Chapter 3: Labor demand (Part 1):
The Mosak model of a firm with increasing marginal cost

1. Profit maximization and production
   A. Profit = total revenue – total costs
      $\Pi = R - C$
      $R = \text{sales revenue} = PQ$
      $Q = f(E, K), \text{ where } E = \text{employees (or "employee hours"), } K = \text{capital}$
      $C = \text{total costs} = rK + wE \text{ where } w = \text{wage rate of labor, } r = \text{rental rate of capital}$
   
   B. note that maximizing profit does not necessarily mean maximizing revenue or minimizing costs
   
   C. however, can view maximizing profit as a two-stage process:
      1. find least-cost way to produce each possible level of output, $Q$
         (this generates a marginal cost schedule and determines demand for $K$ and $E$
         at each level of $Q$)
      2. find the level of $Q$ that, when produced at minimum cost, yields maximum profit
         (find this by finding the level of $Q$ where $MR = MC$ – like in Econ. 102!)
D. Production function: $Q = f(E, K)$ – relationships in a 3-D graph

E. "total product" curves for E and K are vertical "slices" out of the production surface: hold E constant and see how Q varies as K changes ("total product of K" curves); hold K constant and see how Q varies as E changes ("total product of E" curves)
Slope of curve tangent to total product curve
= marginal product of E or K (as appropriate) ("MPE" or "MPK")
= $\frac{\partial Q}{\partial E}$ or $\frac{\partial Q}{\partial K}$

Slope of line from origin to total product curve
= average product of E or K (as appropriate) ("APE" or "APK")
= $\frac{Q}{E}$ or $\frac{Q}{K}$
F. isoquants = horizontal “slice” out of production surface
   = set of \{E, K\} combinations that produce the same level of Q

slope of isoquant = \(- \text{MRTS} = \frac{\text{MPE}}{\text{MPK}} = \frac{(\partial Q/\partial E)}{(\partial Q/\partial K)}\)

to see why, note from production function that, as we move along an isoquant,
\[dQ = \left(\Delta Q/\Delta E\right)dE + \left(\Delta Q/\Delta K\right)dK = 0\]

so solve for \(dK/dE\) to get the slope of the line tangent to the isoquant:
\[
\frac{dK}{dE} = -\frac{\left(\Delta Q/\Delta K\right)}{\left(\Delta Q/\Delta E\right)} = -\frac{(\partial Q/\partial E)}{(\partial Q/\partial K)} = -\text{MPE} / \text{MPK}
\]
2. The short-run employment decision
   
   A. by definition, in the short run, K is fixed
   
   B. maximize profit in the usual way: take derivative of $\prod$ with respect to $E$, and set the result equal to zero

   \[ \prod = R - C = PQ - (rK + wE) = Pf(E, K) - rK - wE \]

   so take the derivative of $\prod$ with respect to $E$ (remember that $K$ is fixed!):

   \[ \frac{\partial \prod}{\partial E} = P \frac{\partial f(E,K)}{\partial E} - w = P \times MPE - w \]

   so when profits are maximized, $P \times MPE - w = 0$, or $MPE = w/P$

   C. this is equivalent to the condition $P = W/MPE$, where $W/MPE = MC$ ($MC = \text{marginal cost of producing output}$!)

   to see this, note that since $K$ is fixed in the short run,

   \[ MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta (rK + wE)}{\Delta Q} = \frac{\Delta (wE)}{\Delta Q} = w \left( \frac{\Delta E}{\Delta Q} \right) = w/MPE \]

   D. $P \times MPE$ is called the “marginal revenue product of labor” – it’s the MPE, expressed in terms of the added revenue that additional labor will generate
E. The equilibrium condition for employment can be shown in various ways, e.g.:

F. Comparative statics: effect on E of changes in $w$ and $P$.
3. The long-run employment decision
   A. here, both E and K are variable
   B. “Mosak model” of a firm with increasing marginal cost –
      assumes a two-stage process:
      * find levels of E and K that minimize the cost of producing each level of Q
        (thereby also determining marginal cost, MC, at each level of Q) –
        shown in isoquant/isocost diagram
      * then, find level of Q that maximizes profits when it’s produced at minimum cost –
        shown in MR/MC diagram
      * note: higher isocost lines imply higher cost levels, and vice-versa

