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Comment on "Modelling nonlinear comovements between time series"

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Abstract

This paper comments on the multivariate GARCH modeling of federal funds and the 3-month Treasury bill rate by Kyrtsou and Vorlow.
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1. Introduction

Nonlinear modeling has contributed important insights to asset pricing. The literature has evolved in the last two decades, moving from its initial focus on deterministic dynamics to parametric and non-parametric modeling and forecasting. A unifying link, Mayfield and Mizrach (1992) have noted, is the entropy of a dynamical system. This property pertains to both deterministic and stochastic systems, linear and nonlinear models, and can be interpreted as a limit to the forecast horizon. From the perspective of the modeler, the difference between a stochastic and deterministic system subject to noise may be of little practical importance.

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The value added of nonlinear models is still frequently questioned. Researchers seek low dimensional nonlinear models to predict both first and second moments. The Markov switching model and its smooth transition brethren have emerged as the most popular in macroeconomics, and the GARCH model, the most widely applied in finance. Mizrach (2008) provides a survey and some perspective on their gradual domination of the literature.

The paper by Kyrtsou and Vorlow falls into the second category. They assess whether a multivariate GARCH model can explain the comovement between the Federal Reserve Board policy instrument, the federal funds rate, and the 3-month Treasury bill rate. Its novelty is to look at a pair of interest rates at the short end of the yield curve. They find surprisingly rich nonlinear dynamics in both the conditional mean and variance.

2. Empirical results

2.1. Volatility modeling

As the authors' Fig. 1 indicates, the two series are closely linked. The levels have a correlation coefficient of 0.994 and the returns 0.748. The presence of nonlinear or non-Gaussian behavior is indicated by negative skewness and kurtosis of over 7.5.

These unconditional moments suggest that the two series, like many financial time series, may be well described by a GARCH model. Kyrtsou and Vorlow first use the multivariate GARCH framework of Engle and Kroner (1995). Kyrtsou and Labys (2006) have offered an alternative GARCH model which they name after Mackey–Glass (MG). In practice, the specification adds autoregressive dynamics to the conditional mean. Both models find a strong bivariate link between the conditional covariances.

It would have been interesting to compare volatility predictions from options prices. The Chicago Board of Trade offers options on fed funds futures. Carlson et al. (2005) have used these series to construct forecast densities for Federal Reserve policy. Options on 13-week Treasury bill futures are traded at the Chicago Mercantile Exchange. To my knowledge, no one has attempted a bivariate model of their implied volatility and covariances.

2.2. Conditional mean specification

The significance of the terms in the MG conditional mean (Table 1a) and the significant AR coefficients in the residuals of the Engle–Kroner model indicate the EK model is misspecified. The higher log likelihood also supports the MG model. The lower Schwarz criterion reveals this does not come at the expense of parsimony.

The authors do not explore whether there is a structural break in the data. Kyrtsou and Vorlow wisely exclude most of the monetarist experience of 1979–1982, but the big decline of August 1982 remains. The fed funds rate falls by 21% (12.59–10.12%) and the T-bill rate by 27% (11.35–8.68%). By not modeling this as a regime change, the interest rate jump shows up as a volatility spike. The volatility shock is the largest in the 25 year sample.

The great moderation of interest rates since the middle 1980s may also constitute a separate regime. Basitha and Startz (2004) show that the Fed began to respond more aggressively to fundamentals during this time. The record low interest rates in 2002–2004 may also be distinct. The second largest percentage change in the Treasury bill rate occurs in

November 2002. While the rate falls only 35 basis points (1.58–1.23%), with rates so low, this is a 25.04% reduction.

Sarno and Thornton (2003) raise another issue: which series is doing the adjusting? The answer may at first seem obvious, that the policy rate should adjust more slowly than the market determined Treasury bill series. Surprisingly, they find that the federal funds rate has a much shorter half-life adjustment to its equilibrium value. Testing alternative lag structures in the Mackey–Glass difference equation is a straightforward way to address this.

The predictable dynamics in the mean also raise the possibility of profitable arbitrage. If lagged short-term rates help to predict current short-term rates, as model MG indicates, a trading strategy might be devised to exploit this. The predictability in the volatility might also be exploited in an options trading context or to improve the Sharpe ratio of a conditional mean predictor. A definitive answer to these questions would require looking out of sample and taking into account transactions costs and bid-ask spreads.

2.3. More complex nonlinearity?

Because the presence of GARCH behavior in finance is so widespread, evidence of more complex nonlinearity would be of great interest. A notable finding is that, for both specifications, the standardized residuals are still non-Gaussian. This is a common result, first observed by Bollerslev (1987), who proposed a GARCH model with student-t innovations.

After fitting an 11 parameter model to the conditional covariance matrix, about one-third of the kurtosis has been removed by either the EK or MG model. The lingering question is what structure, if any, is still there? At a monthly frequency, it seems unlikely to be microstructure noise.

If the residuals were predictable, tests for higher order moment dependence like the BDS could be employed. An alternative non-parametric approach would be to compute estimates of the correlation dimension, before and after shuffling the residuals as in Scheinkman and LeBaron (1989).

3. Conclusions

The modeling of short-term interest rates has been a neglected area of the nonlinear literature. This paper closes that gap. Kyrtsou and Vorlow find AR dynamics in both the conditional mean and variance and demonstrate that a multivariate model is useful for understanding even the short end of the yield curve.

A variety of interesting problems remain open: the structure of implied volatility, the potential for profitable trading on the mean and covariance dynamics, and the nature of the non-Gaussianity in the standardized GARCH residuals.

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