

(Ir)relevance of Imperfect Consumption Insurance to Optimal Monetary Policy

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Abstract

This paper documents implications of imperfect consumption insurance due to asset market friction for optimal monetary policy. I show a central bank should concern more about inflation stabilization because instable aggregate price level leads to undesired consumption dispersion. The result however does not necessarily implies that a central bank will stabilize inflation more relative to the case of perfect consumption insurance. The reason is that while benefit of stable inflation is even larger with asset market friction, the cost of stabilizing inflation becomes also larger due to the endogenous price stickiness amplified by asset market friction.

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1 Introduction

This paper considers implications of imperfect consumption insurance due to asset market friction for optimal monetary policy in the New Keynesian (NK) framework.

Up to my knowledge, most (if not all) of the papers in the literature of optimal monetary policy within NK framework assume perfect consumption insurance through complete asset markets, so that one does not need to pay much attention on distributional implications of monetary policies. In the real world, however, it is often observed that people with bad income shocks also suffer from reduced level of consumption. It seems that most macroeconomists now agree that the asset markets are less ideal in one way or another, although the degree of market incompleteness and its impact on aggregate dynamics are still being debated and has been one of the main research areas in macroeconomics.

The research question that I would like to answer is following: Does our current knowledge on the characteristics of optimal monetary policy need to be changed if we believe that asset markets are less ideal and consequently consumption insurance among households is also less perfect? I believe the answer to this question is not obvious. Assuming complete markets could indeed be a good proxy for the real world and consequently *nothing* on monetary policy may need to be changed. On the other hand, a central bank could do better by adjusting *some* of its policy behaviors. For instance, a central bank may want to have some additional target variables to stabilize other than inflation and output gap. It also may be the case that inflation stabilization (or output stabilization) is much more important for households welfare than one might have thought. I address this question using the simplest possible NK model. The findings are mixed. The asset market friction is relevant in certain dimensions of optimal policy, but at the same time the complete market assumption is indeed a good approximation in some other dimensions.

Incompleteness of asset markets alters equilibrium allocation of economic variables and serves as a source of inefficient market outcomes through two channels. Firstly, for a given level of aggregate output, the cross-sectional consumption distribution among the households is inefficient. Secondly, as shown in Lee (2007), the economy with imperfect consumption insurance is characterized by a greater degree of real rigidity so that the aggregate

output, after either nominal or real shocks hit the economy, would deviate from the efficient level of output by a larger amount and for a longer period of time. Incomplete asset markets therefore affect not only the mean but also the dispersion of the economic variables. I call these two effects on the allocation of economic variables the first and second order effects of incomplete markets.

Many papers in macroeconomics, when studying implications of incomplete markets, often assume there exists only one kind of financial security, short-term riskless bond, and consequently the economy is characterized by imperfect consumption insurance. This paper takes a different approach. I include a friction in asset markets following the same approach taken by Schulhofer-Wohl (2007). There are enough number of state-contingent assets in the model economy. Thus the asset markets are still complete and there still exists a unique stochastic discount factor. I however assume that there is a transaction cost which makes the households to transfer their resources to insure the income risks by a lesser amount than they otherwise would. Consequently, the consumption insurance is less perfect, that is a household relative consumption is affected by its relative income, although there exists a complete set of financial securities. Therefore, although the asset markets are complete in a nominal sense, they are *effectively incomplete*.

Certainly, there are both advantages and disadvantages of the approach taken by this paper. As noted in Schulhofer-Wohl (2007), the model of complete markets with a transaction cost does not really correspond to any real-world institution. The approach also does not try to derive endogenously the transaction cost or explain explicitly where the transaction cost comes from. However, the lack of explicitness can be advantageous. In the real world, there are number of ways that households share risk, and at the same time there are number of reasons that the risk-sharing may not be perfect. Why does relative consumption of a household sometimes fluctuates together with its relative income? Is it because the number of state-contingent asset is not large enough? Is it because people have imperfect information and limited commitment? Is it due to something else? In my view, there is not yet a consensus among economists. While some popular ways of modeling less-ideal asset markets, such as "bond-only economy" or "economy with optimal contract under limited commitment", are more specific about the asset market institution, they inevitably leave

out many aspects of the real world, and it is not clear that they are capturing the most important mechanism that prevents perfect consumption insurance. By the same token, it would be impossible to include all the risk-sharing institutions into one model, and it is certainly true that there are many other ways to share the risks other than trading the riskless bonds. Therefore while the approach taken in the present paper is a shortcut to provide imperfect consumption insurance, I do not see that the other approaches are more realistic. I also believe the alternative approaches would not change the main results of this paper as long as relative income fluctuations cause relative consumption fluctuation as a result of the imperfect asset market institutions.

The existing literature on optimal monetary policies in the sticky-price NK models often takes a linear-quadratic approach. One major advantage of this approach is that one can explicitly derive a central bank's objective function from households' utilities. With the baseline NK model, the welfare-based loss function that a central bank would minimize usually takes the form:

$$-\frac{\Omega}{2} \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2], \quad (1)$$

where π_t and x_t denote inflation and output gap at time t respectively, while Ω , λ and β are constants. Thus if a monetary authority wants to maximize households' welfare, a desirable monetary policy is the one that achieves stable time paths of inflation and output gap, which also happens to be consistent with the traditional policy objective. Of course the objective function is model-specific and derived under the non-trivial assumption that asset markets are complete. It therefore ignores the additional inefficiency, mentioned above, from imperfect consumption insurance. I follow the same linear-quadratic approach as many existing papers in the literature on optimal monetary policy within NK framework. The advantage is clear. I can show most of the main points of this paper analytically.

I summarize the main results here. Firstly, even if the complete market assumption is relaxed, one can derive the objective function of the same form as (1). Up to the second order approximation, a desirable monetary policy therefore would still be the one that stabilizes the inflation and output gap. A central bank does not need to target any other economic variable even though asset market friction causes additional inefficiency. This is because a

degree of inefficient consumption dispersion can be entirely summarized by fluctuations of the current as well as past inflations. It is instability of aggregate price level and inflation that leads to undesired consumption dispersion among the households.

The first result leads us to the second main point of this paper: Even though the policy objective function has the same functional form, a central bank would want to place a larger relative weight on inflation stabilization. It is because stabilizing inflation provides the additional advantage of reducing undesired consumption dispersion. In (1), the optimal relative weight on output gap, λ should be smaller.

Thirdly, even if λ is smaller, it does not necessarily mean that the relative volatility of realized inflation to realized output gap, under the optimal monetary policy, also becomes smaller. The realized relative standard deviation $\frac{std(\pi)}{std(x)}$ indeed gets smaller only if a degree of financial market friction is sufficiently large. The intuition for the third result is following. As mentioned above, the imperfect consumption insurance has two effects on equilibrium allocation: the first and second order effects. Inefficient dispersion of households' consumptions is "the second order effect" since a measure of the dispersion is essentially a second moment. However asset market friction also affects the allocation of aggregate variables due to endogenous price stickiness, which is reflected in a reduced slope of the Phillips curve. I call it "the first order effect". Therefore, while a central bank cares more about inflation-stabilization as reflected in its loss function, the cost of stabilizing inflation also becomes larger as the Phillips curve becomes flatter with friction in asset market. The realized inflation thus could fluctuate either more or less depending on whether the second order effect dominates the first order effect or not. The consumption dispersion would be larger as asset market friction gets bigger and thus the second order effect would dominate with a larger degree of friction.

