INFORMATION FOR THE MIDTERM EXAM (given in class, Thursday, March 23)

INSTRUCTIONS: On the exam, you will be asked to answer any four of six questions like the ones below. (If you answer more than four, your best four answers will be used to determine your exam grade.) Read each question carefully to make sure you understand it. Your answers should be based on the lectures and text for this course. Explain each answer; make sure it responds directly to the question that has been asked. The attached answers to the review questions should give you a good idea of the kind of answers you will be expected to provide. Note: don't peek at the questions (or answers) until you have reviewed the course material thoroughly. Then, use these questions as a practice exam.

COVERAGE: The exam will cover the lectures and readings in Chapters 1-6 of the text. The exam is "open-book, open-notes."


REVIEW SESSION: Tue., 3/21, 7:15-8:35 pm (this is at night!) in regular classroom (HSC 106).

REVIEW QUESTIONS

1. Joan has a fixed income, I, which she uses to buy only two goods, X and Y. The table below shows what she would buy and her resulting utility, U, at different income levels and prices:

<table>
<thead>
<tr>
<th>I</th>
<th>price of X</th>
<th>price of Y</th>
<th>X</th>
<th>Y</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2</td>
<td>2</td>
<td>25</td>
<td>25.00</td>
<td>25.00</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>8</td>
<td>25</td>
<td>6.25</td>
<td>12.50</td>
</tr>
<tr>
<td>200</td>
<td>2</td>
<td>2</td>
<td>50</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>200</td>
<td>2</td>
<td>8</td>
<td>50</td>
<td>12.50</td>
<td>25.00</td>
</tr>
</tbody>
</table>

At the moment, Joan's income (I) is 100, the price of X is 2, and the price of Y is 2. Then the price of Y rises to 8, but Joan's income stays at 200, and the price of X stays at 2. Given the information in the table, first measure the total change in consumption of Y due to this ceteris paribus rise in the price of Y from 2 to 8; and then divide this total change into substitution and income effects. Show all your work; explain your answer. (Note: your answer must give the actual magnitudes of the substitution, income, and total effects on demand for Y of this change in the price of Y. For example, you might say: "The substitution, income and total effects on Y of the rise in the price of Y are -4, -3, and -7, respectively." You should also explain why the magnitudes of these effects are what you say they are.)

2. Lee purchases only two goods, M and N, out of a fixed income Y. There is a decrease in the price of one of these goods - but you don't know which one! As a result of this price decrease, consumption of N rises and consumption of M falls. You know that M is an inferior good. Given only this information, is it possible to say which of the two prices has fallen - the price of M, or the price of N? Explain your answer.

3. Carla earns $100 per month. Telephone calls, T, cost $0.10 per call. In addition to phone calls, Carla consumes one other good, Y, whose price is $1 per unit. At the moment, Carla makes 200 phone calls per month for a total cost of $20, and purchases 80 units of Y for a total cost of $80. Now the phone company introduces a new pricing policy: the first 200 calls will cost $0.10 each, but additional calls beyond the first 200 (e.g., the 201st call, the 202nd call, etc.) will cost only $0.05 each. Everything else stays the same. Under these circumstances, is it possible to say whether Carla will make more than 200 calls under the company's new pricing policy? Explain your answer.
4. A firm's production function is \( Q = LK \), where \( Q = \) output, \( L = \) labor, and \( K = \) capital. The wage rate, \( w \), is $1 per unit of labor; the rental rate of capital, \( r \), is $2 per unit of capital.

A. Find the amounts of \( L \) and \( K \) that will produce an output level \( Q \) of 200 at the lowest possible cost. Show all your work; explain your answer.

B. Assume that the firm is required to produce output of 200, but that the wage rate now rises to $2 per unit of labor. If all other factors remain unchanged, how will this change in the wage rate will affect the firm's least-cost combination of labor and capital? Show all your work; explain your answer.

C. Is production subject to constant, increasing, or decreasing returns to scale? Show all your work; explain your answer.

