Heterogeneous Households, Real Rigidity, and Estimated Duration of Price Contract in a Sticky-Price DSGE Model

Jae Won Lee*

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Abstract

This paper introduces heterogeneous households into an otherwise standard sticky-price model with industry-specific labor markets. Households are heterogeneous because each household possesses a labor skill specialized exclusively for a certain industry and asset markets are incomplete. I show that the household heterogeneity amplifies price stickiness endogenously: firms adjust prices more slowly in response to economic shocks for a given nominal rigidity. The main economic mechanism is an idiosyncratic wealth effect on labor supply, which leads to a less elastic industry labor supply and a more elastic marginal cost schedule. To quantify the importance of the household heterogeneity in amplifying stickiness, I estimate and compare the representative and heterogeneous household models. The quantitative exercise shows the heterogenous household model performs better than its representative counterpart in accounting for the persistent aggregate dynamics of the U.S., while being more consistent with empirical evidence on nominal rigidity at the aggregate and sectoral levels, thanks to the stickiness endogenously delivered by the model. (*JEL C51, E13, E31, E32, E44, J20*)

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*E-mail: jwlee@econ.rutgers.edu, Department of Economics, Rutgers University, New Brunswick, NJ 08901. I am greatly indebted to Chris Sims for his valuable advice. I also thank Nobuhiro Kiyotaki, Ricardo Reis, Sam Schulhofer-Wohl, Lars Svensson, Per Krusell, Roberto Chang, Tack Yun, Jinil Kim, Jonathan Heathcote, Carlos Carvalho, Jae Sim, Woong Yong Park, and participants of seminars at Princeton, Rutgers, Ohio State, Indiana, UC-Davis, Georgetown, FRB-Boston and the Federal Reserve Board for helpful comments and discussions. A part of this project was completed while I was visiting the FRB. I wish to thank the International Finance Division of the FRB for its hospitality.

1 Introduction

The first generation of dynamic stochastic general equilibrium (DSGE) models, best examplifid by Kydland and Prescott (1982) and King, Plosser and Rebelo (1988), is abstract from an important reality: hetrogeneities of houesholds. That has changed over the last two decades. There is now a large body of literature that investigates how heterogenity can affect the aggregate dynamics of equilibrium prices and qauntities.[1] Most of important works in the literature, however, assume that prices are fully flexible, which makes it impossible to study if the heterogeneity can non-trivially affect the aggregate equilibrium through nominal rigidity.

In this paper I introduce heterogeneous households due to incomplete asset markets into an otherwise prototype *sticky-price* DSGE model which has become one of the workhorse models for the analysis of monetary policy and business cycles.[2] The households in the model are heterogeneous because each household possesses a labor skill specialized exclusively for a certain industry and asset markets are incomplete. Following the tradition of the first generation DSGE models, most sticky-price models also do not account for household heterogeneities. The representative-household abstraction would be a useful approximation if household heterogneities would not have first-order importance for the aggregate dynamics in the sticky-price models.

I show that this is not the case. The introduced household heterogeneity, through interactions with nominal rigidity, do affect the equilibrium dynamics of key macroeconomic variables quantitatively as well as qualitatively. The main finding is that the household heterogeneity amplifies the price stickiness endogenously: the prices adjust more slowly in response to economic shocks for an exogenously given nominal rigidity. The endogenously amplified price stickiness is often referred to as "*real rigidity*" or "*endogenous stickiness*" in the literature to distinguish it from the nominal rigidity exogenously given to sticky-price models (Ball and Romer, 1990 and Chari et al., 2000). The household heterogeneity thus provides a new explanation to large and persistent real effects of monetary shocks and inertial inflation that are often observed in many economies even if the economies are characterized by weak nominal rigidity (i.e. prices adjust relatively frequently).

This finding appears to reconcile the inconsistency between micro and macro observations. At the *macro* level, sticky-price DSGE models often require a large degree of nominal rigidity to generate persistent dynamics of aggregate inflation and output. In contrast, *micro* level data indicate that the nominal rigidity is not strong enough.[3] For example, Bills and Klenow (2004) and

[1] This research agenda is relatively young, but is growing rapidly, partly due to the development of faster computing machines. Important early contributions include Huggett (1993), Aiyagari (1994), Krusell and Smith (1998), and many other articles cited in the review paper by Heathcote et al.(2009).

[2] The standard sticky-price models (or New Keynesian models) are extensively discussed in many graduate level textbooks such as Woodford (2003), Walsh (2003), and Gali (2008). Aoki (2001), Benigno (2004), Clarida, Gali, and Gertler (1999, 2002), Erceg, Henderson, and Levin (2000), Benigno and Woodford (2007) and others document various issues in monetary policies in New Keynesian framework. See Christiano et al. (2005) and Smet and Wouters (2003, 2007) for leading examples of medium-scale sticky-price DSGE models. In this paper, I use the two terms, "sticky-price models" and "New Keynesian (NK) models" interchangeably.