There is a large number of papers that study optimal monetary policy taking linear-quadratic approach within a NK framework. The literature includes Rotemberg and Woodford (1997), Aoki (2001), Benigno (2004), Clarida, Gali, and Gertler (2002), Erceg, Henderson, and Levin (2000), and Benigno and Woodford (2007). All of these papers derive the welfare-based loss functions in various contexts. This paper takes the same methodology

but in a different economic environment.¹

2 Model

I describe the model economy in this section. The model is similar to the standard NK model in Woodford (2003, chapter 3). The only difference is the existence of a cost of transferring resources among the households as in Schulhofer-Wohl (2007). As a result, the asset markets are *effectively incomplete* and households are not able to insure their income risks perfectly.

2.1 Household

There is a continuum of industries indexed by $i \in [0, 1]$, each of which produces a different type of product. Each industry i is represented by one firm called type- i firm. Different labor skills are required to produce different types of product, that is labor skill is industry-specific. In each industry i , there is a representative household called type- i household. The type- i household possesses a labor skill specialized exclusively for industry i , and thus supplies labor service to type- i firm.

Type- i household maximizes the following discounted expected utility function:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t(i)^{1-\sigma} - 1}{1-\sigma} - \frac{H_t(i)^{1+\varphi}}{1+\varphi} \right] \right),$$

where $C_t(i)$ denotes type- i household's consumption, $H_t(i)$ denotes the hours of labor services supplied to industry i , β is the discount factor, φ is the inverse of Frisch elasticity of labor supply and σ is the coefficient of relative risk aversion. The preference parameters β , φ and σ are non-negative.

The household's dynamic budget constraint is given by

$$E_t [Q_{t,t+1} B_{t+1}(i)] + P_t C_t(i) + \underbrace{P_t \phi k(C_t(i), X_t(i))}_{\text{real cost}} = B_t(i) + W_t(i) H_t(i) + \int_0^1 \Pi_t(i) di,$$

¹For an overview of optimal monetary policy from NK perspective, see Clarida, Gali, and Gertler (1999) and Woodford (2003).

The literature review is incomplete and still in progress.

where P_t denotes aggregate price level, $W_t(i)$ is the competitive wage rate in industry i and $\Pi_t(i)$ is dividends from capital ownership of type- i firm. A household's *total* income at time t is the sum of *labor* income $W_t(i)H_t(i)$ and *capital* income $\int_0^1 \Pi_t(i)di$. I let $X_t(i)$ denote type- i household's *total* real income at time t :

$$X_t(i) \equiv \frac{W_t(i)H_t(i) + \int_0^1 \Pi_t(i)di}{P_t}.$$

Unlike labor income, a household's capital income comes from every industry, that is each household holds a mutual fund so that the economy's total profit is equally distributed among the households. Consequently, the income differential between any two households is entirely due to a difference in labor income not capital income. The equal capital ownership is a simplifying assumption that does not affect the main results of this paper.

Households can issue nominal securities with arbitrary patterns of state-contingent payoffs. In budget constraint, $B(i)$ denotes type- i household's holding of one period state contingent nominal securities and $Q_{t,t+1}$ is a unique stochastic discount factor. At time t when the household makes its portfolio decision, $B_{t+1}(i)$ is a random variable that can have different values depending on the state realized at time $t+1$. Households completely specify whole distribution of $B_{t+1}(i)$ at time t , taking the market prices for the state-contingent payoffs as given.

Making consumption different from its income is costly for households. If a household's consumption $C(i)$ is different from its income $X(i)$ then the amount $\phi k(C(i), X(i))$ of consumption good is taken away from the household's resource. I assume $\phi \geq 0$. An important special case is when $\phi = 0$. The model presented here is then the same as the standard NK model. Following Schulhofer-Wohl (2007), I let the transaction cost function $k(\cdot)$ have the following form:

$$k(C, X) = \frac{C}{2} \left(\log \frac{C}{X} \right)^2.$$

However, the specific function given above is not necessary because any convex transaction cost would bring us the same result.

A household's first order conditions are

$$\begin{aligned} C_t(i)^{-\sigma} &= \lambda_t(i) P_t \{1 + \phi k_C(C_t(i), X_t(i))\} \\ Q_{t,t+1} &= \frac{\beta \lambda_{t+1}(i)}{\lambda_t(i)} \\ H_t(i)^\varphi &= \lambda_t(i) W_t(i) \{1 - \phi k_X(C_t(i), X_t(i))\}, \end{aligned}$$

where $\lambda_t(i)$ is the Lagrange multiplier and k_C and k_X are the partial derivatives of k with respect to the level of consumption and income:

$$\begin{aligned} k_C &\equiv \frac{\partial k}{\partial C} = \left(\log \frac{C_t(i)}{X_t(i)} \right) + \frac{1}{2} \left(\log \frac{C_t(i)}{X_t(i)} \right)^2 \\ k_X &\equiv \frac{\partial k}{\partial X} = -\frac{C_t(i)}{X_t(i)} \left(\log \frac{C_t(i)}{X_t(i)} \right). \end{aligned}$$

Substituting out the multiplier from the first order conditions yields

$$Q_{t,t+1} = \beta \frac{P_t C_t(i)^\sigma \{1 + \phi k_C(C_t(i), X_t(i))\}}{P_{t+1} C_{t+1}(i)^\sigma \{1 + \phi k_C(C_{t+1}(i), X_{t+1}(i))\}}, \quad (2)$$

$$\frac{W_t(i)}{P_t} = H_t(i)^\varphi C_t(i)^\sigma \frac{\{1 + \phi k_C(C_t(i), X_t(i))\}}{\{1 - \phi k_X(C_t(i), X_t(i))\}}. \quad (3)$$

To issue a riskless one-period nominal bond is equivalent to a household's choosing a flat distribution in which $B_{t+1}(i) = 1$ for all possible states of economy at time $t + 1$. Therefore the gross nominal interest rate should be given by $R_t^{-1} = E_t [Q_{t,t+1}]$ by no arbitrage condition. The last equation and (2) together give a consumption Euler equation

$$R_t^{-1} = \beta E_t \left[\frac{P_t C_t(i)^\sigma \{1 + \phi k_C(C_t(i), X_t(i))\}}{P_{t+1} C_{t+1}(i)^\sigma \{1 + \phi k_C(C_{t+1}(i), X_{t+1}(i))\}} \right]. \quad (4)$$

One can show that in the case of no transaction cost where $\phi = 0$, the economy is characterized by perfect consumption insurance. With a normalizing assumption on the distribution of the households' initial wealth, one necessarily obtains that

$$C_t(i) = C_t(j) \quad \forall i, j \in [0, 1], \quad (5)$$

which should hold for every time period t and also for every possible state of the economy. However, (5) no longer holds when ϕ is positive. A household relative consumption is higher when its relative income is higher, and consequently cross-household consumption distribution departs from the efficient distribution that would be realized when $\phi = 0$.