5. The utility \( U \) of a consumer who purchases only two goods is given by \( U(x, y) = 2x^{1/2} + 2y \), where \( x \) and \( y \) are the amounts of the two goods purchased. Suppose that the consumer's income is $200; the price of \( y \) is $40/unit; and the price of \( x \) is $20/unit. Now suppose that the price of \( x \) falls to $10/unit.

A. Calculate the change in utility that would accrue to the consumer from this price decrease.

B. Calculate the compensating variation and equivalent variation for the change in the price of \( x \) from $20/unit to $10/unit.

6. In the faraway country of Erewhon, each household has the same tastes and income, and each household consumes 800 gallons of gasoline per year. Then the government introduces a $0.20 per gallon tax on gasoline, combined with a $160 annual gasoline tax rebate to each household. (Note that $160 = 800 \times 0.20.) Thus, each household would pay $0.20 in tax for each gallon of gasoline it consumes; but it would also receive a check for $160 from the government.

A. Relative to the situation before the tax-and-rebate program took effect, will Erewhon households consume more, less, or the same amount of gasoline per year after the program takes effect? Explain your answer.

B. Relative to the situation before the tax-and-rebate program took effect, will Erewhon households consume more, less, or the same amount of all other goods (other than gasoline) per year after the program takes effect? Explain your answer.

C. Relative to the situation before the tax-and-rebate program took effect, will Erewhon households be better off, worse off, or have the same utility after the program takes effect? Explain your answer.

**ANSWERS TO THE REVIEW QUESTIONS FOR THE MIDTERM**

(Note: It's best NOT to look at these answers – or even look at the questions – until after you've done some serious reviewing. The treat the questions and answers like a real exam: give yourself 80 minutes, sit down, write out at least the outline of your answers, and then check them against what you see here.)

1. The effects on \( Y \) of the rise in the price of \( Y \) are as follows:

<table>
<thead>
<tr>
<th>effect</th>
<th>amount</th>
<th>source of calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>6.25 – 25.00 = -18.75</td>
<td>first and second rows of table</td>
</tr>
<tr>
<td>substitution</td>
<td>12.50 – 25.00 = -12.50</td>
<td>first and fourth rows of table</td>
</tr>
<tr>
<td>income</td>
<td>-18.75 – (-12.50) = -6.25</td>
<td>total effect minus substitution effect</td>
</tr>
</tbody>
</table>
To see why, note to the following:

(a) A ceteris paribus increase in the price of Y doesn't change income I, but it does change utility U. To get the effect of higher Y price on Y demand, compare the first and second rows of the table in the question. The first row shows demand when price of Y = 2; the second row shows demand when the price of Y rises to 8, with income constant. Since Y falls from 25 to 6.25, the total effect on Y of the rise in the price of Y is therefore 6.25 - 25 = -18.75.

(b) To calculate the substitution effect of the higher price of Y, we need to know how Y demand would change if we raised the price of Y and simultaneously raised I by enough to keep utility constant. (That's because the substitution effect is the effect of changing relative prices while staying on the same indifference curve.) To do this, compare the first and fourth rows in the table. Comparing these two rows, we see that the price of Y rose from 2 to 8, but income rose by enough (from 100 to 200) to keep utility the same in both cases (U = 25). This kind of constant-utility price increase reduces Y from 25 to 12.50, i.e., changes Y by -12.50.

(c) As usual, the total effect of the price increase is just the sum of the substitution effect and the income effect. The total effect = -18.75; the substitution effect = -12.50; so -18.75 = -12.50 + income effect; so income effect = -18.75 + 12.50 = -6.25.