4. Stage 1: Cost minimization
   A. use isocost to show production costs: \( C = rK + wE \)
   B. solve for K, to get K on the vertical axis and L on horizontal axis (like isoquants):
      \[
      K = \frac{C}{r} - \frac{w}{r}E
      \]
C. cost minimization: find lowest isocost compatible with a given isoquant – this isocost just touches (is tangent to) that isoquant, at the least-cost \{K,E\} point at the tangency: isocost slope = isoquant slope

so \[-w/r = -\frac{\text{MPE}}{\text{MPK}},\]

or \[\frac{\text{MPE}}{w} = \frac{\text{MPK}}{r}\]

(another “equal bang per buck” criterion)

non-tangency generally implies that the firm could cut costs by changing E or K: e.g., if \[\frac{\text{MPE}}{w} > \frac{\text{MPK}}{r}\] (see point D), then raising E and reducing K, moving “southeasterly” to point M, will cut costs without reducing Q
D. in this way, we find...

the least-cost input combination that will produce each level of Q (i.e., input levels $E^*$ and $K^*$)
the minimized cost of producing each level of Q, i.e., $C^* = rK^* + wE^*$
marginal (minimized) cost of producing additional output, $\frac{\Delta C^*}{\Delta Q}$ – i.e., i.e., the firm’s marginal cost curve!

E. isoquant/isocost diagram shows change in cost $C^*$ at successive levels of output $Q$
thus, it shows marginal cost $MC$!
5. Stage 2: Profit maximization
   A. maximizing profits requires producing more output, so long as MR > MC
   B. in perfect competition, MR = P
   C. in Stage 1, we determined MC
   D. so, determine profit-maximizing level of Q, = Q*, by finding where MR = MC
      then, find profit-maximizing input levels and costs by going back to isoquant for Q*
      and finding the least-cost \{E, K\} combination that produces Q* at lowest cost C*
      so, now we have determined Q*, E*, K*, and C*
6. Comparative statics and the long-run demand for labor
   A. when \( w \) or \( r \) changes, must consider both stages (Stages 1 and 2)
   B. In Stage 1, a higher price for an input reduces demand for that input, and increases demand for the other input, at any given level of \( Q \) this is the **substitution effect**
   (e.g., higher \( w \) reduces \( E^* \), raises \( K^* \) at each \( Q \); higher \( r \) raises \( E^* \), reduces \( K^* \) at each \( Q \))

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\begin{align*}
\text{slope} &= \frac{w_2}{r} \\
W_2 > W_1
\end{align*}
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C. In Stage 1, a higher price for an input will **raise** MC, *if* that input is “normal” (this is the “normal” case, the one we would expect)

* L is normal: changing output from Q to Q’ raises use of L  
  (true both for shift in budget line from AA to BB, or from CC to DD)
* MC rises when w rises: MC at lower w is DC; MC at higher w is AB; AB > DC
D. In Stage 1, a higher price for an input will **reduce** MC, **if** that input is “inferior”
  
  * L is inferior: changing output from Q to Q’ **reduces** use of L
    (true both for shift in budget line from AA to BB, or from CC to DD)
  
  * MC falls when w rises: MC at lower w is DC; MC a higher w is AB; AB < DC
E. In Stage 2,
if the input is “normal,” the rise in input’s price raises MC
...which in turn must reduce optimal Q (i.e., reduce the scale of operations)
...and since the input is normal, this must also reduce demand for the input
so, if the input is normal, the *scale effect* of the input price rise
is to reduce demand for the input

if the input is “inferior,” the rise in input’s price reduces MC
...which in turn must raise optimal Q (i.e., raise the scale of operations)
...and since the input is inferior, this must also reduce demand for the input
so, if the input is inferior, the *scale effect* of the input price rise
is to reduce demand for the input
6. Summary of the Mosak model of a firm with increasing MC (consider the effect of a rise in w, to be specific)

A. Stage 1: **substitution effect**
   - at each level of Q, rise in w reduces E, raises K
   - if E is normal, rise in w raises MC
   - if E is inferior, rise in w reduces MC