[3] The inconsistency is perhaps summarized best by Altig et al. (2004): *"Macroeconomic and microeconomic data paint conflicting pictures of price behavior. Macroeconomic data suggest that inflation is inertial. Microeconomic data indicate that firms change prices frequently."*

Nakamura and Steinsson (2008) (henceforth BK and NS respectively) document that firms update their prices less than every 2 quarters,[4] while the estimated frequency of price changes is often greater than one year when standard sticky-price models are taken to the aggegate time series data. To highlight and isolate quantitative importance of the household heterogeneity in reconciling the seeming inconsistency, I fit two versions of a sticky-price DSGE model, with and without household heterogeneity, to the U.S. time series data employing Bayesian method. I refer to these models as the heterogeneous household and representative household models respectively (or the $\mathcal{HH}$ and $\mathcal{RH}$ models for short). The quantitative exercise shows that the household heterogeneity does help the sticky-price model to match the empirical evidence on frequencies at the aggregate and sectoral levels, thanks to the endogenous stickiness delivered by the household heterogeneity,

As a benchmark, I first estimate the models under the assumption that every firm in the economy has a same degree of nominal rigidity (i.e. same frequency of price changes), which has been a conventional approach in the literature. The estimation results indicate that the firms update their prices every 4.65 quarters in the case of representative household. In contrast, the price adjustments are more frequent in the heterogenous household model: the average duration of price contract is estimated as between 1.51 and 3.81 quarters, depending on a degree of financial market frictions. Therefore the heterogeneous household model appears to be more capable of explaining the persistent aggregate dynamics while being more consistent with empirical evidence of nominal rigidity (or frequency of price changes).

I then relax the special assumption of the identical nominal rigidity and consider a more general case in which the model economy consists of multiple sectors with potentially different frequencies of price changes as in Carvalho (2006), Benigno (2004), and Aoki (2001). I view the multiple-sector case is more interesting for several reasons. First, the empirical studies, such as BK and NS, show that nominal rigidities (i.e. frequencies of price changes) are different across sectors. It might be interesting to see if the sticky-price DSGE models can match not only the average level of the empirical frequencies but also the empirical frequencies at the sectoral level. Second, even when we mainly focus on the average frequency (or "*aggregate frequency*") of price changes as in many existing papers, the estimated average while allowing the firms to have different frequencies is generally different from the one estimated imposing the counter-factual restriction that every firm has an identical degree of nominal rigidity. Indeed the empirical papers, such as BK and NS, first estimate sectoral frequencies and then make inference for the *aggregate* frequency of price changes by taking weighted mean or median of the estimated sectoral frequencies. Therefore, estimating the models with multiple sectors would better mimic the estimation procedure employed in those papers. Third, there is a growing interest in differential responses of economic variables to various economic shocks (e.g. Boivin et al., 2009). Therefore, "multiple-sector-sticky-price DSGE models" are interesting in their own right, regardless of interest in the model-implied frequncies of price changes.

[4] This empirical finding was obtained without excluding temporary sales.

As conjectured above, allowing different degrees of nominal rigidity across the sectors turns out to have a non-trivial implication for inference of *aggregate frequency* of price changes. The estimated duration of prices has decreased in each of the $\mathcal{RH}$ and $\mathcal{HH}$ models. The average duration of price implied by weighted average of the estimated sectoral frequencies is 1.74 quarters in the $\mathcal{RH}$ model and 1.39 quarters in the $\mathcal{HH}$ model. Therefore even the $\mathcal{RH}$ model is characterized by frequent price changes and thus becomes more consistent with the micro evidence. This result is mainly driven by the fact that the sectoral heterogenity in nominal rigidity itself can generate the real rigidity aside from the household heterogeneity. However the $\mathcal{HH}$ model still performs better at matching the empirical frequencies at both aggregate and sectoral levels. I show that the $\mathcal{HH}$ model is broadly more consistent with cross-sector distribution of frequencies of price changes than its representative-household counterpart, although the difference is not as big as in the benchmark single-sector case.[5]

The main economic mechanism, through which the household heterogeneity endogenously amplifies the price stickiness is, an *idiosyncratic wealth effect* on labor supply that makes each individual household's labor supply inelastic to a change in real wage, which in turn makes each firm's marginal cost more elastic to a change in the firm's price and output.