2. It is not possible to say here which of the two goods has fallen in price. In other words, we will always get the result observed here (M falls, N rises) if PM falls; but we might also get it if PN falls. With only the information given in this question, there is therefore no way to tell which price actually fell. To see why, note several things. First, since M is inferior, N must be normal. (There are only two goods, so they can't both be inferior -- if that were true, a rise in income would result in less total expenditure!) Now analyze how a rise in PM or PN will affect consumption of M and N, using income and substitution effects:

<table>
<thead>
<tr>
<th>case</th>
<th>effect on</th>
<th></th>
<th>effect</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>substitution</td>
<td>income*</td>
<td>total</td>
</tr>
<tr>
<td>PM falls</td>
<td>M</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>PN falls</td>
<td>M</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>+</td>
<td>+</td>
<td>+</td>
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</table>

* Since M is inferior, a price decline has a negative income effect on demand for M.
Since M is inferior, N must be normal, so a price decline must have a positive income effect on demand for N.

Now, a decrease in PN will always generate the result that has occurred here (a decline in M and a rise in N). But this doesn't allow us to rule out the other possibility, i.e., that the price of M has fallen. That's because we could also get the result that has occurred here when the price of M falls, provided (i) the positive substitution effect on M is weaker than the income effect, and (ii) the negative substitution effect on N is weaker than the income effect.

3. Carla will definitely make more than 200 calls after the new pricing structure is introduced. See Figure 3. To begin with, Carla faces budget line abc and is in equilibrium at point b. The cut in price for all calls after the first 200 changes the budget line from abc to abde. At least some points on the new budget line between point b and point d must provide higher utility for Carla than her current equilibrium, point b; and, of course, all points on the new budget line between b and d involve purchases of T in excess of 200. Intuitively, the idea is that the company's new pricing structure reduces the marginal cost of telephone calls after the first 200 calls, increases the "bang per buck" spent on telephone calls after the first 200, and must therefore increase purchases of such calls. (Note that Carla would never locate on
segment ab of the budget line; after all, even before the change in pricing, she preferred point b to any other point on segment ab to the left of b; and the new pricing structure doesn't change any of that.)

4. There's one key respect in which the model of cost-minimizing production differs from the model of utility-maximizing consumption: a cost-minimizing firm wants to produce by getting on the lowest possible isoquant that's consistent with being on a given isocost; whereas a utility-maximizing consumer wants to get on the highest possible indifference curve consistent with being on a given budget line. Thus, in this sense, the two models are opposites of each other (do you see why?). However, in many other respects, the two models have remarkably similar implications.

In particular, in the model of cost-minimizing production, we have two unknowns (in this case, K and L) that we solve for using two equations. Here, the first equation is the "equal bang per buck" criterion, which in this case says that the marginal product of K relative to its price must equal the marginal product of L relative to its price. (Of course, this is formally identical to the "equal bang per buck" criterion in consumer behavior theory.) The other equation we need is the production function, expressing output as a function of K and L. (Note that this is formally identical to the utility function, giving utility as a function of two goods, say, A and B.)

In this problem, the production function is \( Q = LK \). Thus, the marginal product of labor is \( MPL = \frac{\partial Q}{\partial L} = \delta(LK)/\delta L = K \), and the marginal product of capital is \( MPK = \frac{\partial Q}{\partial K} = \delta(LK)/\delta K = L \). This gives us the "equal bang per buck" criterion for this problem: \( MPL/w = MPK/r \) or, substituting in for \( MPL \) and \( MPK \), we have \( K/w = L/r \). Note that this is always true for this production function; whenever \( w \) and \( r \) change, we can just plug in their new values.

(A) Here, \( w = 1 \) and \( r = 2 \), so plug these prices into the equal bang per buck criterion to get

\[
\frac{K}{1} = \frac{L}{2}
\]

Solve this for \( L \), to obtain \( L = 2K \). Now plug this into the production function \( Q = LK \), remembering that in this case \( Q = 200 \): \( Q = LK \) and \( 200 = (2K)K = 2K^2 \). Now divide both sides by 2, and then take the square root, to get \( K = 10 \). Since \( L = 2K \), \( L = 20 \). So \( K = 10, L = 20 \).
(B) Since \( w \) rises to 2, plug these prices into the equal bang per buck criterion, to get

\[
\frac{K}{2} = \frac{L}{2}
\]