B. Stage 2: **scale effect (or “output effect”)**
   - if E is normal, rise in w raises MC, which reduces optimal Q, which reduces E (K rises if it’s inferior, falls if it’s normal)
   - if E is inferior, rise in w reduces MC, which raises optimal Q, which reduces E (K falls if it’s inferior, rises if it’s normal)

C. so, regardless of whether E is normal or inferior,
   - rise in w *always reduces* E via **both** the substitution and **scale** effects

D. total effect (substitution effect + scale effect)?
   - effect of higher w on L is always determinate!
   - regardless of whether L is normal or inferior, higher w *always reduces* demand for L via both the substitution effect and the scale effect
   - ... so the long-run demand-for-curve is downward-sloping
E. Effect of higher w on K (a “cross-price effect”): can’t always be determined a priori

**substitution effect:** higher w always raises K

**scale effect:**
- (a) if L is normal, higher w raises MC, reduces Q*
  - (1) if K normal, scale effect reduces K
  - (2) if K inferior, scale effect raises K
- (b) if L is inferior, higher w reduces MC, raises Q*
  since there are only two inputs, K must be normal (can you see why?)
  since K must be normal, scale effect on K is positive

In case a(2) and case (b), both substitution and scale effects on K are positive, so here we can be sure that K will rise when w rises.

In case a(1), substitution effect on K is negative while scale effect on K is positive, so here we can’t say what will happen to K on balance.
7. Comparative statics in the Mosak model: Effect of a change in P

A. Change in P (ceteris paribus) doesn’t change w or r, but does change P
   So best way to analyze effect of change in P (ceteris paribus) is to look first at Stage 2,
   then at Stage 1: change in P changes optimal Q; change in optimal Q changes K*, E*

B. Stage 2: When P changes, P = MC equilibrium occurs at a higher level of Q

C. Stage 1: Increase in Q raises E if E is normal, reduces E if E is regressive (“inferior”)
   Increase in Q raises K if K is normal, reduces K if K is regressive (“inferior”)

D. so, effect of change in P on an input depends on whether it’s normal or regressive
   e.g., consider two cases: (i) both inputs are normal or (ii) K normal and E regressive
8. The elasticity of demand for labor with respect to the wage
   A. elasticity of labor demand w.r.t. \( w = \)

\[
\eta_{E,w} = \frac{\% \Delta \text{ labor demand}}{\% \Delta \text{ wage}}
\]

B. “some consensus” that...
   * short-run elasticity is between -0.5 and -0.5
   * long-run elasticity is roughly -1.0

9. The elasticity of capital-labor substitution, \( \sigma \)
   A. measures “flatness” of an isoquant
      (a flatter isoquant \( \Rightarrow \) higher \( \sigma \); a curvier isoquant \( \Rightarrow \) lower \( \sigma \))
   measures ease of substituting inputs at a given level of output
   (higher \( \sigma \) \( \Rightarrow \) easier to substitute)
   related to size of substitution effect of an input price increase
   (higher \( \sigma \) \( \Rightarrow \) bigger substitution effect)

B. definition (for the case of two inputs):

\[
\sigma = \frac{\% \Delta (K/E)}{\% \Delta (\text{MPE}/\text{MPK})}
\]

and since \( \text{MPE}/\text{MPK} = w/r \) in equilibrium, we can also write \( \sigma = \frac{\% \Delta (K/E)}{\% \Delta (w/r)} \) at given \( Q \)
10. Examples of the elasticity of substitution and isoquant curvature

A. perfect substitutes: \( \sigma = \infty \), isoquants are straight lines
   
x units of K are equivalent to one unit of L (like 2 nickels = 1 dime!)

B. perfect complements ("fixed coefficients," "Leontief" production function):
   \( \sigma = 0 \), isoquants are rectangles; K and L must be used in fixed proportions
   (e.g., 1 shovel: 1 worker)
11. Factor demands with many inputs

A. Production function now becomes

\[ Q = f(x_1, x_2, x_3, \ldots, x_N; z_1, z_2, z_3, \ldots, z_K) \]

(There are \( N \) inputs \( x \) and \( K \) other factors \( z \) (e.g., temperature, rainfall, etc.) affecting production of \( Q \).)