One of the key features of the standard sticky-price models is monopolistic competition with differentiated goods, each of which requires a different labor skill to be produced (Woodford 2003). Since labor markets are segmented for different types of goods and the households can be employed in different labor markets (because they have different skills each other), the households are heterogenous in their labor incomes. In the representative household model, this household heterogeneity becomes irrelevant as the household comsumption levels are equalized through trades of state-contingent assets. Consequently a change in real wage that applies to a household do not affect that household's consumption level. In contrast, when risk-sharing is imperfect due to some frictions in asset markets as in the heterogeneous household model, each household's consumption depends positively on its labor income and thus on the wage rate the household faces. This feature of the model makes the wage elasticity of labor supply smaller, relative to the case of representative household, due to the *wealth effect*: for instance, when the wage rate increases, household consumption level also rises, and consequently the household has less incentive to supply labor.

To see how the less elastic labor supply due to the idiosyncratic wealth effect influences a firm's pricing decision, it is helpful to consider a firm hit by a shock that reduces the firm's marginal cost (a positive productivity shock for instance). The profit maximizing firm then has an incentive to lower its price, which would induce more demand for its product. To meet the increased demand and thus produce more, the firm would demand more labor hours, which would shift the labor demand curve to the right. This would raise the equilibrium wage rate and consequently the firm's marginal cost, which can offset the initial decrease in the marginal cost. However, the later

[5] In the case of durable sectors, the model is not as successful in matching the empirical frequencies as in the other sectors. One of the main reasons is that the model treats all the goods in the economy symmetrically as if they were non-durable goods.

increase in the marginal cost must be larger when there is a friction in asset markets because the labor supply curve is steeper due to the wealth effect. The firm therefore decides to reduce its price by a smaller amount than it would do if the asset markets were perfect as in the standard representatvie household model. In summary, imperfect risk-sharing among the heterogeneous households can explain inertial aggregate inflation and persistent business cycles, even if firms change prices relatively frequently, because when the firms change prices they do so by a smaller amount.

This paper fits well into the growing literature studying aggregate implications of heterogeneous households in quantitative macroeconomic models and also the literature on the real rigidity.[6] These two research agendas have been growing rapidly yet without much interaction each other. This paper builds a bridge between these two active research areas by showing the household heterogeneity can be a source of real rigidity, through its impact on the household labor supply, and thus can nontrivially affect the aggregate dynamics.

Building on the papers on real rigidity, some authors recently examine the consequences of various sources of real rigidity for model-implied nominal rigidity, that is, frequency of price changes and implied duration of price contracts estimated in the models. Some important works include Altig et al. (2004), Eichenbaum and Fisher (2007), and Woodford (2005). Besides the source of real rigidity, the current paper differs from the earlier works in estimation method. I employ the likelihood-based Bayesian method, which allows utilizing the entire predictions from general equilibrium of the models, which proves to be important. Instead of utilizing the whole equilibrium conditions, Eichenbaum and Fisher (2007) focus on the aggregate supply equation (i.e. Phillips curve), and estimate frequency of price changes with generalized method of moments. Altig et al. (2004), on the other hand, choose the parameters that minimize the distance between the model and VAR based impulse responses. Another key difference is that this paper allows for multiple sectors with potentially different degrees of nominal rigidity, which turns out to have non-trivial consequences for estimation.

In terms of estimation method, the current paper builds on the literature that estimates DSGE models using the likelihood-based Bayesian method.[7] Relative to the earlier papers, this paper focuses more on consequences of the household heterogeneity on model-implied nominal rigidity. In addition, up to my knowledge, this paper is one of the earliest attempts to estimate the DSGE model with multiple sectors as well as heterogeneous households.

The rest of the paper is organized as follows. In the next section, I present a simple static model to present the main theoretical results in the most explicit way. Section 3 introduces full-blown

[6] Some of the earlier works on the real rigidity include Ball and Romer (1990), Kimball (1995), Basu (1995), and Bergin and Feenstra (2000). These papers have identified various sources that can amplify monetary non-neutrality. Chari et al. (2000) also have stressed importance of endogenous stickiness. They argue that sticky-price models need to amplify the price stickiness endogenously to explain persistent aggregate dynamics with a reasonable degree of nominal rigidity

[7] For recent contributions, see Smet and Wouters (2003, 2007), Rabanal and Rubio-Ramirez (2005), Lubik and Schorfheide (2005), and many other papers cited in An and Schorfheide (2006).

DSGE models with Calvo-style sticky goods prices. The DSGE models developed here will be the basis for the quantitative exercises in following sections. Section 4 details derivations and properties of the New Keynesian (NK) Phillips curve and discuss how asset market frictions increase the degree of real rigidity. In section 5, I estimate the representative and heterogeneous household models and compare the two based on the estimation results. Section 6 summarizes the results and concludes.