Solve this for \( L \), to obtain \( L = K \). Now plug this into the production function \( Q = LK \), remembering that we still have \( Q = 200 \): \( Q = LK \) and so \( 200 = K(K) = K^2 \). Now take the square root, to get \( K = 14.14 \). Since \( L = K \), \( L = 14.14 \) and \( L = K \), so \( L = 14.14 \) also. So \( K = 14.14 \) and \( L = 14.14 \). Note that this change in \( K \) and \( L \) is what we would expect: the rise in \( w \), at a fixed level of output, produces only a pure substitution effect on demands for \( K \) and \( L \). \( K \) rises (from 10 to 14.14); \( L \) falls (from 20 to 14.14).

(C) Production in this case involves **increasing returns to scale**: if we multiply both inputs by the same percentage, then, in this case, we multiply output by a **greater** percentage. For example, the output level \( Q_1 \) produced by input levels \( K_1 \) and \( L_1 \) is equal to \( Q_1 = K_1L_1 \). Now suppose we increase both \( K \) and \( L \) by \( "x" \) percent. This is equivalent to multiplying both \( K \) and \( L \) by \((1+x)\). So the new levels of \( K \) and \( L \) are \( L_2 = (1+x)L_1 \) and \( K_2 = (1+x)K_1 \), respectively. The new level of output produced by these new input levels is given by the production function as \( Q_2 = K_2L_2 = [(1+x)K_1][(1+x)L_1] = (1+x)^2K_1L_1 = (1+x)^2Q_1 \). Thus, \( Q_2 = (1+x)^2Q_1 \), and so multiplying both \( K \) and \( L \) by the same percentage, \((1+x)\), ends up multiplying \( Q \) by a **greater** percentage, \((1+x)^2\). This is **increasing returns to scale** (check the definition in Chapter 6!)

5. Remember the strategies for solving problems of this kind. First, it helps to draw a picture so that you can see what you're doing. Second, review the boxes in the study guide: p. 117 (obtaining demand curves); p. 122 (income and substitution effects); and p. 128 (compensating and equivalent variation).

A. We first need to get the utility-maximizing values of \( x \) and \( y \) for each set of prices, and then work out the level of utility associated with each. To do this, we will use two equations (the equal bang per buck criterion, and the budget constraint) in two unknowns (\( x \) and \( y \)). Once we know \( x \) and \( y \), we can then determine the level of utility using the utility function, our third equation.

To get the equal bang per buck criterion, we first need to know the marginal utilities of \( x \) and \( y \). From the utility function, \( MU_x = \partial U/\partial x = (1/2)x^{1/2}y^{1/2} = x^{-1/2} \) and \( MU_y = \partial U/\partial y = 2 \). We can now plug these into the "equal bang per buck" criterion, which requires that \( MU_x/P_x = MU_y/P_y \), to get \( x/P_x = 2/P_y \). (Note that we can use this no matter what the prices \( P_x \) and \( P_y \) happen to be; just plug in the appropriate values of the prices.)

First, for \( I = 200 \), \( P_x = 20 \) and \( P_y = 40 \), the equal bang per buck criterion here requires \( x^{1/2}/20 = 2/40 \), so solve this for \( x \) to obtain \( x^{1/2} = 40/40 \), or \( x = 1 \). This gives us one of our unknowns, \( x \); use the budget constraint equation to get the other unknown, \( y \), by substituting \( x \) in: that is, \( I = P_x x + P_y y \), so \( 200 = 20(1) + 40y \), so solve for \( y \) to obtain \( y = 4.5 \). Finally, plug \( x = 1 \) and \( y = 4.5 \) into the utility function to get the level of utility at this equilibrium: \( U = 2(1)^{1/2} + 2(4.5) = 11 \).