I can loglinearize the equilibrium conditions around the symmetric efficient non-stochastic steady state. The first order approximation of (2), (3) and (4) would then look like

$$c_t(i) = c_{t+1}(i) + \frac{1}{\sigma + \phi} (q_{t,t+1} - \pi_{t+1}) + \frac{\phi}{\sigma + \phi} (x_t(i) - x_{t+1}(i)), \quad (6)$$

$$w_t(i) - p_t = \varphi h_t(i) + \sigma c_t(i), \quad (7)$$

$$c_t(i) = E_t c_{t+1}(i) - \frac{1}{\sigma + \phi} (r_t - E_t \pi_{t+1}) + \frac{\phi}{\sigma + \phi} (x_t(i) - E_t x_{t+1}(i)). \quad (8)$$

I use the lowercase letters to denote percentage deviations from the steady state.² From (6), one can derive an analytical expression for a household's consumption as a function of its income.

Proposition 1. *Up to the first order approximation, the relationship between a household's relative consumption and relative income is given by*

$$c_t^R(i) = \frac{\phi}{\sigma + \phi} x_t^R(i) \quad (9)$$

for every time period t and for every state of the economy.

Proof. See the Appendix. □

The letters with superscript R , $c_t^R(i)$ and $x_t^R(i)$, denote type- i household's consumption and income relative to aggregate level of consumption and of income. The proposition indicates that a household's consumption moves in the same direction with its income as long as ϕ is positive.

Schulhofer-Wohl (2007) had estimated the ratio of transaction cost to risk aversion $\omega \equiv \phi/\sigma$ and shows that a reasonable ratio would be in the range of 0.117 to 0.205 if one

²For instance,

$$c_t(i) \equiv \log C_t(i) - \log C,$$

where C is the common steady state level of consumption of households.

accounts for the heterogeneous risk and time preferences among the households. If one does not account for the heterogeneous preferences as in this paper, the estimated ω becomes larger and it can be greater than 0.5. Schulhofer-Wohl (2007)'s estimate of transaction cost parameter is considered one of the smallest. I set ω to be 0.2 as a benchmark to be conservative. To put the magnitude of the estimated ω into context, I can rewrite (9) as

$$c_t^R(i) = \frac{\omega}{1 + \omega} x_t^R(i). \quad (10)$$

With a benchmark value of $\omega = 0.2$, (10) implies that a household's consumption would be higher by 16.7% than the economy's average consumption, if its real income were larger by 100% than the economy's average income.

2.2 Firms

This subsection describes the supply side of the economy. As mentioned above, there is a continuum of industries indexed by $i \in [0, 1]$, each of which produces a distinguished type of product $Y_t(i)$. In each industry i , there is a representative firm called type- i firm.

The differentiated products $\{Y_t(i)\}_{i \in [0,1]}$ are used to produce final consumption good Y_t , through a CES technology given by

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}.$$

The corresponding price index P_t for the final consumption good is

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}},$$

where $P_t(i)$ is the price of type- i product. The optimal demand for each type of good is then given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t. \quad (11)$$

Type- i firm's production function is given by

$$Y_t(i) = A_t H_t(i), \quad (12)$$

where A_t denotes the level of economy-wide technology. Each firm maximizes its profit with the given technology (12) while facing the demand function for its product (11).

If the prices were flexible, there would be an efficient equilibrium in which the firms choose a common optimal price and the households choose a common level of consumption. The natural output, the level of aggregate output that would be realized with flexible prices, is then given by

$$Y_t^N = A_t^{\frac{1+\varphi}{\sigma+\varphi}}, \quad (13)$$

or taking log gives

$$y_t^N = \left(\frac{1+\varphi}{\sigma+\varphi} \right) a_t. \quad (14)$$

Obtaining (13) and (14), I have implicitly assumed that there exists a constant lump-sum subsidy to employment so that the natural output is also an efficient level of aggregate output. In principle even the natural level of output is inefficient due to imperfect competition. The existence of subsidy is therefore a simplifying assumption so that I can ignore the inefficiency due to imperfect competitiveness of the intermediate goods markets and instead can focus on the welfare loss from imperfect price adjustment and from financial market friction.

As emphasized by Lee (2007), with the absence of idiosyncratic shocks, the asset market incompleteness by itself does not generate inefficient equilibrium outcomes when prices are flexible: because the disaggregate prices and outputs of the firms are symmetric, the consumption levels of the households should also be symmetric. Consequently the aggregate variables are not affected by the asset market friction.

When the prices are not flexible, however, even the aggregate shocks act like idiosyncratic shocks as the timing of price adjustments are not synchronized among the firms and consequently the labor incomes across the households are different. In consequence, the consumption distribution must be different from the efficient consumption distribution that

would be realized if there were no friction in asset market (or the prices were fully flexible).

As in the standard NK models, I model the nominal rigidity following Calvo (1983) and Yun (1996). The firms adjust their prices with probability $1 - \alpha$ each period. When they have a chance to re-optimize, they set the prices that maximize expected profits. A firm that adjusts its prices at time t solves the following profit maximization problem:

$$\max_{P_t^*(i)} E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \tilde{\Pi}_{t+k}(i),$$

where

$$\begin{aligned} \tilde{\Pi}_{t+k}(i) &= P_t^*(i) Y_{t+k}(i) - (1-s) \frac{W_{t+k}(i)}{A_{t+k}} Y_{t+k}(i) \\ &= P_t^*(i) \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta} Y_{t+k} - (1-s) \frac{W_{t+k}(i)}{A_{t+k}} \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta} Y_{t+k}. \end{aligned}$$

In the equation above, s is the employment subsidy. The first order condition is

$$0 = E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta} Y_{t+k} \left\{ P_t^*(i) - \frac{W_{t+k}(i)}{A_{t+k}} \right\}. \quad (15)$$

By loglinearizing the first order condition above, I can obtain the generalized NK Phillips curve which nests the case of perfect consumption insurance as a special case.

Proposition 2. *Consider the heterogeneous household model with imperfect consumption insurance described in this paper. The aggregate output and inflation must satisfy a Phillips curve (or an aggregate supply relation) of the form*

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - y_t^N),$$

where

$$\begin{aligned} \kappa &\equiv \left\{ \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \right\} \left\{ \frac{\sigma + \varphi}{1 + \theta(\varphi + \sigma\gamma)} \right\}, \\ \gamma &\equiv \frac{\phi(1+\varphi)}{\sigma(1-\phi) + \phi}. \end{aligned}$$

The functional form of the Phillips curve therefore is not changed from the baseline NK model. However the slope of the Phillips curve, κ gets smaller as the degree of the transaction cost ϕ gets larger as long as $(\sigma - 1)\phi \leq \sigma$.

The inequality $(\sigma - 1)\phi \leq \sigma$ is a necessary and sufficient condition that makes γ positive and it is assumed throughout this paper. Since I only consider the case where $\sigma \in [0, 6]$, the condition is satisfied as long as $\phi \leq 1.2$.