2 Static Model

A main advantage of the model presented in this section is that I can show the results analytically. On the other hand, the model is too simple to be used for a serious quantitative analysis. A full-blown DSGE model for that purpose is presented in the next section.[8]

The economy is composed of a continuum of industries indexed by $i \in [0, 1]$. Each industry i produces a distinguished type of product $Y(i)$. In each industry i, there is a representative firm which I refer to as "type-i firm." The differentiated goods, $\{Y(i)\}$ are aggregated to produce final consumption good Y, through a Dixit-Stiglitz aggregator:

$$Y = \left(\int_0^1 Y(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where θ is the elasticity of substitution between two products and is assumed to be greater than one. The corresponding price index, P for the final consumption good is

$$P = \left(\int_0^1 P(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \tag{2}$$

where $P(i)$ is the price of type-i good. The optimal demand for each type of good minimizes total expenditure PY:

$$Y(i) = \left(\frac{P(i)}{P} \right)^{-\theta} Y. \tag{3}$$

Each firm has a linear production technology:

$$Y(i) = H(i), \tag{4}$$

where $H(i)$ denotes type-i labor that can produce only type-i goods and cannot produce the other types of goods: labor markets are industry-specific. Type-i firm chooses its price $P(i)$ to maximize the profit:

$$\Pi(i) = P(i)Y(i) - W(i)H(i),$$

subject to the demand function, (3). I let $W(i)$ denote the competitive wage rate in industry i.

[8] Those who have little interest in analytical results can skip this section and go to section 3 without loss of continuity.

The optimality condition then can be obtained as

$$\frac{P(i)}{P} = \delta \frac{W(i)}{P}, \tag{5}$$

where $\delta = \theta / (\theta - 1)$ is the firm's desired mark-up.

In each industry i, there is a representative household which I referred to as "type-i household." Type-i household possesses a labor skill specialized exclusively for industry i. Households choose its consumption level, $C(i)$ and labor hours, $H(i)$ to maximize utility:

$$\log C(i) - H(i),$$

subject to budget constraint which equate consumption with income:

$$\begin{aligned} PC(i) &= W(i)H(i) + \Pi(i) + K(i), \\ K(i) &\equiv \lambda \int_{z \neq i} P(z)Y(z)dz; \quad \lambda \in [0,1]. \end{aligned} \tag{6}$$

In addition to labor income $W(i)H(i)$, type-i household receives profit, $\Pi(i)$ from industry i for simplicity.[9] In the budget constraint, I include another term, $K(i)$, which is exogenously given, to denote a pre-arranged financial portfolio. Introducing $K(i)$ provide a convenient way to nest both complete and incomplete market economies within a single framework. Although there are no state-contingent assets, the pre-arranged financial contracts, when $\lambda = 1$, guarantee that households have the same amount of income they would have if the asset markets were complete. The economy is, therefore, *effectively under complete markets* when $\lambda = 1$. On the other hand, the asset markets are effectively incomplete when $\lambda < 1$. A household's first order condition is given by

$$C(i) = \frac{W(i)}{P}, \tag{7}$$

which characterizes a houshold labor supply.

The model is completed by imposing the quantity equation:

$$PY = M, \tag{8}$$

where M denotes exogenous money supply. The quantity equation can be rationalized by introducing a cash-in-advance constraint which is omitted here for simplicity.

Given M, the equilibrium is characterized by allocations of quantities and prices at industry level, $\{C(i), Y(i), H(i), P(i), W(i)\}_{i \in [0,1]}$, and two aggregate variables: output and price level, $\{Y, P\}$ that satisfy the followings:

1. *Definitions of the aggregates: (1) and (2),*

[9] This assumption is not necessary to show the results. But it makes the exposition simpler. In the next section, I consider a more general case.

2. *Firms' optimality conditions and production functions: (3), (4) and (5),*

3. *Households' budget constraints and optimality conditions: (6) and (7),*

4. *Quantity equation: (8)*

5. *Market clearing condition:* $\int_0^1 C(i)di = Y$.

For a benchmark case, I first consider the economy where prices are completely flexible in that firms observe money supply, M before setting prices. It turns out that the asset market completness (or the household heterogeneity) does not affect equilibrium outcome: there exists a unique symmetric equilibrium regardless of asset market completeness. The two main reasons for this result are the symmetric nature of the model and absence of idiosyncratic shcoks. In the absence of idiosyncratic shocks, *ex-ante* symmetric firms chose the same price and produce the same amount of goods. Consequently, the households' incomes are symmetric and there is no need for state-contingent asset markets.

Proposition 1 ***(Irrelevance of heterogeneous households under flexible prices)*** *If prices are flexible in the sense that firms can adjust prices in the event of changes in the exogenous variable,* M*, then there exists a unique symmetric equilibrium in which* $P(i) = P$ *and* $Y(i) = C(i) = Y$ *for all* $i \in [0, 1]$*. Moreover, money is neutral: the equilibrium output,* Y *is determined independently from* M*. The output and price level are explicitly given by*

$$Y = \frac{1}{\delta} \text{ and } P = \delta M.$$

The proof of this (and all other propositions) is in the appendix. In what follows I show "the irrelevance result" no longer holds when not every firm adjusts its price in response to a change in economic environment.