Next, for \( I = 200 \), \( P_x = 10 \) and \( P_y = 40 \), the equal bang per buck criterion here requires \( x^{1/2}/10 = 2/40 \), so solve this for \( x \) to obtain \( x^{1/2} = 1/2 \), or \( x = 4 \). (Note that when you square both sides of \( x^{1/2} \) you get \( x^{1} = 1/x = 1/4 \), so \( x = 4 \).) This gives us one of our unknowns, \( x \); use the budget constraint equation to get the other unknown, \( y \), by substituting \( x \) in: that is, \( I = P_x x + P_y y \), so \( 200 = 10(4) + 40y \), so solve for \( y \) to obtain \( y = 4 \). Finally, plug \( x = 4 \) and \( y = 4 \) into the utility function to get the level of utility at this equilibrium: \( U = 2(4)^{1/2} + 2(4) = 12 \).

Thus, utility changes from 11 to 12, i.e., increases by 1, when the price of \( x \) falls from 20 to 10.

B. **Compensating variation (CV)**: Here we need to calculate the change in consumer income that would get utility back to its old level, but with the new prices of \( x \) and \( y \). (Thus, we "compensate" for or "offset" the reduction in the price of \( x \) by taking away enough income to drive utility back to its old level.) The equal bang per buck criterion still applies, and since we know what utility has to be (i.e., its
original level, \( U = 11 \)), we can use the utility function. This gives us two equations in two unknowns, which we can use to solve for \( x \) and \( y \). We can then use the third equation, the budget constraint, to work out how much income we need to buy these levels of \( x \) and \( y \). The difference between this new level of income and the original level is the CV.

With the new prices \( P_x = 10 \) and \( P_y = 40 \), the equal bang per buck criterion requires \( x^{1/2}/10 = 2/40 \), so solve this for \( x \) to obtain \( x^{1/2} = 1/2 \), or \( x = 4 \). Utility should be restored to its old level, 11, so plug \( U = 11 \) and \( x = 4 \) into the utility function (our second equation) to get \( 11 = 2(4)^{1/2} + 2(y) \). Solve this for \( y \), to get \( y = 3.5 \). Finally, use the budget constraint to work out how much income we need to buy \( x = 4 \) and \( y = 3.5 \) at the new prices \( P_x = 10 \) and \( P_y = 40 \): \( I = 10(4) + 40(3.5) = 180 \). The difference between this and the original level of income = 180 – 200 = -20. Thus, \( CV = -20 \): after the price of \( x \) falls from 20 to 10, we can get utility back to its original level (at the new prices) by reducing income by 20.

C. Equivalent variation (EV): Here we need to calculate the change in consumer income that would get utility to its new level, but with the old prices of \( x \) and \( y \). (Thus, instead of raising utility by reducing prices, we obtain an "equivalent" effect by adding enough income to raise utility up to its new level.) The equal bang per buck criterion still applies, and since we know what utility has to be (i.e., its new level, \( U = 12 \)), we can use the utility function. This gives us two equations in two unknowns, which we can use to solve for \( x \) and \( y \). We can then use the third equation, the budget constraint, to work out how much income we need to buy these levels of \( x \) and \( y \). The difference between this new level of income and the original level is the CV.

With the old prices \( P_x = 20 \) and \( P_y = 40 \), the equal bang per buck criterion requires \( x^{1/2}/20 = 2/40 \), so solve this for \( x \) to obtain \( x^{1/2} = 1 \), or \( x = 1 \). Utility should be kept at its new level, 12, so plug \( U = 12 \) and \( x = 1 \) into the utility function (our second equation) to get \( 12 = 2(1)^{1/2} + 2(y) \). Solve this for \( y \), to get \( y = 5 \). Finally, use the budget constraint to work out how much income we need to buy \( x = 1 \) and \( y = 5 \) at the old prices \( P_x = 20 \) and \( P_y = 40 \): \( I = 20(1) + 40(5) = 220 \). The difference between this and the original level of income = 220 – 200 = 20. Thus, \( EV = 20 \): instead of reducing the price of \( x \) from 20 to 10, we can get utility to its new level (at the old prices) by increasing income by 20.