The proposition suggests that the economy with imperfect consumption insurance is characterized by a greater degree of real rigidity. This result is consistent with the one from Lee (2007) although the asset market institutions that prevent perfect consumption insurance are different.³

In the special case in which $\phi = 0$, the slope of the Phillips curve would be the same as the one in the benchmark complete market model. On the other hand, suppose the coefficient of relative risk averse σ is 3, which is not an unreasonable value, and let $\phi = 0.6$ so that $\omega = 0.2$. Then the slope is only about 37% of the slope under perfect consumption insurance.⁴ If a higher value of either σ or ϕ were used, then the slope would get even smaller. This suggests that the real rigidity due to the asset market friction can potentially be substantial. Figure 1 plots the slope of the Phillips curve for $\phi \in [0, 1.2]$.

3 Equilibrium

Equilibrium is characterized by allocation of the resources and prices that satisfy the households' optimality conditions and budget constraints, the firms' optimality conditions, the monetary policy (to be specified later) and finally the market clearing conditions:

$$\int_0^1 \{C_t(i) + \phi k(C_t(i), X_t(i))\} di = Y_t, \quad \int_0^1 B_t(i) di = 0$$

for every time t and every state of the economy.

In the log-linear approximation, however, if we were only interested in the dynamics of the aggregate variables $\{y_t, \pi_t, r_t\}$, then only two equations, the aggregate demand and

³Lee (2007) assumed that there was only one type of asset: riskless bond.

⁴Unless noted otherwise, I set β to be 0.99, φ to be 1, θ to be 6, and α to be 0.5 throughout the paper.

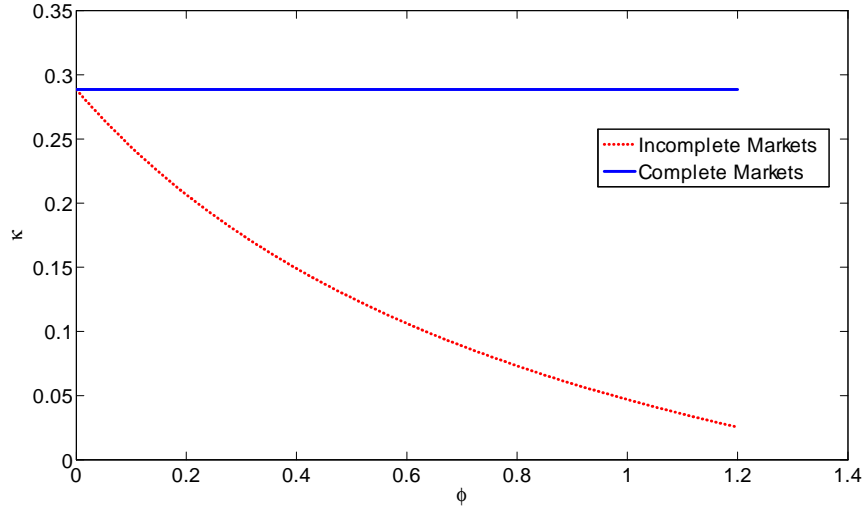


Figure 1: Slope of the Phillips Curve

supply relations:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (y_t - y_t^N),$$

together with a characterization of monetary policy would completely determine the dynamics of those variables. The aggregate demand equation (or the IS equation) can be derived by integrating the households' Euler equations (8) over unit interval.

This suggests that once we correctly adjust the slope of the Phillips curve, the asset market imperfection does not play any role as long as one's focus is only on the aggregate dynamics up to first order approximation. Especially one does not have to keep track of the cross-sectional distribution of the households' consumptions and asset holdings.

4 Monetary Policies

4.1 A Utility-Based Welfare Criterion

A desirable monetary policy would be one that minimizes certain loss function. The loss function that has been most extensively used in academic literature is one or another vari-

ation of (1), the expected value of a discounted sum of weighted average of terms quadratic in the deviation of inflation from a certain target rate and some measure of output gap over time. While that kind of policy objective conforms with the conventional wisdoms, it is not long ago that it was given a theoretical justification. Economists have shown that, at least in the baseline NK model, the conventional loss function can be derived by taking the second order approximation of the households' expected utilities. One of the earliest papers that have developed this method is Rotemberg and Woodford (1997). Woodford (2003, Ch 6) also discusses the method in a great detail. That approach not only provides a theoretical justification for the traditional loss function but also gives a theoretically consistent relative weight on each target variable.

There is a situation when the quadratic approximation method does not correctly rank alternative policies as discussed in Kim and Kim (2003). The model presented here, however is not subject to that criticism because the non-stochastic steady state allocation of the model is efficient.

I suppose the government's policy objective is to maximize sum of the households' intertemporal utilities:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \int_0^1 \left[\frac{C_t(i)^{1-\sigma} - 1}{1-\sigma} - \frac{H_t(i)^{1+\varphi}}{1+\varphi} \right] di \right), \quad (16)$$

In what follows, I derive a quadratic loss function that is the second order approximation of (16), following the method presented in Woodford (2003).

In the benchmark NK model, output gap and inflation enter the government quadratic loss function: welfare loss depends on the variability of both output gap and the rate of inflation. The time variance of the output gap matters for welfare because the households wish to keep the output at the efficient level which corresponds here to the flexible-price level of output. However the cross-sectional dispersion of outputs matters as well, due to the relative price distortion. Not only the prices do not adjust continuously, but also the timings of price adjustments among different firms are not synchronized so that an unstable aggregate price level leads to inefficient price dispersion. Thus stabilizing aggregate inflation is also important.

If the complete market assumption is relaxed, not only the cross-sectional distribution of the productions but also the consumption distribution among the households would be inefficient over the business cycle. If anything, one might imagine that the relative price dispersion would lead to a relative income dispersion and consequently a consumption dispersion which is undesirable because the households' consumptions should be identical one another under the efficient consumption distribution. This provides an additional case for stable inflation. More specifically one can show a degree of inefficient consumption dispersion can be entirely summarized by fluctuations of the current as well as the past inflations.

Proposition 3. *Up to the second order approximation, cross-sectional dispersion of the current consumptions is given by a weighted sum of terms quadratic in the current and the past inflations:*

$$\frac{1}{(\theta\gamma)^2} \text{Var}_i [c_t(i)] = \sum_{s=0}^t \alpha^{t-s} \left(\frac{\alpha}{1-\alpha} \right) \pi_s^2 + \alpha^{t+1} \text{Var}_i [p_{-1}(i)] + O(\|\xi\|^3),$$

where $\text{Var}_i[\cdot]$ is the cross-sectional variance and $O(\|\xi\|^3)$ denotes all relevant terms that are of third or higher order.

Proof. See the Appendix. □

It is therefore quite intuitive that the government should place an even larger weight on inflation stabilization if consumption insurance is less than perfect. The following proposition shows that a greater attention to the inflation stabilization is indeed reflected in the utility-based loss function.