B. “All of the key results derived in the simpler case of a two-factor production function continue to hold”:

- Short- and long-run demand curves for an input are always downward-sloping w.r.t. the input’s price
- Input price rises generate both a substitution effect and an output (“scale”) effect, and both are always negative
- Long-run input demand curves are more elastic than short-run demand curves (basically, because [more] substitution with other inputs can occur in the long run)
- Cost minimization still requires getting an “equal bang per buck” from each input \( i \), equating the ratio of its marginal product \( MP_i \) to its price \( w_i \) to the same ratio for each and every other input \( j, k, \ldots, \) etc.:

\[
\frac{MP_i}{w_i} = \frac{MP_j}{w_j} = \frac{MP_k}{w_k} \quad \text{for all inputs } i, j, k
\]

- Profit maximization still requires equating each input’s price \( w_i \) to its marginal revenue product, and still requires equating output price \( P \) to marginal cost:

\[
w_i = P \times MP_i \quad \text{and } P = \frac{w_i}{MP_i} \quad (= MC) \quad \text{for all } i, j, k
C. Some key results for the two-input case must be modified for the n-input case
• Several (even many) inputs can be inferior
• However, there must be at least one normal input (i.e., all inputs can’t be inferior)
• Several (even many) inputs can be complementary (like bread and butter?)
  (so that a rise in the price of one input can reduce demand for another, complementary, input, even at a fixed level of output)
• However, at least one pair of inputs must be substitutes (like butter and margarine?)
  (so that a rise in the price of one such input must raise demand for the other input, even at a fixed level of output – as in the simple model with only two inputs)
• If w rises and all other input prices stay the same, substitution is (still) away from E, but is towards the composite of all other inputs: use of substitute inputs will rise, but use of complementary inputs will fall along with E

D. example: three-input model – unskilled workers, skilled workers, and capital
• Demand for unskilled workers seems to be more elastic than demand for skilled workers (so, employment for unskilled workers may be more volatile – big swings)
• Unskilled labor and capital seem to be substitutes
• Skilled labor and capital seem to be complementary – they “go together”
  (rise in price of one of these will reduce demand for the other, even at constant Q)
• e.g., 10% reduction in r will reduce employment of unskilled by 5%, and raise employment of skilled by 5%
Chapter 3, Part 2: The Hicks-Allen Model

1. Hicks-Allen model of input demands of an industry with constant returns to scale
   A. Production function is assumed to have constant returns to scale (CRTS):
      an x% increase in all inputs ⇒ an x% increase in output
      (i.e., entire scale of operations changes)
   B. but note that this means that, with CRTS,
      * all inputs are normal!
      * an x% increase in all inputs will also raise total production cost (TC) by x%
      * so that average total cost (ATC) = TC/Q will remain unchanged
      * but, in turn, this means that marginal cost must equal ATC and is also constant!
         (If MC > ATC, then ATC would rise; if MC < ATC, then ATC would fall)
   C. thus, with CRTS, MC is constant – which raises an awkward question:
      if MC is constant, and if we have perfect competition (so that MR = P is also constant),
      how can we have MR = P equal to MC?
      if MR = P > MC, MC won’t rise as firm increases Q: Q would rise without limit
      if MR = P < MC, MC won’t fall as firm decreases Q: Q would fall to zero
   D. so if a competitive firm has CRTS, the size of the firm is indeterminate!
   E. simple solution: finesse the problem by interpreting the model as referring to
      an industry of perfectly-competitive producers with downward-sloping demand curve
      for the industry’s output – intersection occurs where P = MC!
Stage 2 in the Hicks-Allen model of an industry with CRTS:
MC is constant with respect to output;
industry demand curve shows consumer demand for industry output
and the price $P$ consumers will pay for each level of $Q$
industry is composed of perfectly-competitive firms that all produce
where $P = MC$
therefore industry output is $Q^*$ (see above)

(Stage 1 in Hicks-Allen model is the same as Stage 1 in the Mosak model.)
2. Input demands in the Hicks-Allen model: A closer look
   A. under CRTS, isoquants have the same slope along a constant ray from the origin (each ray is the expansion path of the industry as Q rises with factor prices constant) this is why, if Q increases (ceteris paribus), all inputs must increase by the same % but note that this means that, under CRTS, all inputs are normal!

