009 + 1.394 (m_{t-1} - m_{t-5}) $$

$$ -0.515 (m_{t-2} - m_{t-6}) $$

(5.23)

$$ m_t - m_{t-1} = 0.0005 + 0.619 (m_{t-1} - m_{t-2}) $$

$$ + 0.178 (m_{t-2} - m_{t-3}) $$

(1.32)

$$ + 0.129 (m_{t-3} - m_{t-4}). $$

(1.07)

The Q-statistics are 39.88 (21 degrees of freedom) and 38.90 (20 degrees of freedom), respectively. t-ratios are in parentheses.

### Table 2. — Forecasts 1980:1–1983:4

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean Error</th>
<th>RMSE</th>
<th>RMSE%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I</td>
<td>0.0360</td>
<td>0.0419</td>
<td>4.65</td>
</tr>
<tr>
<td>2. II</td>
<td>-0.1321</td>
<td>0.1363</td>
<td>15.11</td>
</tr>
<tr>
<td>3. III</td>
<td>0.0017</td>
<td>0.0255</td>
<td>2.83</td>
</tr>
<tr>
<td>4. I-II</td>
<td>-0.1276</td>
<td>0.1320</td>
<td>14.63</td>
</tr>
<tr>
<td>5. I-II-II</td>
<td>0.0644</td>
<td>0.0906</td>
<td>10.04</td>
</tr>
<tr>
<td>6. I-III</td>
<td>-0.0030</td>
<td>0.0239</td>
<td>2.65</td>
</tr>
<tr>
<td>7. II-III</td>
<td>0.0594</td>
<td>0.0836</td>
<td>9.27</td>
</tr>
<tr>
<td>8. ARIMA (3,1,0)</td>
<td>-0.0199</td>
<td>0.0338</td>
<td>3.75</td>
</tr>
<tr>
<td>9. ARIMA (2,4,0)</td>
<td>-0.0022</td>
<td>0.0359</td>
<td>3.98</td>
</tr>
<tr>
<td>10. L95 (3,1,0)</td>
<td>0.0449</td>
<td>0.0635</td>
<td>7.04</td>
</tr>
<tr>
<td>11. U95 (3,1,0)</td>
<td>-0.0849</td>
<td>0.0902</td>
<td>10.00</td>
</tr>
<tr>
<td>12. L95 (2,4,0)</td>
<td>0.0776</td>
<td>0.1002</td>
<td>11.10</td>
</tr>
<tr>
<td>13. U95 (2,4,0)</td>
<td>-0.0820</td>
<td>0.0874</td>
<td>9.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OLS Combinations</th>
<th>RMSE</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3</td>
<td>0.0000</td>
<td>0.0132</td>
</tr>
<tr>
<td>8, 9</td>
<td>0.0000</td>
<td>0.0205</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
vided by an OLS combination of models I, II and III, even though II and III are statistically representative of discontinuous models. Identifying a structural break becomes crucial with this technique as we want the least squares to properly adjust for individual bias. No case can be made for simply ignoring the changes in regime.

The agreeable feature of the new combining technique is that the old dictum “garbage in = garbage out” no longer seems necessarily the case. Short of more elaborate techniques, such as ARCH\(^5\) to obtain more sample information out of the error pattern, there seems to be a compelling case to use this method to forecast from discarded models.

V. Concluding Remarks and Implications

The finding of an earlier structural break in the real money demand relationship seems to mitigate the importance of the missing money in the mid-1970s. The tests of section III cast doubt on whether there ever was a consensus.

As to the causes of this break, Slovin and Shushka (1975) anecdotally linked this movement to changes in Regulation Q. Hafer and Hein (1979) also found a similar CUSUM pattern in their test of the Garcia-Pak (1979) equation, even though they concluded that the Goldfeld equation is stable.\(^6\)

The second break does seem to coincide with the episode of the missing money. Nearly uniform negative residuals for the six years after 1973:4 confirm that there was no straw man. The third break would have to be seen as a result of the new Federal Reserve Policy. The surprising aftermath of this third break is that regime IV may represent a return to the pre-1966 model with a more reasonable speed of adjustment.

This last finding, we suggest, is a source of some tempered nihilism. This research, we think, does not set an agenda to go back and conduct trial-and-error tests for specifications that might turn out to be stable forecasting equations. Given our ex post goals, this would only introduce an additional source of bias, as noted by Cooley and LeRoy (1981). Rather, we should recognize that the Lucas critique of forecasting equations is very relevant to the area of money demand. Stability should not be expected across policy regimes or within periods of substantial regulatory change.

Advances in optimal forecast theory have, on the other hand, made it possible for us to make better sense of the future from a stormy past. We need not wait for data points to accumulate in a new regime to construct credible forecasts. If our goal is efficient forecasting, perhaps we should be less wedded to theory and utilize more time-series models to help us detect such changes and incorporate the information that they can tell us about a series.\(^7\)

REFERENCES


\(^5\) Autoregressive conditional heteroskedasticity.

\(^6\) Their approach is fatally flawed by the fact that they assume \(\rho\) to be constant over the entire interval. \(\rho\), however, varies in the recursive-estimation process by 1.422 (min. −0.864 for 1961:2 and max. 0.58 for 1975:1).

\(^7\) This last point should not imply that we are closing the door to hypothesis testing using structural equations. However, we wish to distinguish the role of structural, behavioral models in empirical investigations from the strict forecasting function. The latter, we believe, may be better served with a continued move toward time-series modeling.