Chapter 2: Labor Supply

1. Terms and concepts
   P = Population
   L = Labor force
   = E + U (employed + unemployed)

   L/P = labor force participation rate
   U/L = unemployment rate
   E/P = employment-population ratio

   unemployed = not working, but looking *actively* for work
   "hidden" unemployed = not working, would like to work,
   but not looking actively

   over time, LFPR's, hours of work have...
   fallen for men, risen for women (for more, see Blau & Kahn reading)

2. Basic model of labor-leisure choice: optimization subject to constraints
   (maximize utility subject to budget constraint and time constraint)
   (note: basic model has many extensions)
3. Utility function: $U = f(C, L, \ldots)$
   $C$ = consumption, $L$ = leisure,
   $\ldots$ = many other things (to simplify, assume they’re constant)

A. utility surface graphs the function
B. slice into the utility surface vertically to get total-utility curves:
  U vs. C with L constant, U vs. L with C constant

Slope of total-utility curve
  = marginal utility
  = \( \frac{\partial U}{\partial L} \) in graph of U vs. L  (see graph on the left)
  = \( \frac{\partial U}{\partial C} \) in graph of U vs. C  (see graph on the right)
C. slice into the utility surface horizontally to get the indifference curve:
   shows C vs. L with U constant
   • indifference curves have negative slope
   • higher indifference curves have higher U
   • indifference curves don't intersect
   • indifference curves are convex to origin

D. slope of indifference curve = marginal rate of substitution = -MUL/MUC
   Now, U = f(C, L),
   so \( dU = (\partial U/\partial C)dC + (\partial U/\partial L)dL \)
   since \( dU = 0 \) along an indifference curve, \( (\partial U/\partial C)dC + (\partial U/\partial L)dL = 0 \)
   so, solve for \( dC/dL \) to get \( dC/dL = -(\partial U/\partial L)/(\partial U/\partial C) \)

NB: convexity of indifference curves implies diminishing MRS
   (i.e., MRS gets smaller in absolute value/flatter in slope as L rises,
    C falls as we move along a given indifference curve)

NB: MRS measures "subjective value of L" relative to "subjective value of C"
   = the "value of time to the individual" (relative to consumption)
   e.g., high MRS (high indifference curve slope) means
   you'd require large increase in C to be willing to give up a unit of L
E. different workers have different tastes, so their indifference curves have different shapes so indifference curves for different persons can certainly cross

(indifference curves for the same person cannot cross – that would imply inconsistent tastes)
4. Time and budget constraints

A. Time constraint: Total available time T per period is divided between work hours H and "leisure" (nonwork) hours L:
   \[ T = H + L \quad \text{so} \quad H = (T - L) \]

B. Budget constraint: Total expenditure cannot exceed income (NB: can expand the analysis to allow for borrowing and saving, but not yet)
   \[ P = \text{cost of consumer goods per unit} \]
   \[ W = \text{wage rate per hour} \]
   \[ V = \text{nonwork income (dividends, etc.)} \]
   
   so budget constraint says that
   \[ PC \leq WH + V \]
   
or
   \[ PC \leq W(T - L) + V \quad \text{(substitute in the time constraint)} \]

C. rewrite budget constraint with C on left-hand side, L on right-hand side (like the indifference curve):
   \[ C \leq \frac{W}{P} (T-L) + \frac{V}{P} \quad \text{or} \quad C \leq w(T-L) + v \]
   
   where \( w = \frac{W}{P} = \text{“real” wage} \), \( v = \frac{V}{P} = \text{“real” nonwork income} \)
   (thus, no “money illusion”)

5. See graph of budget line:

when \( L = 0, \ C = wT + v \) = "full income" (= max. possible consumption \( C \))
when \( L = T, \ C = v \) (= real nonwork income)

note that \( wH = w(T-L) \) = real earnings, and \( wH + v \) = real income
6. Budget line changes when \( w \) or \( v \) changes