   B. thus, an input price rises will always raise MC (because each input is normal), so an input price rise will always reduce output Q so the scale (or “output”) effect of the input price rise will always reduce demand for the input
3. Elasticities of demand for inputs in the Hicks-Allen model

“cross-wage” elasticity of demand for input $X_1$ with respect to $w_2$ ( = price of input $X_2$) :

$$\frac{\% \Delta X_1}{\% \Delta w_2} = s_2 (\sigma_{12} - \eta) = s_2 \sigma_{12} - s_2 \eta = \text{substitution effect} + \text{scale effect}$$

where $s_2$ = cost of $X_2$ as a percentage of total cost (= “share” of $X_2$ in total costs)  
(note that the “s” here refers to the input whose price went up, #2!)

$\sigma_{12}$ = elasticity of substitution between inputs $X_1$ and $X_2$  
(positive if are substitutes, negative if are complements)

$\eta$ = elasticity of consumer demand for output (= - $\Delta Q/% \Delta P$), always $> 0$

“own-wage” elasticity of demand for input $X_1$ with respect to $w_1$ ( = price of input $X_1$) :

$$\frac{\% \Delta X_1}{\% \Delta w_1} = s_1 (\sigma_{11} - \eta) = s_1 \sigma_{11} - s_1 \eta = \text{substitution effect} + \text{scale effect}$$

where $s_1$ = cost of $X_1$ as a percentage of total cost (= “share” of $X_1$ in total costs)  
(note that the “s” here refers to the input whose price went up, #1!)

$\sigma_{11}$ = “own-elasticity of substitution,” and is always negative

(For example, if there are only two inputs, they must be substitutes, so $\sigma_{12} > 0$;  
and in this case, it can be shown that $\sigma_{11} = -(s_2/s_1) \sigma_{12} < 0$
4. Marshall’s “Rules of Derived Input Demand” (from Hicks-Allen model)

A. Elasticity of demand for an input will be greater (more negative)...

- the greater is the elasticity of substitution, \( \sigma \) (higher \( \sigma \) \( \rightarrow \) bigger substitution effect)
- the greater is the elasticity of consumer demand for the product \( \eta \) (higher \( \eta \) \( \rightarrow \) larger scale effect)
- the greater is the elasticity of supply of other factors of production (this makes it easier to substitute other factors for the input whose price went up)
- the greater than input’s share in total costs, \( s \) (provided the elasticity of product demand \textbf{exceeds} the elasticity of factor substitution – see footnote in text!)

B. Note, e.g., how unions and others seek to reduce the elasticity of demand for labor (make it less negative) by focusing on factors highlighted by these rules:

- fight technological advances (reduce \( \sigma \))
- limit availability of competing products (reduce \( \eta \))
- immigration curbs, “prevailing wage” laws (reduce elasticity of supply of competing factors)
- organize small elite groups of workers (low share of total costs)
5. Econometric studies of input demands
   A. start with a production function, cost function or some other building block to get an estimating equation for econometric analysis
   B. e.g., “Cobb-Douglas” production function:

\[ Q = A(X_1)^{\alpha_1} (X_2)^{\alpha_2} (X_3)^{\alpha_3} \ldots (X_N)^{\alpha_N} \]

so \( MP_1 = \frac{\partial Q}{\partial X_1} = \alpha_1 A(X_1)^{\alpha_1-1} (X_2)^{\alpha_2} (X_3)^{\alpha_3} \ldots (X_N)^{\alpha_N} = \frac{\alpha_1}{\alpha_1} \frac{Q}{X_1} \)

C. “equal bang per buck” requires \( \frac{\alpha_1}{\alpha_1} \frac{Q/X_1}{w_1} = \frac{\alpha_2}{\alpha_1} \frac{Q/X_2}{w_2} = \frac{\alpha_3}{\alpha_1} \frac{Q/X_3}{w_3} = \ldots \)

so rearrange terms to get 

\[ X_2 = \left(\frac{\alpha_2}{\alpha_1}\right) \left(\frac{w_1}{w_2}\right) X_1 \]
\[ X_3 = \left(\frac{\alpha_3}{\alpha_1}\right) \left(\frac{w_1}{w_3}\right) X_1 \text{ etc. etc.} \]

so now substitute these expressions for \( X_2, X_3, \text{ etc.} \), back into the production function and rearrange terms, to get an expression for \( X_1 \) in terms of factor prices (\( w \)) and output (\( Q \)) – an estimating equation, suitable for a regression (see next page)