Let us consider the simplest case. Suppose that the government announces a certain level of $M = \bar{M}$. Some firms, called sticky-price firms, believe the announcement and set their prices accordingly, while the other firms, called flexible-price firms, wait until they observe actual M.[10] The common price set by sticky-price firms, which believe M would be equal to $\bar{M}$ with probability one, must be $\delta\bar{M}$ as shown in Proposition 1. If the government indeed keeps its promise, the remaining firms also set the price to $\delta\bar{M}$ and the equilibrium output would be equal to δ^{-1}, that is, with no surprise in monetary policy, the aggregate output would be equal to the flexible-price level of output.

When the government deviates from its announcement, however, money is no longer neutral due to the predetermined prices. The degree of non-neutrality hence would depend on how much the flexible-price firms would respond to a change in M. I show below that the responsiveness is smaller, and thus the monetary non-neutrality is larger, when the asset markets are incomplete.

[10] This amounts to assume that the firms, which set the prices before realization of M, form a subjective probability distribution of M that places entire mass on $\bar{M}$ after the announcement. It is may not be the most realistic or elegant way to model firms' belief. The assumption however helps us to see the main results explicitly

To see first the impact of the wealth effects on a firm's pricing decision, it is useful to combine (7) with (5) and then to express type-i household conumption $C(i)$ in terms of type-i firm's price $P(i)$, employing the household budget constraint, the definition of nominal profit and the demand function for $Y(i)$:

$$\begin{aligned} P(i) &= \delta W(i) = \delta PC(i) = \delta \left\{ P(i)Y(i) + \lambda \int_{z \neq i} P(z)Y(z)dz \right\} \\ &= \delta \left\{ (1-\lambda) P(i)Y(i) + \lambda PY \right\} = \delta \underbrace{\left\{ (1-\lambda) \left(\frac{P(i)}{P} \right)^{1-\theta} PY + \lambda PY \right\}}_{PC(i)} \end{aligned} \tag{9}$$

Note that type-i firm's real marginal cost is type-i household's consumption. As apparent from the second line in (9), when the asset markets are incomplete (i.e. when $\lambda < 1$) , a firm's changing price affect its empolyees' consumption, which in turn influences the marginal cost (i.e. the wage $W(i)$), throgh the wealth effects on labor supply, in such a way to discourage the firm to change its price. In contrast, a household consumption would not be affected by a firm's price if the asset markets were complete,[11] and thus the wealth effects would not arise.

To study how the optimal price P^*, that are commonly chosen by the flexible-price firms, responds to M, I simply replace $P(i)$ with P^* and PY with M in (9), which leads to

$$P^* = \underbrace{(1-\lambda)\,\delta \left(\frac{P^*}{P} \right)^{1-\theta}}_{(i)} M + \underbrace{\lambda\delta}_{(ii)} M. \tag{10}$$

The equation (10) implicitly determines the equilibrium level of P^* given $\{M, P\}$. A response of P^* to a change in M can be characterized by the sum of the two coefficients, (i) and (ii) on M. The first coefficient (i) is however decreasing in P^* given P since $\theta > 1$, which dampens a response of P^* to a change in M. This "dampening effect" disappers when the asset markets are effectively complete as the first coefficient becomes zero when $\lambda = 1$, and hence the prices respond more to a change in monetary policy.

Finally the price level can be obtained by aggregating the price set by sticky-price firms, $\delta \bar{M}$ and the price set by flexible-price firms shown in (10), through the price index, (2). Under complete asset markets (i.e. when $\lambda = 1$), (10) implies $P^* = \delta M$, and the price level is consequently given by

$$P_C = \left(n \left[\delta M \right]^{1-\theta} + (1-n) \left[\delta \bar{M} \right]^{1-\theta} \right)^{\frac{1}{1-\theta}},$$

where P_C denotes the price level when the asset markets are complete, and n denotes the fraction of the flexible-price firms. Under incomplete markets, however, λ is less than 1. Let us consider a special case in which $\lambda = 0$ (i.e. financial autarky). Then the price level P_{IC} under incomplete

[11] When $\lambda = 1$, $C(i) = \delta Y$, as can be seen on the second line in (9).

markets is

$$P_{IC} = \left(n \left[[\delta M]^{\frac{1}{\theta}} \, P_{IC}^{\frac{\theta-1}{\theta}} \right]^{1-\theta} + (1-n) \left[\delta \bar{M} \right]^{1-\theta} \right)^{\frac{1}{1-\theta}} .$$

Unlike P_C, it is hard to obtain an explicit solution for P_{IC} in terms of the exogenous variables M and $\bar{M}$ only, even in this simple special case. It is still possible, however, to show that the price level under incomplete markets does not adjust as much as when the asset markets are complete.