Proposition 4. *The discounted sum of the utilities of households (16) is given by*

$$-\frac{\Omega}{2} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}$$

where

$$L_t = \pi_t^2 + \lambda x_t^2 + t.i.p + O(\|\xi\|^3),$$

$$\lambda = \frac{(1-\alpha)(1-\alpha\beta)(\sigma+\varphi)}{\alpha\{1+\varphi\theta+\theta\sigma\gamma^2\}} \frac{1}{\theta},$$

$$\Omega \equiv YU_C(Y) \frac{(1-\alpha)(1-\alpha\beta)}{\alpha\{1+\varphi\theta+\theta\sigma\gamma^2\}} \frac{1}{\theta},$$

and *t.i.p* stands for the terms independent of monetary policy.

Proof. See the Appendix. □

The effect of the imperfect asset markets is reflected by the term $\theta\sigma\gamma^2$ in the denominator of the relative weight on output gap λ . A positive γ due to transaction costs makes λ smaller. Figure 2 shows the optimal relative weight, λ for $\phi \in [0, 1.2]$ and $\sigma = 3$. The proposition and figure suggest that a monetary authority should concern more about stabilizing inflation if it believes that unstable inflation affects the households' wealth and consumption distribution in an inefficient way.

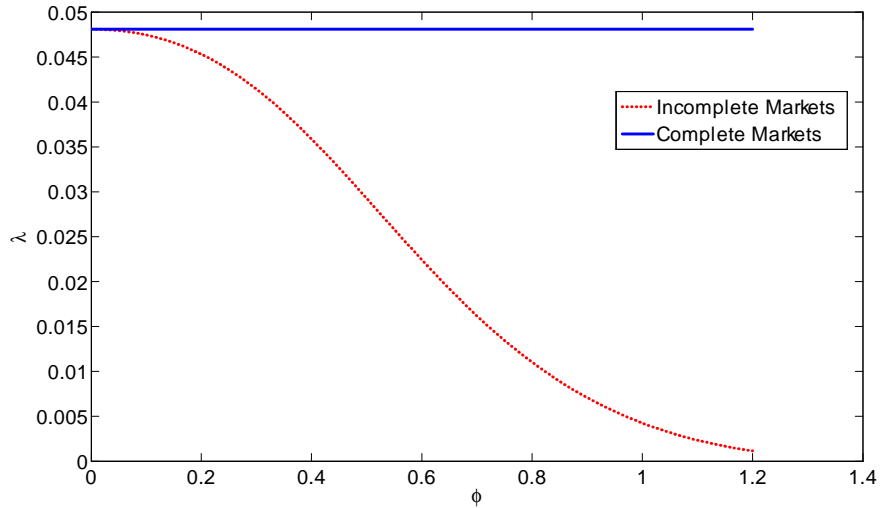


Figure 2: Relative Weight

4.2 Optimal Monetary Policy

I analyze optimal monetary policy under discretion rather than commitment since it is easier to show the main points. A monetary authority takes private sector's expectations as given and minimizes the loss function period by period. More specifically a central bank

tries to pick a pair of inflation and output gap that minimizes

$$\pi_t^2 + \lambda x_t^2,$$

and that also satisfies the constraint

$$\pi_t = \kappa x_t + \beta \pi^e.$$

The first order condition of this static problem is

$$\pi_t^* = -\frac{\lambda}{\kappa} x_t^*,$$

which should hold for all t . A pair $\{\pi_t^*, x_t^*\}$ is the equilibrium inflation and output gap under optimal discretionary monetary policy.

In this specific setup, there is no policy trade-off between output gap and inflation stabilization. The central bank can simply choose $\pi_t^* = x_t^* = 0$ and this pair minimizes the loss function while satisfying the constraint (i.e. Phillips curve).

It is however no longer the case if there is a supply (or cost-push) shock as in many other papers in the literature of optimal monetary policy. The Phillips curve then would look like

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t,$$

where u_t is the cost-push shock. There are more than one way to introduce this shock, which I do not discuss here. The constraint that the central bank faces would then be

$$\pi_t = \kappa x_t + \beta \pi^e + u_t.$$

In this case, both inflation and output gap would fluctuate even under optimal monetary policy and thus have non-zero time variances.

What would happen to the *relative* volatility of realized time path of inflation to realized time path of output gap under discretionary optimal monetary policy? Does the asset market friction affects the *optimal relative volatility* of equilibrium inflation? The answer

must be yes in principle because I have shown that a central bank would care more about stable inflation if consumption insurance were less than perfect.

One measure of optimal relative volatility would be the ratio of standard deviation of inflation to standard deviation of output gap:

$$ORV \equiv \frac{std(\pi_t^*)}{std(x_t^*)},$$

where ORV stands for *optimal relative volatility*. One might naturally conjecture that ORV would become smaller as the degree of asset market friction gets larger because a central bank would care more about inflation stabilization. However it is not necessarily true. In fact there is an analytical expression for ORV and it is given by

$$ORV = \frac{\lambda}{\kappa} = \left(\frac{1 + \theta\varphi + \theta\sigma\gamma}{1 + \theta\varphi + \theta\sigma\gamma^2} \right) \frac{1}{\theta}.$$

Under perfect consumption insurance (i.e. $\phi = 0$), γ is also equal to zero so that

$$ORV_{\phi=0} = \frac{1}{\theta},$$

where $ORV_{\phi=0}$ denotes the value of ORV when $\phi = 0$. Therefore ORV is smaller than $ORV_{\phi=0}$ if and only if $\gamma > 1$ and vice versa. Other parameters being fixed, γ is increasing in the degree of asset market friction, ϕ . More specifically one can show the following.

Proposition 5. *Suppose there exists a friction in the asset markets so that $\phi > 0$. Then the relative volatility of inflation to output gap under discretionary optimal monetary policy should satisfy the following properties:*

$$\begin{aligned} \phi > \frac{\sigma}{\sigma + \varphi} &\implies ORV < ORV_{\phi=0} \\ \phi < \frac{\sigma}{\sigma + \varphi} &\implies ORV > ORV_{\phi=0} \\ \phi = \frac{\sigma}{\sigma + \varphi} &\implies ORV = ORV_{\phi=0}. \end{aligned}$$

Proof. To be added. In fact it is a matter of simple algebra. □

Therefore a central bank would let the equilibrium inflation be relatively less volatile in the incomplete market economy only if the degree of asset market friction were sufficiently large, even though the relative weight on the inflation in the welfare loss function is always larger in the case of incomplete markets.

An intuition is following. As mentioned above, the imperfect consumption insurance has two effects on equilibrium allocation. Firstly, for a given aggregate output and price level, it affects the cross-sectional distributions of resources, especially the consumption distribution among the households. The undesired consumption dispersion is the reason for a larger relative weight on inflation in the welfare loss function. I would call it "the second order effect" of imperfect consumption insurance since the dispersion of consumptions is a second moment. However the asset market friction also affects the allocation of aggregate variables due to the endogenously amplified price stickiness, which is reflected in a reduced slope of the Phillips curve. I call it "the first order effect". Therefore, while a central bank cares more about inflation as reflected in its loss function, the cost of inflation stabilization also becomes larger as the Phillips curve becomes flatter with asset market friction. The realized inflation thus could fluctuate more or could fluctuate less depending on whether the second order effect dominates the first order effect or not. The consumption dispersion would become larger as the degree of asset market friction gets bigger. Therefore the second order effect would be dominant with a large value of ϕ .