(analogous expressions can be obtained for every other \( X \))
\[
Q = AX_i^\alpha_i \left( \frac{x_2}{x_1} \frac{w_i}{w_2} X_1 \right)^{a_2} \left( \frac{x_3}{x_1} \frac{w_i}{w_3} X_1 \right)^{a_3} \ldots \\
\text{so } Q = A \left( \frac{x_2}{x_1} \right)^{a_2} \left( \frac{x_3}{x_1} \right)^{a_3} \ldots \left( \frac{w_i}{w_2} \right)^{a_2} \left( \frac{w_i}{w_3} \right)^{a_3} \ldots X_1 \\
\text{so } X_1 = A^{-a} \left( \frac{x_1}{x_2} \right)^{a_2} \left( \frac{x_1}{x_3} \right)^{a_3} \ldots \left( \frac{w_i}{w_2} \right)^{-a_2} \left( \frac{w_i}{w_3} \right)^{-a_3} \ldots Q^a \\
\text{where } a = 1 / (a_1 + a_2 + a_3 + \ldots) \\
a_2 = x_2 / a_1, \quad a_3 = x_3 / a_1, \quad \ldots \\
\text{so } \log X_1 = a_0 + b_2 \log \left( \frac{w_i}{w_2} \right) + b_3 \log \left( \frac{w_i}{w_3} \right) + \ldots \\
+ c \log Q \\
\text{where } a_0 = \log \left\{ A^{-a} \left( \frac{x_1}{x_2} \right)^{a_2} \left( \frac{x_1}{x_3} \right)^{a_3} \ldots \right\} \\
b_2 = -a_2, \quad b_3 = -a_3, \quad \ldots \\
c = 1 / (a_1 + a_2 + a_3 + \ldots)
Chapter 3, Part 3: Applications and public policy

1. Affirmative action and the least-cost input combination
   A. two inputs, M and F (or W and B, etc.) – imperfect substitutes (convex isoquants)
   B. nondiscriminatory firm makes input decisions only on input prices and
      marginal products – thus, achieves the least-cost input combination at point P
   C. discriminatory firm acts as marginal product of one type of labor (M or W, etc.)
      is higher than the true amount, by an amount d ("discrimination coefficient") –
      thus, this firm operates at point P and has has (e.g.)

   \[
   \frac{MPM + d}{WM} = \frac{MPF}{WF} \quad \text{so that} \quad \frac{MPM}{WM} < \frac{MPF}{WF}
   \]

   ...and therefore \(\frac{MPM}{MPF} < \frac{W_M}{W_F}\) (see next page)

D. thus, the discriminatory firm loses money! (doesn’t achieve least-cost combination)

E. also implies that requiring an increase in the ratio of F workers to M workers
   ("affirmative action")...
   • can reduce costs at each level of Q for the discriminatory firm
     (provided F/M is set at the right level – see next page)
   • will raise costs at each level of Q for the nondiscriminatory firm (see next page)
     (unless F/M is set at the right level)
**Discriminatory Firm:**

- Operates at \( P \)
- Cost is \( C_1 \)
- \( F/M \) ratio is low

\[
\frac{\text{MPM}}{\text{MPF}} < \frac{\text{w}_m}{\text{w}_F} = \frac{\text{MPM} + a}{\text{MPF}}
\]

Affirmative action requires firm to operate at \( Q \) (higher \( F/M \), lower costs)

**Non-Discriminatory Firm:**

- Operates at \( P \), cost is \( C_1 \)
- \( F/M \) ratio is OK \( \left( \frac{\text{w}_m}{\text{w}_F} = \frac{\text{MPM}}{\text{MPF}} \right) \)

If Affirmative Action sets \( \text{FM} \) "too high," firm is required to operate at \( Q \), where \( \frac{\text{w}_m}{\text{w}_F} < \frac{\text{MPM}}{\text{MPF}} \)