- When \( v \) changes, budget line shifts up but its slope stays the same.
- When \( w \) changes, budget line slope changes, but its location at \( L = T \) stays the same.
Note that, when $w = W/P$ rises, the shift in the budget line means that…

(a) more \{C, L\} combinations are available – individual is better off
   * the “income effect”

(b) the budget line’s slope changes – price of L relative to C changes
   * the “substitution effect”

In contrast, when $v = V/P$ rises, the shift in the budget line means only that

more \{C, L\} combinations are available – individual is better off
   * the “income effect”
(but note that here, the budget line slope doesn’t change)
7. Equilibrium: the hours-of-work decision

A. Constrained utility maximization involves getting on the highest indifference curve that is consistent with the budget line.

B. At an "interior optimum" (with $0 < L < T$), we have $\text{MRS} = \frac{W}{P}$ or $\text{MRS} = w$ or $\frac{\text{MUL}}{\text{MUC}} = \frac{W}{P}$
or $\frac{\text{MUL}}{W} = \frac{\text{MUC}}{P}$ (e.g., point E)
(an "equal bang per buck" criterion)
(NOTE: later on, we consider "corner optimum," with $L = T$ and $H = 0$)
C. in contrast, consider point A
here, MRS > W/P, or MUL/W > MUC/P
so L has bigger “bang” per dollar of cost than does C
so at point A, we would want more L, less C
so we would move away from point A towards point E

likewise, at point D, MUL/W < MUC/P
so at point D, we would want less L, more C
so we would move from point D towards point E
8. Comparative statics: Effect of a rise in nonwork income, \( V/P = v \)

A. rise in \( v \) shifts budget line up, but doesn’t change budget line slope

B. in response to the rise in \( v \), the individual will move somewhere between point \( m \) and point \( n \) (other points would involve less utility)

C. if \( L \) is a “normal” good, then \( L \) will rise
   if \( L \) is an “inferior” good, then \( L \) will fall
C. if \( C \) is a “normal” good, then \( C \) will rise
   if \( C \) is an “inferior” good, then \( C \) will fall

D. both \( C \) and \( L \) can’t be inferior (why?)

E. so there are three possible outcomes:
   1. \( C \) rises, \( L \) rises (both are normal) (this case seems the most plausible)
   2. \( C \) rises, \( L \) falls (\( C \) normal, \( L \) inferior)
   3. \( C \) falls, \( L \) rises (\( C \) inferior, \( L \) normal)
9. Comparative statics: Effect of a rise in the wage, $W$ (ceteris paribus)
   A. rise in $W$ changes slope of budget line –
      moves budget line like a windshield wiper
      (budget line stays “anchored” at the no-work point,
       where $W$ isn’t relevant)
   B. the income effect of a rise in $W$
      the rise in $W$ makes the individual better off:
      more $\{C, L\}$ points are now available
      so utility rises, just as if $V$ or $v = V/P$ had increased
   C. to measure the “pure income effect” of a rise in $W$,
      raise $V$ (or $v$) by just enough to increase $U$ by the same amount
      as will occur due to the wage increase – BUT,
      keep the slope of the budget line constant
      (note that the income effect on $H$ and $L$ of a higher $W$ could be either
      positive or negative – depends on whether $L$ is normal or inferior)
D. the **substitution effect** of a rise in $W$
the rise in $W$ increases the slope of the budget line – makes it steeper, increasing the “price of leisure”

E. to measure the “pure substitution effect” of the higher $W$,
increase $W$ by as much as will occur due to the wage increase, BUT, keep utility constant
(note that substitution effect of higher $W$ must always raise $C$ and $H$, and must always reduce $L$)
Income and substitution effects: another example

Wage falls, equilibrium changes from $e_1$ to $e_2$

Substitution effect = $e_1 \rightarrow e_s$
Income effect = $e_s \rightarrow e_2$

Subst. effect on $L$ (+)
Income effect on $L$ (+)

Income and substitution effects: one more example

Wage rises, equilibrium changes from \( e_1 \) to \( e_2 \)

Income effect: \( e_1 \rightarrow e_2 \)

Substitution effect: \( e_2 \rightarrow e_2 \)
F. Total effect of the rise in W is the \textit{sum} of the income and substitution effects

G. note that substitution effect involves a change in W with U constant, whereas income effect involves a change in U with W constant ➔ of course, a change in the wage changes both U and W!