Figure 3 shows ORV for some alternative values of σ and ϕ . As mentioned above, ORV is smaller than $ORV_{\phi=0}$ only if ϕ is sufficiently large. Another observation worth mentioning is that the effect of asset market friction on equilibrium volatility of output and inflation is larger when the households are more risk-averse. This is because both the first and the second order effects are amplified as σ gets larger for a given degree of asset market friction. The first order effect is amplified because the wealth effect on labor supply becomes more important as σ gets larger. On the other hand, the second order effect becomes more important because the households dislike less-insured consumptions even more and thus a consumption dispersion reduces households' welfare even more.

For certain parameter values, the imperfect consumption insurance does not make a big difference *quantitatively* in terms of equilibrium time paths of inflation and output gap

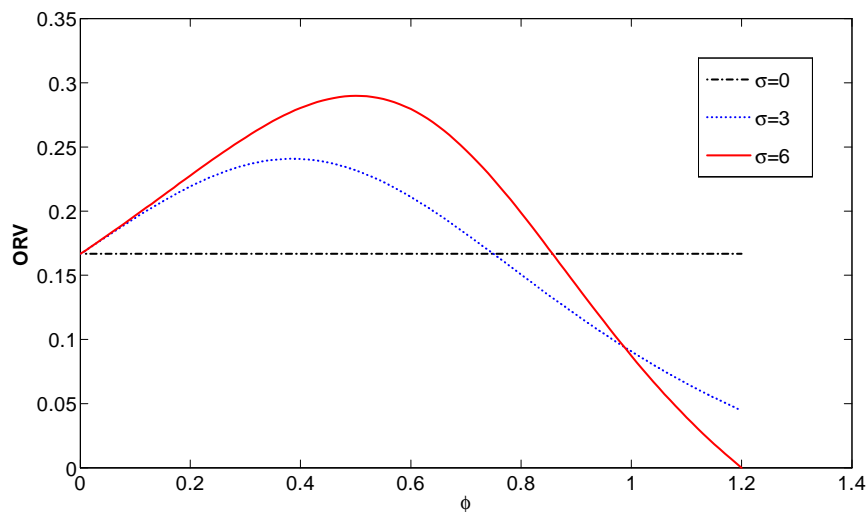


Figure 3: Optimal Relative Volatility

under optimal monetary policy. In fact there is even a case in which asset market friction does not make any difference at all. In those cases, conducting monetary policy ignoring the asset market friction incidentally cause little problem due to the identical end-result, even though the difference in the optimal weight, λ is substantial between the complete and incomplete market economy. Those are the situations in which the first order and the second order effect are completely or nearly canceled out.

However, there are situations in which asset market friction makes a non-trivial difference even in terms of the realized output gap and inflation. Let us consider the case in which the households are somewhat more risk-averse and set σ to be 6, which is a relatively large, yet not unreasonable value. Then I consider two calibrations for the asset market friction: $\phi = 0.6$ and $\phi = 1.2$. These are legitimate degrees of asset market friction as $\omega = \phi/\sigma$ are 0.1 and 0.2 respectively, which are in the reasonable range according to Schulhofer-Wohl (2007)'s estimation. From Figure 3, $ORV_{\phi=0.6}$ is about 0.3 suggesting that the output gap should fluctuate more than inflation only by 333% rather than by 600% as in the complete market economy. On the other hand, when $\phi = 1.2$, the second order effect is so dominant that a central bank wants to dampen the movement of inflation completely while leaving the output gap fluctuations uncontrolled. Therefore the quantitative importance of asset market

friction for monetary policy crucially depends on a couple of parameters: the measure of risk aversion and the measure of asset market friction.

5 Conclusion

I study some implications of asset market friction for the optimal monetary policy within a simple NK framework. I show that the asset market friction could matter and that there could be a welfare loss if a central bank conducts monetary policy pretending the asset markets are complete.

I also have briefly discussed when asset market friction is likely and not likely to have quantitatively meaningful implications for the optimal policy. However I deliberately use the simplest possible model with the purpose of showing the results in a clear way. The paper therefore is not intended for a serious quantitative experiment. An elaborative quantitative study with more realistic models such as the ones in Christiano et al. (2005) or Smets and Wouters (2003), is left as a future research.

Appendix

A Proof

A.1 Proof of Proposition 1

From (6) one can derive the following equation:

$$c_t^R(i) - \frac{\phi}{\sigma + \phi} x_t^R(i) = c_{t+1}^R(i) - \frac{\phi}{\sigma + \phi} x_{t+1}^R(i), \quad (17)$$

which must hold for every time period t and for every state of economy. (17) implies that $c_t^R(i) - \frac{\phi}{\sigma + \phi} x_t^R(i)$ should be some constant. Let

$$c_t^R(i) - \frac{\phi}{\sigma + \phi} x_t^R(i) = z,$$

for some constant z . Then it is necessary that $z = 0$ because $\int_0^1 c_t^R(i) di = \int_0^1 x_t^R(i) di = 0$.

A.2 Proof of Proposition 2

Loglinearizing a firm's first order condition (15) gives

$$\begin{aligned} E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [p_t^*(i) - p_{t+k}] &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [w_{t+k}(i) - a_{t+k} - p_{t+k}] \\ &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [w_{t+k}(i) - a_{t+k} - p_{t+k}] \\ &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [\varphi h_{t+k}(i) + \sigma c_{t+k}(i) - a_{t+k}] \\ &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [\varphi y_{t+k}(i) + \sigma c_{t+k}(i) - (1 + \varphi)a_{t+k}] \\ &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [\varphi y_{t+k}^R(i) + \sigma c_{t+k}^R(i) + (\sigma + \varphi)y_{t+k} - (1 + \varphi)a_{t+k}] \\ &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\varphi y_{t+k}^R(i) + \frac{\sigma\phi}{\sigma + \phi} x_{t+k}^R(i) + (\sigma + \varphi)y_{t+k} - (1 + \varphi)a_{t+k} \right] \end{aligned} \quad (18)$$

Note that

$$x_t^R(i) = \left(\frac{1 + \varphi}{1 - \frac{\sigma\phi}{\sigma + \phi}} \right) y_t^R(i). \quad (19)$$

By substituting (19) into (18), I can rewrite the linearized first order condition as

$$\begin{aligned} & E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [p_t^*(i) - p_{t+k}] \\ = & E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [(\varphi + \sigma\gamma) y_{t+k}^R(i) + (\sigma + \varphi) y_{t+k} - (1 + \varphi) a_{t+k}], \end{aligned} \quad (20)$$

where

$$\gamma = \frac{\phi}{\sigma + \phi} \frac{1 + \varphi}{1 - \frac{\sigma\phi}{\sigma + \phi}} = \frac{\phi(1 + \varphi)}{\sigma + \phi - \sigma\phi}.$$

Note that linearizing the demand function gives

$$y_{t+k}^R(i) = -\theta [p_t^*(i) - p_{t+k}] \quad (21)$$

Then substituting (21) into (20) gives

$$\begin{aligned} & (1 + \theta(\varphi + \sigma\gamma)) E_t \sum_{k=0}^{\infty} (\alpha\beta)^k [p_t^*(i) - p_{t+k}] \\ = & (\sigma + \varphi) E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[y_{t+k} - \left(\frac{1 + \varphi}{\sigma + \varphi} \right) a_{t+k} \right]. \end{aligned} \quad (22)$$

It is now a standard procedure to obtain the following Phillips curve from (22).