Proposition 2 ***(Stronger Non-neutralities)*** *Let P_F and Y_F denote price level and output that would be realized when the prices were completely flexible (i.e. $Y_F = 1/\delta$ and $P_F = \delta M$ as shown in Proposition 1). Also let Y_C and Y_{IC} denote the aggregate output under complete markets and under incomplete markets respectively. If $M > \bar{M}$, then $P_{IC} < P_C < P_F$ and $Y_{IC} > Y_C > Y_F$, and vice versa.*

Proposition 3 ***(Stronger Comovements)*** *Let Y_1 and Y_2 denote a common level of outputs produced by sticky-price and flexible-price firms respectively. Then the difference between the two sectoral outputs, $|Y_1 - Y_2|$ is smaller under incomplete markets than under complete markets.*

The two propositions suggest that asset market incompleteness has not only an aggregate but also a distributional implication. Proposition 3 especially suggests that a model with heterogeneous households and incomplete asset markets can generate stronger comovement between sectors. Nakamura and Steinsson (2009) have documented that the model with intermediate inputs is more successful, relative to a standard model *without* intermediate input, in accounting for the strong comovement, which is a key feature of business cycles (Lucas, 1977; Stock and Watson, 1999). The household heterogeneity plays the same role here.

The results in the propositions are not limited to the case of a monetary policy shock. For instance, the same arguments could be made for aggregate productivity shock if an exogenous productivity were added to the model. In the next section, I investigate *how much* the theoretical results obtained here influence the aggregate and sectoral dynamics with a more quantitatively oriented model.

3 Sticky-Price DSGE Model

I incorporate the ideas explored in the previous section into a prototype DSGE model. Some of the model settings are same as those in the static model. I however describe every detail for completeness.

There is a continuum of industries indexed by $i \in [0, 1]$, in which there is a representative firm refered to as "type-i firm". Firms produce differentiated gooda that are aggregated into final consumption goods. Labor markets are industry-specific: a distinguished labor skill is required to produce each type of good. Households are heterogeneous in labor skills: "type-i household" possesses labor skill specialized for industry i, and thus the household supplies labor to type-i firm.

The economy is divided into a finite number of mutually exclusive sectors indexed by $j \in \{1, 2, \cdots, J\}$, and the sectors are characterized by potentially different degrees of nominal rigidity, $\{\alpha_j\}_{j=1}^J$. I use $\mathcal{I}_j$ to denote the set that contains the industires that belong to sector j. The length of the interval $\mathcal{I}_j$, denoted by n_j, gives the size of the sector.[12] One important special case is when the degrees of nominal rigidities are identical across the sectors (i.e. $\alpha_j = \alpha \; \forall j$). The model then becomes the standard single-sector model.

3.1 Households

Type-i household seeks to maximize a discounted sum of utilities of the form

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\log C_{j,t}(i) - \Xi_t^{-\varphi} \frac{H_{j,t}(i)^{1+\varphi}}{1+\varphi} \right] \right),$$

where $C_{j,t}(i)$ denotes type-i household's consumption and $H_{j,t}(i)$ denotes the hours of labor services supplied to industry i in sector j. There are two aggregate preference shocks denoted by Γ_t and Ξ_t. They serve as aggregate demand and aggregate supply shocks respectively. The parameter β is the discount factor, and φ is the inverse of the Frisch elasticity of labor supply.

The flow budget constraint of the household is given by

$$C_{j,t}(i) + \frac{B_{j,t}(i)}{P_t} + \tilde{\epsilon} \left(\frac{B_{j,t}(i)}{P_t Y_t} \right)^2 = \frac{R_{t-1} B_{j,t-1}(i)}{P_t} + \tau_t + \frac{W_{j,t}(i) H_{j,t}(i)}{P_t} + \frac{K_{j,t}(i)}{P_t}, \tag{11}$$

where τ_t denotes the government net transfer, R_t denotes the gross nominal interest, $W_{j,t}(i)$ is the competitive nominal wage rate in industry i, and P_t is the aggregate price level to be defined below.