10. Corner solutions and the decision to work

A. “corner solution” is an equilibrium with $H = 0$, $L = T$ (fulltime “leisure,” zero hours of work)

B. note that this does NOT necessarily involve a tangency – in a corner solution we locate at the “no-work point” with $\text{MRS} \geq \frac{W}{P}$, where $L = T$, $H = 0$, and $C = \frac{V}{P}$. 

![Diagram of corner solutions with and without tangency]
C. **reservation wage**: wage rate that makes the individual indifferent between not working and working ($H = 0$ vs. $H > 0$):

reservation wage is equal to the slope of the indifference curve (\(=\) MRS) **at the “no-work” point**
11. The labor supply curve
   A. Change \( w \) (or \( W \)), ceteris paribus, and see how \( H \) changes
   B. corner solution: for all values of \( W \) below the reservation wage,
      \( H = 0 \) and \( L = T \) – the individual doesn’t work
   C. interior solution: for all values of \( W \) above the reservation wage,
      \( H > 0 \) and \( L < T \) – the individual does work
   D. so labor supply schedule will look as shown below left
      (note that \( W \) is on the vertical axis, \( H \) is on the horizontal axis):

   E. Shape of labor supply curve above the reservation wage depends on
      income and substitution effects
      e.g., “backward-bending” labor supply curve – see above right:
      at lower values of \( W \), substitution effect of higher \( W > \) income effect,
      so \( H \) rises as \( W \) rises; then, at higher values of \( W \), the income effect is
      stronger than substitution effect, so \( H \) falls as \( W \) rises
12. Empirical analysis of labor supply

A. run a regression for hours of work, e.g.,
   \[ H = a + bW + cV + \text{other variables} + e \]  
   (e = error term/unobservables)

   (the other variables – age, education, etc. – are interpreted as representing factors that shift the intercept, e.g., “taste shifters”)

B. if \( L \) is normal, \( c < 0 \)
   if income effect of wage increase > substitution effect, \( b < 0 \)
   if income effect of wage increase < substitution effect, \( b > 0 \)

C. to measure the income effect of a higher \( W \):
   a rise in \( W \) (at constant \( H \)) raises income by \( H \times dW \);
   a rise in \( V \) (at constant \( W \)) changes labor supply by \( dH = c \times dV \)
   so the income effect of a wage increase is given by
   \[
   dH_i = \frac{\text{change in } H \text{ due to higher income}}{dW} = c \times \frac{\text{change in income due to higher } W}{dW}
   \]

D. so we get the \textbf{substitution effect} by \textit{subtracting} the income effect (above) from the total effect:

   \[
   dH_s = \frac{dH - dH_i}{dW} = b - cH
   \]
   (note that theory says this must be positive)
12. Empirical analysis of labor supply (continued)

E. many challenges in empirical analyses of labor supply: e.g.,

* data on \( W \) not available for people who don’t work (thus, difficult to include non-workers in the analysis)

* labor supply schedule is “segmented” (not a straight line):
  flat, with \( H = 0 \), for all wages below the reservation level
  curved (?), with \( H > 0 \), for all wages above the reservation level

* the labor supply equation may be affected by omitted-variables bias (i.e., \( e \) and \( W \) could be correlated for given values of \( V \) and the other variables):

\[
H = a + bW + cV + \text{other variables} + e
\]

F. Female labor supply seems to be lower, but more elastic, than male labor supply
Female labor supply has risen sharply over time in most developed economies (U.S., Europe, etc.)
Male labor supply remained basically the same over time in most economies; at older ages, male labor supply has fallen
13. Welfare programs and work incentives