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa (y_t - y_t^N) \\ \kappa &= \left\{ \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \right\} \left\{ \frac{\sigma + \varphi}{1 + \theta(\varphi + \sigma\gamma)} \right\}. \end{aligned}$$

A.3 Proof of Proposition 3 & 4

I follow Woodford (2003) in deriving the utility-based loss function. I take a Taylor expansion of each term of the utility function. Taking a second order linear expansion around the

steady state, I obtain

$$U(C_t(i)) = U(C) + U_C(C_t(i) - C) + \frac{1}{2}U_{CC}(C_t(i) - C)^2 + O(\|\xi\|^3) \quad (23)$$

where I denote $O(\|\xi\|^3)$ the all relevant terms that are of third or higher order, and $U(C_t(i)) = \frac{C_t(i)^{1-\sigma}-1}{1-\sigma}$. I also take a second order Taylor expansion of C_t . Then I have

$$C_t(i) = C \left(1 + c_t(i) + \frac{1}{2}c_t(i)^2 \right) + O(\|\xi\|^3) \quad (24)$$

where $c_t(i) \equiv \log \frac{C_t(i)}{C}$ as in the text. This implies

$$C_t(i) - C = Cc_t(i) + \frac{1}{2}Cc_t(i)^2 + O(\|\xi\|^3) \quad (25)$$

Substituting (25) into (23) gives

$$U(C_t(i)) = U(C) + U_C Cc_t(i) + \frac{1}{2}U_C Cc_t(i)^2 + \frac{1}{2}U_{CC}C^2c_t(i)^2 + O(\|\xi\|^3) \quad (26)$$

Note that $U(C)$ is independent of monetary policy. We rewrite (26) as

$$U(C_t(i)) = U_C C \left\{ c_t(i) + \frac{1}{2}c_t(i)^2 + \frac{1}{2} \frac{U_{CC}C}{U_C} c_t(i)^2 \right\} + t.i.p + O(\|\xi\|^3)$$

where $t.i.p$ denotes all the terms independent of monetary policy. From the utility function I assume in the text, I have $\frac{U_{CC}C}{U_C} = -\sigma$. Also, in the steady state, $C(i) = Y$. Thus I finally obtain

$$U(C_t(i)) = U_C Y \left\{ c_t(i) + \frac{1}{2}(1 - \sigma)c_t(i)^2 \right\} + t.i.p + O(\|\xi\|^3) \quad (27)$$

Integrating (27) gives

$$\int_0^1 U(C_t(i)) di = U_C Y \left\{ E_i [c_t(i)] + \frac{1}{2}(1 - \sigma)E_i [c_t(i)^2] \right\} + t.i.p + O(\|\xi\|^3)$$

Recall that the economy's resource constraint is given by

$$\int C_t(i)di = Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}. \quad (28)$$

Taking the second order expansions gives

$$\begin{aligned} E_i [c_t(i)] &= y_t + \frac{1}{2}y_t^2 - \frac{1}{2}E_i [c_t(i)^2] + O(\|\xi\|^3) \\ &= y_t + \frac{1}{2}y_t^2 - \frac{1}{2}Var_i [c_t(i)] - \frac{1}{2}E_i [c_t(i)]^2 + O(\|\xi\|^3) \end{aligned} \quad (29)$$

$$E_i [c_t(i)]^2 = y_t^2 + O(\|\xi\|^3) \quad (30)$$

Substituting (29) and (30) into above, I obtain

$$\begin{aligned} \int_0^1 U(C_t(i))di &= U_C Y \left\{ y_t + \frac{1}{2}y_t^2 - \frac{1}{2}\sigma E_i [c_t(i)^2] \right\} + t.i.p + O(\|\xi\|^3) \\ &= U_C Y \left\{ y_t + \frac{1}{2}y_t^2 - \frac{1}{2}\sigma \left(Var_i [c_t(i)] + E_i [c_t(i)]^2 \right) \right\} + t.i.p + O(\|\xi\|^3) \\ &= U_C Y \left\{ y_t + \frac{1-\sigma}{2}y_t^2 - \frac{\sigma}{2}Var_i [c_t(i)] \right\} + t.i.p + O(\|\xi\|^3). \end{aligned} \quad (31)$$

I also take a second order Taylor expansion of $V(H_t(i)) = \frac{H_t(i)^{1+\varphi}}{1+\varphi}$, which gives

$$V(H_t(i)) = V(H) + V_H(H_t(i) - H) + V_{HH}(H_t(i) - H)^2 + O(\|\xi\|^3) \quad (32)$$

The second order approximation of $H(i)$ is:

$$H_t(i) = H \left(1 + h_t(i) + \frac{1}{2}h_t(i)^2 \right) + O(\|\xi\|^3) \quad (33)$$

Substituting (33) into (32) gives

$$V(H_t(i)) = V_{HH} \left\{ h_t(i) + \frac{1}{2}h_t(i)^2 + \frac{1}{2}\frac{V_{HHH}}{V_H}h_t(i)^2 \right\} + t.i.p. + O(\|\xi\|^3) \quad (34)$$

Since $\frac{V_{HHH}}{V_H} = \varphi$, I rewrite (34) as

$$V(H_t(i)) = V_{HH} \left\{ h_t(i) + \frac{1}{2}(1 + \varphi)h_t(i)^2 \right\} + t.i.p. + O(\|\xi\|^3) \quad (35)$$

From the production function, I have

$$\begin{aligned} y_t(i) &= a_t + h_t(i) \implies \\ h_t(i) &= y_t(i) - a_t \end{aligned} \quad (36)$$

Substituting (36) into (35), I obtain

$$\begin{aligned} V(H_t(i)) &= V_H H \left\{ y_t(i) - a_t + \frac{1}{2}(1 + \varphi) [y_t(i)^2 + a_t^2 - 2a_t y_t(i)] \right\} + t.i.p. + O(\|\xi\|^3) \\ &= V_H H \left\{ y_t(i) + \frac{1}{2}(1 + \varphi)y_t(i)^2 - (1 + \varphi)a_t y_t(i) \right\} + t.i.p. + O(\|\xi\|^3) \end{aligned} \quad (37)$$

because $\frac{1}{2}(1 + \varphi)a_t^2 - a_t$ is *t.i.p.* By integrating (37) over $i \in [0, 1]$, I obtain

$$\int_0^1 V(H_t(i)) di = V_H H \left\{ \begin{array}{l} E_i [y_t(i)] + \frac{1}{2}(1 + \varphi)Var_i [y_t(i)] + \\ \frac{1}{2}(1 + \varphi)E_i [y_t(i)]^2 - (1 + \varphi)a_t E_i [y_t(i)] \end{array} \right\} + t.i.p. + O(\|\xi\|^3) \quad (38)$$