Households do not trade state-contingent assets. Instead, they borrow and lend through trading riskless nominal bonds (IOUs). I use $B_{j,t}(i)$ to denote a household's holding of the bond at time t. A convex cost, $\tilde{\epsilon} \left(\frac{B_{j,t}(i)}{P_t Y_t} \right)^2$, which I refer to as "participation cost", is introduced in the budget constraint for two reasons. First, the cost term, when $\tilde{\epsilon} > 0$, makes the model stationary and induce a unique steady state so that it makes sense to solve the model by linearization around a steady state equilibrium.[13] I specify the participation cost in such way that the *ex-ante* symmetric households hold zero net borrowing in the steady state. Second, the cost term intends to capture some important frictions in the private bond market. As extensively discussed in Heaton and Lucas (1996), there are certain frictions even in the supposedly riskless bond markets. For instance, there is a substantial spread between the lending and borrowing rates, reflecting the fact that lenders are paying a monitoring cost to prevent defaults. Also households may face a borrowing constraint that makes it difficult to borrow a large amount of fund due to some unobserved frictions. Thus a private bond is only *ex-post* riskless in the sense that it is riskless only after the market agents pay

[12] Since the sectors are mutually exclusive, it must follow that $\bigcup_{j=1}^{J} \mathcal{I}_j = [0, 1]$, where $\mathcal{I}_j$, $j = 1, ..., J$, are disjoint sets, and that $\sum_{j=1}^{J} n_j = 1$.

[13] See Schmitt-Grohe and Uribe (2003) for further discussions.

the participation costs. Recognizing this type of frictions, Heaton and Lucas (1996) have included a cost term of similar form in their model to investigate the effects of bond market frictions on asset prices and the equity premium. I interpret the "participation cost" as a reduced form that makes the bond market less ideal: the cost term serves as a convenient shortcut for many (observed and/or unobserved) frictions in the bond market, which allows me to avoid making the model unnecessarily complicated. Alternative ways to model asset market incompleteness however would not change the main result of this paper as long as a houshold consumption depend positively on its labor income due to the imperfect asset market institutions. The parameter, $\tilde{\epsilon}$, which is assumed to be positive, controls the magnitude of the cost. Two interesting limiting cases would arise when $\tilde{\epsilon}$ gets closer to zero and to infinity. If $\tilde{\epsilon}$ were close to zero, a household would borrow and lend frictionlessly against its future income to smooth consumption. On the other hand, if $\tilde{\epsilon}$ were sufficiently large, households would decide not to trade bonds.[14]

In addition to the labor income, $W_{j,t}(i)H_{j,t}(i)$, each household also earns capital income, $K_{j,t}(i)$. The monopolistically competitive firms make positive profits which are distributed to households. It is useful to consider two extreme cases. In one extreme, type-i household has entire ownership of the firms in industry i, but has zero shares in the other industries. Although this case may not be far from reality for much of the households, it may be too extreme to capture the reality. In the other extreme, the households have the same ownership of every firm in the economy, so that the economy's whole profit is equally distributed among the households. This case is also unrealistic. A more realistic case would be somewhere between these two extremes as shareholders, in the real world, often own a disproportionately larger amount of shares in the industry in which they are employed. With this regard, I specify $K_{j,t}(i)$ in such way that it nests intermediate cases:

$$K_{j,t}(i) \equiv \chi \left(\sum_{k=1}^{J} \int_{\mathcal{I}_k} \Pi_{k,t}(z) dz \right) + (1-\chi)\, \Pi_{j,t}(i), \qquad 0 \leq \chi \leq 1,$$

where $\Pi_{k,t}(z)$ denotes nominal profit of type-z firm in sector k. The parameter, χ controls intensity of concentration of a household's portfolio on its own industry. The two extreme cases arise when $\chi = 0$ and $\chi = 1$. For example, a household's capital income is perfectly diversified over the different industries when $\chi = 1$. The parameter χ must be different across the households in principle. It is therefore difficult to decide what value of χ would be appropriate in representing the entire economy, so that it can be used in macroeconomic models. Although I use 1 as a benchmark value in a calibration exercise, the parameter χ is estimated along with the other parameters in a later section.

The wage in each industry is fully flexible, and both the households and firms in the given

[14] The participation cost should be distinguished from a "trading cost" which might occur where there is a change in bond holding, $\Delta B_{j,t}(i)$. As noted in Heaton and Lucas, while the trading cost is more important in the stock markets (for instance one has to pay commissions to a financial agency when buying or selling stocks), the participation cost is more relevant in bond market.

industry take the wage as given. A household's first order condition is then given by

$$1+2\tilde{\epsilon}\frac{B_{j,t}(i)}{(P_tY_t)^2}=\beta R_t E_t\left[\left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)\left(\frac{C_{j,t}(i)}{C_{j,t+1}(i)}\right)\left(\frac{P_t}{P_{t+1}}\right)\right], \tag{12}$$

$$\left(\frac{H_{j,t}(i)}{\Xi_t}\right)^{\varphi}C_{j,t}(i)=\frac{W_{j,t}(i)}{P_t}. \tag{13}$$