A. typical AFDC program: grant that is reduced, dollar for dollar, as income rises (equivalent to a 100% marginal “tax” rate!)
B. Negative income tax (NIT): income “guarantee,” with 50% “tax rate” (benefits cut by 50 cents for each dollar earned) as earnings increase, subsidy gradually falls; subsidy equals zero at the “breakeven” level of real income.
C. such programs sharply reduce incentives to work – e.g., AFDC and NIT both raise income and reduce the (net) wage below, note the situation for someone getting NIT payments who is initially below the breakeven level of income – provided L is normal, this person must reduce H, raise L

Someone initially at E will definitely take the NIT and locate along MP after NIT.
D. such programs also encourage “opting in” (or “dropping down”) for persons *not* initially on welfare

(e.g., persons above break-even with relatively flat indifference curves could go on NIT by sharply reducing H, yet still be better off)
E. “MINIT” (negative income tax with minimum work hours requirement) provides a grant and requires minimum H (note that the budget line below starts at H = 30/week)

people working less than 30 hours/week have a strong incentive to work just a little more, in order to get the subsidy however, people already working more than 30 hours/week have some incentive to work somewhat less (as with a NIT)
F. another program: EITC (see text, esp. Figs. 2.16-17) raises the (net) wage for lowest earners – should raise labor force participation
“flat” zone at intermediate income levels where grant is constant – pure income effect tending to reduce hours of work
“tapering off” zone at higher income levels where grant gradually falls to zero – similar to NIT
G. Empirical evidence on the EITC: “difference-in-difference” regression

\[ \text{% in labor force} = a + b_1 \text{AFTER} + b_2 \text{TREATMENT} + b_3 [\text{AFTER} \times \text{TREATMENT}] + \ldots + e \]

\text{AFTER} = 1 \text{ if date is after passage of EITC, } = 0 \text{ otherwise}
\text{TREATMENT} = 1 \text{ if eligible to receive EITC, } = 0 \text{ otherwise}

eligible (\text{TREATMENT} = 1):
\[ \text{new % in LF} = a + b_1 + b_2 + b_3 \quad (\text{TREATMENT} = 1, \text{AFTER} = 1) \]
\[ \text{old % in LF} = a + b_2 \quad (\text{TREATMENT} = 1, \text{AFTER} = 0) \]
\[ \text{difference, new} - \text{old %} = b_1 + b_3 \]

not eligible (“controls,” \text{TREATMENT} = 0):
\[ \text{new % in LF} = a + b_1 \quad (\text{AFTER} = 1) \]
\[ \text{old % in LF} = a \quad (\text{AFTER} = 0) \]
\[ \text{difference, new} - \text{old %} = b_1 \]

“difference-in-difference” = difference for \text{TREATMENT} - \text{CONTROLS}
\[ = (b_1 + b_3) - b_1 = b_3 \]

So the effect of the “treatment” is given by the coefficient \( b_3 \), i.e., by the coefficient on the \( \text{AFTER} \times \text{TREATMENT} \) interaction term
A few caveats:
• What if the effect of time \( (b_1) \) isn’t the same for treated and the controls?
• Spillovers: what if the “experiment” affected the controls as well as the treated group

Effects of EITC:
“difference in difference” estimate of effect of EITC found that EITC raised labor force participation of eligible women by 2.4%, relative to ineligible women (see Table 2.5)

<table>
<thead>
<tr>
<th>group</th>
<th>% participating in the labor market before EITC</th>
<th>% participating in the labor market after EITC</th>
<th>difference</th>
<th>difference-in-differences</th>
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<td>“Treatment” (eligible)</td>
<td>72.9</td>
<td>75.3</td>
<td>2.4</td>
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<td>(unmarried women w/ children)</td>
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<tr>
<td>“Controls” (ineligible)</td>
<td>95.2</td>
<td>95.2</td>
<td>0.0</td>
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<tr>
<td>(unmarried women w/o children)</td>
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