Finally, compare your answers to part B (CV = -20) and part C (EV = 20), and note that CV and EV are opposite in sign but equal in magnitude. What is it about the utility function that causes this to be true? If you don't know the answer, better review Chapter 5 again!

6. See Figure 6. All households are identical, so consider a single household (its response will be representative of the response of all households). Let utility depend on gasoline consumption \( G \) and consumption of all other goods, treated as a composite commodity \( C \). Let the original budget line be ab and the original indifference curve \( U^* \), with initial equilibrium at point E. As noted in the question, at this initial equilibrium, \( G = 800 \). The price of \( C \) is \( P_C \), and the price of \( G \) is initially \( P_G \). Then the government introduces a tax of 0.20 per gallon, so \( P_G \) rises from \( P_G \) to \( P_G' = P_G + 0.20 \). Thus, the budget line shifts from ab to ac, as shown in Figure 6. (Note that the slope of ac is steeper than the slope of ab, reflecting the fact that the price of gas has gone up.)

However, that's not the end of the story, because, in addition to the tax, the government also sends each household a rebate check for $160. Note that since initial consumption of \( G \) is 800 gallons and since the tax is 0.20 per gallon, this rebate is exactly enough to make it possible for the household to buy (again) its original equilibrium combination of \( G \) and \( C \), if it chooses to do so. Thus, after the rebate, the new budget line is de, which passes through point E (the original equilibrium) and is parallel to the budget line ac (which reflects the new, post-tax price of \( G \), but which does not include the rebate).

Thus, when all is said and done, the combination of the gas tax of $0.20 per gallon and the rebate of $160 shifts the budget line from ab to de. This is crucial to understanding the rest of the question.

(A) We can be sure that the household will consume less gas after the tax-and-rebate program than it did previously. To see this, note that after the budget line shifts from ab to de, the new budget line slope
exceeds the marginal rate of substitution at point E (where the MRS at E is equal to the slope of the original budget line, ab). This means that G now produces less "bang per buck" than C (do you see why?), and thus that the household will move in a northwesterly direction along budget line de, reducing G and increasing C. Equivalently, the fact that the budget line is now de means that there are now some points on the new budget line – specifically, between E and f – that the household can now afford, and which will certainly provide higher utility than U*.

(B) For the same reasons, we can be sure that the household will consume more of the composite good C than it did previously. (Remember also that, by the composite commodity theorem, "consumption" of C actually means increased expenditure on C.) We know that the household will move from E to some point between E and f; all such points involve greater C.

(C) For the same reasons, we can be sure that the household will be better off (enjoy utility greater than U*) after the tax-and-rebate plan takes effect: as just shown, the household can now afford to locate somewhere between E and f on the new budget line de, and points between E and f must offer higher utility than U*.

Note that this situation is very similar to the situation involving the consumer price index (CPI), as discussed in class and in the text (see Chapter 5). The CPI measures the cost of buying a fixed market-basket of goods. As we saw in our discussion of the CPI, increasing income to match the increase in the cost of buying a fixed market-basket of goods will actually give the household more income than it would need simply to offset the increase in prices, and will actually make the household better off than it was before the price increase took effect. In other words, the increase in the cost of buying a fixed market-basket of goods is more than sufficient to offset the impact of inflation on utility, and is greater than the "compensating variation" needed to offset that impact.

The tax-and-rebate program represents the same kind of situation: the program raises the cost of buying a given combination of C and G, but then gives the household exactly enough money to allow it to go on purchasing the original combination of C and G (i.e., to stay at the point "E" in Figure 6). But, as Figure 6 shows, this will actually increase the household's utility to a level beyond the initial level U*.

As an aside, note that this question is concerned with ideas that are virtually the same as those that arise in Question 3: in both cases, there is a change in the consumer's budget line that allows the consumer to remain at the original equilibrium (point b in Question 3, point E in Question 6), but which also changes prices in a way that creates incentives for the consumer to change his/her consumption and therefore move away from the original equilibrium. Just because you can afford to go on purchasing a given market basket after prices change doesn't mean that it is always optimal to do so.