Taking a second order approximation of the aggregators gives

$$y_t = E_i [y_t(i)] + \frac{1}{2} \left(\frac{\theta - 1}{\theta} \right) Var_i [y_t(i)] + O(\|\xi\|^3)$$

This gives the two equations

$$E_i [y_t(i)] = y_t - \frac{1}{2} \left(\frac{\theta - 1}{\theta} \right) Var_i [y_t(i)] + O(\|\xi\|^3) \quad (39)$$

$$E_i [y_t(i)]^2 = y_t^2 + O(\|\xi\|^3) \quad (40)$$

We substitute (39) and (40) into (38), obtaining

$$\begin{aligned} \int_0^1 V(H_t(i)) di &= V_H H \left\{ y_t + \frac{1}{2}(1 + \varphi)y_t^2 - (1 + \varphi)a_t y_t + \frac{1}{2}(\varphi + \theta^{-1}) Var_i [y_t(i)] \right\} \\ &\quad + t.i.p. + O(\|\xi\|^3) \end{aligned}$$

Now recall that $H = Y/A$, and from the household's labor supply relation, we have

$$-\frac{V_H}{U_C} = \frac{W}{P} = A = \frac{Y}{H}$$

Thus, it follows that

$$-V_H H = U_C Y$$

This implies

$$\begin{aligned} & \int_0^1 \{U(C_t(i)) - V(H_t(i))\} di \\ &= U_C Y \left\{ \begin{array}{l} \frac{1-\sigma}{2} y_t^2 - \frac{\sigma}{2} \text{Var}_i [c_t(i)] - \frac{1}{2}(1+\varphi) y_t^2 \\ +(1+\varphi) a_t y_t - \frac{1}{2}(\varphi + \theta^{-1}) \text{Var}_i [y_t(i)] \end{array} \right\} + t.i.p. + O(\|\xi\|^3) \\ &= U_C Y \left\{ \begin{array}{l} \frac{1-\sigma}{2} y_t^2 - \frac{\sigma}{2} \text{Var}_i [c_t(i)] - \frac{1}{2}(1+\varphi) y_t^2 \\ +(1+\varphi) a_t y_t - \frac{1}{2}(\varphi + \theta^{-1}) \text{Var}_i [y_t(i)] \end{array} \right\} + t.i.p. + O(\|\xi\|^3) \\ &= -\frac{U_C Y}{2} \left\{ \begin{array}{l} (\sigma + \varphi) (y_t - y_t^N)^2 \\ + (\varphi + \theta^{-1}) \text{Var}_i [y_t(i)] + \sigma \text{Var}_i [c_t(i)] \end{array} \right\} + t.i.p. + O(\|\xi\|^3) \end{aligned}$$

The demand for $Y_t(i)$ is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t$$

Then

$$\log Y_t(i) - \log Y_t = -\theta (\log P_t(i) - \log P_t)$$

This implies that

$$\begin{aligned} \text{Var}_i [\log Y_t(i) - \log Y_t] &= \theta^2 \text{Var}_i [\log P_t(i) - \log P_t] \\ \implies \text{Var}_i [\log Y_t(i) - \log Y] &= \theta^2 \text{Var}_i [\log P_t(i) - \log P] \\ \implies \text{Var}_i [y_t(i)] &= \theta^2 \text{Var}_i [p_t(i)] \end{aligned}$$

On the other hand, we have

$$\begin{aligned} c_t^R(i) &= \frac{\phi}{\sigma + \phi} x_t^R(i) + O(\|\xi\|^2) \\ x_t^R(i) &= \frac{1 + \varphi}{1 - \frac{\sigma\phi}{\sigma + \phi}} y_t^R(i) + O(\|\xi\|^2) \end{aligned}$$

This implies that

$$c_t^R(i) = \gamma y_t^R(i) + O(\|\xi\|^2)$$

Thus

$$\begin{aligned} \text{Var}_i [c_t(i)] &= \gamma^2 \text{Var}_i [y_t(i)] + O(\|\xi\|^3) \\ &= (\theta\gamma)^2 \text{Var}_i [p_t(i)] + O(\|\xi\|^3) \end{aligned} \quad (41)$$

Thus

$$\begin{aligned} &\int_0^1 \{U(C_t(i)) - V(H_t(i))\} di \\ &= -\frac{UCY}{2} \left\{ (\sigma + \varphi) (y_t - y_t^N)^2 + \theta^2 \{(\varphi + \theta^{-1}) + \sigma\gamma^2\} \text{Var}_i [p_t(i)] \right\} \\ &\quad + t.i.p. + O(\|\xi\|^3), \end{aligned} \quad (42)$$

When the prices are staggered as in the discrete time Calvo fashion, Woodford (2003) has shown that

$$\begin{aligned} \Delta_t &= \alpha\Delta_{t-1} + \frac{\alpha}{1-\alpha}\pi_t^2 + O(\|\xi\|^3) \implies \\ &= \alpha^{t+1}\Delta_{-1} + \sum_{s=0}^t \alpha^{t-s} \left(\frac{\alpha}{1-\alpha} \right) \pi_s^2 + O(\|\xi\|^3), \end{aligned} \quad (43)$$

where $\Delta_t \equiv \text{Var}_i [p_t(i)]$ is a measure of price dispersion. Therefore the proposition 3 is just proved by (41) and (43). If a new policy is conducted from $t \geq 0$, the first term, $\alpha^{t+1}\Delta_{-1}$, is independent of policy. If we take the discounted sum over time, we obtain

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 + t.i.p. + O(\|\xi\|^3) \quad (44)$$

Now plugging (44) into (42), the sum of households' utilities is given by

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \int_0^1 \{U(C_t(i)) - V(H_t(i))\} di \\ = & -\frac{U_C Y}{2} \left[\sum_{t=0}^{\infty} \beta^t (\sigma + \varphi) (y_t - y_t^N)^2 + \frac{\alpha \theta \{(1 + \varphi \theta) + \theta \gamma^2 \sigma\}}{(1 - \alpha)(1 - \alpha \beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 \right] \\ & + t.i.p. + O(\|\xi\|^3) \end{aligned}$$

And finally, I obtain

$$\sum_{t=0}^{\infty} \beta^t \int_0^1 \{U(C_t(i)) - V(H_t(i))\} di = -\frac{\Omega}{2} \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) + t.i.p + O(\|\xi\|^3)$$

where

$$\begin{aligned} \lambda &= \frac{(1 - \alpha)(1 - \alpha \beta)(\sigma + \varphi)}{\alpha \{1 + \varphi \theta + \theta \sigma \gamma^2\}} \frac{1}{\theta}, \\ \Omega &\equiv Y U_C(Y) \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha \{1 + \varphi \theta + \theta \sigma \gamma^2\}} \frac{1}{\theta}. \end{aligned}$$

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