Costs are \( C_2 > C_1 \)
2. Adjustment costs and input demands
   A. when input prices change, firms don’t usually change input levels immediately, because of adjustment costs, e.g.:
      hiring costs: advertise; test and process applicants, put new hires on payroll, etc.
      firing costs: process terminated workers, pay severance/unemployment benefits, etc.
   B. some adjustment costs are fixed (independent of number of workers hired/fired) – e.g., an ad in the newspaper seeking applicants
   C. other adjustment costs are variable (depend on number of workers hired/fired) – e.g., preparing pink slips, processing new hires
   D. to limit adjustment costs, firms may…
      • expand/contract employment over time more slowly than if there were no such costs
      • use “labor hoarding” during a recession (why lay off workers, if cost of hiring replacements later on will be high?)
      • hold off hiring new workers even when the economy is expanding, if there would be a high cost of laying them off later (e.g., as in Europe)
   E. when there are fixed adjustment costs, firms will either…
      (a) not change employment at all in response to a small change in W, P, or Q, etc.
         (why spend money taking out a newspaper ad if the firm needs only a few more workers?), or
      (b) make a large change in employment in response to a slightly larger change in W, P or Q
      (c) thus, pattern will show long periods of stable employment, with occasional big discontinuous changes in employment
3. “Rosie the Riveter as an instrumental variable” (or, did the folks at MIT get it right?)

A. the identification problem: can \( \{w, E\} \) points be identified as a demand curve?
   or as a supply curve? Or as neither?

\[ \Rightarrow \] problem is that when both \( S \) and \( D \) curves shift, the observed \( \{w, E\} \) points
   don’t necessarily trace out either a demand curve or a supply curve

B. to identify the demand curve, we need to find an “instrumental variable”
   that shifts the supply curve but not the demand curve
   (likewise, to identify \( S \), we need an IV that shifts \( D \) but not \( S \)) (see next page)
C. in WWII, mobilization of men shifted S curve for women out

\textit{(without shifting D curve for women??)} –

thus, mobilization of men could be an IV for estimating female D curve:

- 1\% higher mobilization of men \(\rightarrow\) 2.58\% lower female wage \((w)\) growth
- 1\% higher mobilization of men \(\rightarrow\) 2.62\% higher female employment \((E)\) growth

so, elasticity of demand for female labor = \(\frac{\%\Delta E}{\%\Delta W} = \frac{2.62}{-2.58} = -1.02\)
4. Effects of minimum wages – theory

A. **simple theory**: binding minimum wage (a “price floor”): reduces employment (likewise, a price ceiling *also* reduces employment! see diagram to right!)

B. **more complicated theory**: two sectors, one covered by minimum wage law, the other sector not covered by the law – then in response to a higher minimum,
   - some workers in *covered* sector lose their jobs
   - some workers displaced from the *covered* sector try to get new jobs in the uncovered sector
   - some workers in the *uncovered* sector may quit to try to get better-paying jobs in the *covered* sector (“wait unemployment”)
   - thus, uncovered-sector wage could fall or rise relative to its previous level
5. More complicated theory of minimum wage effects – continued

In equilibrium, expected wage should be the same in covered and uncovered sector:

\[ \pi W_{\text{min}} = W_u \]

where \( \pi \) = probability of getting minimum wage job

\( W_{\text{min}} \) = minimum wage

\( W_u \) = wage in uncovered sector

(So, in equilibrium, since \( \pi < 1 \), we must have \( W_{\text{min}} > W_u \).)
6. Effects of minimum wage – evidence
   A. Econometric studies of minimum wage use regression analysis to elasticity of
demand for teenage labor with respect to minimum wage, e.g.,

   \[ \text{Teen employment} = a + b \text{ Min. wage} + c_1 X_1 + c_2 X_2 + c_3 X_3 + ... + e \]

   where \( X \) are other measured variables affecting demand for teenage labor
   and \( e \) is the regression error term (including missing/unmeasured variables)

   B. \( \frac{\partial \text{Teen employment}}{\partial \text{Minimum wage}} = b \),
   so elasticity of demand for teen labor w.r.t. minimum wage
   \[ = b \times (\text{min wage/teen employment}) \]

   C. typical estimate of this elasticity is between -0.1 and -0.3,
   **BUT** the estimates vary a lot between studies, and the \( X \) variables may not fully take
   account of all factors (besides minimum wage) that affect demand for teen labor
   also, possible missing-variables bias: \( e \) may be correlated with Min. wage
   e.g., if lawmakers raise minimum wage only during booms
   then \( e \) and minimum wage will be positively correlated
   and so OLS estimate of \( b \) will be upward biased (i.e., less negative)
6. Effects of minimum wage – evidence (continued)

D. “Revisionist” studies – typically attempt to find “experimental” situations instead of using conventional time-series data; find a roughly zero effect of minimum wage on teen employment!

E. Card and Krueger “revisionist” study of NJ and PA finds zero employment effect
- 1992: NJ raised its minimum wage to $5.05/hr; PA kept its minimum at $4.25/hr
- C&K got data on both sides of the border, did “difference-in-difference” analysis

<table>
<thead>
<tr>
<th></th>
<th>NJ</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>before rise in NJ min wage</td>
<td>20.4</td>
<td>23.3</td>
</tr>
<tr>
<td>after rise in NJ min wage</td>
<td>21.0</td>
<td>21.2</td>
</tr>
<tr>
<td>difference, after – before</td>
<td>+0.6</td>
<td>-2.1</td>
</tr>
<tr>
<td>difference-in-difference</td>
<td></td>
<td>+2.7</td>
</tr>
</tbody>
</table>

So the +2.7 diff-in-diff is POSITIVE (though not statistically significant) – thus, no convincing evidence to suggest that in NJ employment fell relative to PA, despite the rise in the NJ minimum wage
7. How do “difference-in-difference” analyses work?

A. “Experimental” and “control” observations are observed over time:
   * experimentals may always have been/may always be different from the controls
   * simple before-and-after comparisons may be affected by changes caused by factors other than the experiment itself (e.g., a boom or recession)
   * assume that changes in “other factors” affect experimentals and controls equally and that the experiment doesn’t affect the controls as well as the experimentals

B. regression setup: \[ Y = \beta_0 + \beta_1 \text{Experimental} + \beta_2 \text{Time} + \beta_3 [\text{Experimental} \times \text{Time}] + e \]

where Experimental = 1 if experimental (e.g., where Minimum Wage rose)
   = 0 if not experimental

Time = 1 if a period after experiment (e.g., after minimum rose)
   = 0 if a period before experiment (e.g., before minimum rose)

for experimentals: Experimental = 1, Time = 1 or 0

new \( Y = \beta_0 + \beta_1 + \beta_2 + \beta_3 \)
old \( Y = \beta_0 + \beta_1 \) so for experimentals, change in \( Y = \text{new} Y - \text{old} Y = \beta_2 + \beta_3 \)

for controls: Experimental = 0, Time = 1 or 0

new \( Y = \beta_0 + \beta_2 \)
old \( Y = \beta_0 \) so for controls, change in \( Y = \text{new} Y - \text{old} Y = \beta_2 \)

So the difference between the experimental difference and the control difference

(= the “difference-in-difference estimate”)

= \((\beta_2 + \beta_3) - \beta_2 = \beta_3\), the coefficient on the interaction term [Experimental \( \times \) Time] in the regression!
8. Explaining Card & Krueger
   A. Adverse effect of small change in minimum wage likely to be small, hard to detect?
   B. C&K data have lots of measurement error; using company data (more reliable?) produces zero or slightly negative estimates of min. wage effects (...but so what?)
   C. C&K focus only on existing restaurants, so they might overlook effect of min. wage in deterring firms from expanding or opening new restaurants in NJ
   D. Hike in min. wage may affect local restaurants more than chains, yet C&K only studied the response of chains (Roy Rogers, etc.) –
      (1) local restaurant employment could drop sharply due to the minimum
      (2) chain employment picks up some of the slack
      (3) so chain employment might rise due to the minimum, even though total employment (chains + local restaurants) falls
   E. timing: C&K may not get the timing right (response to the minimum may not occur until later – or could even occur before it takes effect!)

9. What if C&K are right?
   A. If they’re right, what explains their results? Why doesn’t employment drop? (maybe adjustment costs keep a big response from occurring?)
   B. if employment doesn’t change much when minimum rises, what else happens? (something’s gotta give...)
      * do firms’ profits drop?
      * do firms’ prices rise?
      * any other changes?