In contrast to the $\mathcal{HH}$ model discussed so far, a representative household supplies every type of labors in the $\mathcal{RH}$ model. The representative household maximizes the discounted sum of utilities:

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\Gamma_t\left[\log C_t-\Xi_t^{-\varphi}\sum_{j=1}^{J}\int_{\mathcal{I}_j}\frac{H_{j,t}(i)^{1+\varphi}}{1+\varphi}di\right]\right),$$

subject to budget constraint:

$$C_t+\frac{B_t}{P_t}=\frac{R_{t-1}B_{t-1}}{P_t}+\tau_t+\sum_{j=1}^{J}\int_{\mathcal{I}_j}\frac{W_{j,t}(i)H_{j,t}(i)}{P_t}di+\sum_{j=1}^{J}\int_{\mathcal{I}_j}\frac{\Pi_{j,t}(i)}{P_t}di.$$

After imposing the market clearing condition, $C_t=Y_t$, the first order conditions are given by

$$1=\beta(1+i_t)E_t\left[\left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)\left(\frac{Y_t}{Y_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\right],$$

$$\left(\frac{H_{j,t}(i)}{\Xi_t}\right)^{\varphi}Y_t=\frac{W_{j,t}(i)}{P_t}. \tag{14}$$

It is helpful to compare (14) to (13) in developing intuition on how the household heterogeneity leads to a greater degree of real rigidity. In the $\mathcal{HH}$ model, type-i household's consumption $C_{j,t}(i)$ depends positively on its labor income due to imperfect risk-sharing. Thus $C_{j,t}(i)$ depends positively on real wage, $\frac{W_{j,t}(i)}{P_t}$ as well as labor hour, $H_{j,t}(i)$, which makes the wage elasticity of labor supply smaller. In consequence, type-i firm's marginal cost gets more sensitive to a change in the firm's price and production. On the other hand, there is no such channel in the $\mathcal{RH}$ model since each industry is so small that industry wage rate $\frac{W_{j,t}(i)}{P_t}$ or labor hour $H_{j,t}(i)$ do not affect directly the aggregate output, Y_t.

3.2 Firms

The aggregate output, Y_t, which is consumed by households, is produced by perfectly competitive firms using the sectoral goods, $\{Y_{j,t}\}_{j=1}^{J}$ with a Dixit-Stiglitz production technology:

$$Y_t=\left(\sum_{j=1}^{J}\left(n_jD_{j,t}^R\right)^{1/\eta}Y_{j,t}{}^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)}, \tag{15}$$

where η is the elasticity of substitution among the sectoral outputs, and $D^R_{j,t}$ is given by $D^R_{j,t} \equiv D_{j,t}/D_t$, which can be interpreted as a sector-specific demand shock relative to overall strength of the demand, $D_t \equiv \sum_{j=1}^{J} n_j D_{j,t}$. The appropriate price index for the final consumption good is found as the minimum cost that should be paid by the firms for producing one unit of the consumption good and is given by

$$P_t = \left(\sum_{j=1}^{J} \left(n_j D^R_{j,t}\right) P_{j,t}^{1-\eta} \right)^{1/(1-\eta)} . \tag{16}$$

Given the aggregate consumption good Y_t, and the price levels, $P_{j,t}$ and P_t, the optimal demand for a sectoral good would be the one to minimize the total expenditure $P_t Y_t$:

$$Y_{j,t} = n_j D^R_{j,t} \left(\frac{P_{j,t}}{P_t} \right)^{-\eta} Y_t \quad \forall j. \tag{17}$$

Similarly to the final consumption good, each sectoral good, $Y_{j,t}$ is also an aggregate of the goods $\{Y_{j,t}(i)\}_{i \in \mathcal{I}_j}$ that are produced by the firms in sector j, and is given by

$$Y_{j,t} = \left(\left(\frac{1}{n_j} \right)^{1/\theta} \int_{\mathcal{I}_j} Y_{j,t}(i)^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)} \quad \forall j, \tag{18}$$

where θ is the elasticity of substitution between different types of goods and is larger than one. The corresponding price indexes for a sectoral good is given by

$$P_{j,t} = \left(\frac{1}{n_j} \int_{\mathcal{I}_j} P_{j,t}(i)^{1-\theta} di \right)^{1/(1-\theta)} , \forall j. \tag{19}$$

Given $Y_{j,t}$, the optimal demand for type-i good $Y_{j,t}(i)$ would be

$$Y_{j,t}(i) = \frac{1}{n_j} \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\theta} Y_{j,t}. \tag{20}$$

Firms in industry i (i.e. type-i firm) employ labors supplied by type-i household. A firm's production function is given by

$$Y_{j,t}(i) = A_{j,t} H_{j,t}(i), \tag{21}$$

where $A_{j,t}$ is an exogenous sector-specific productivity.

The prices are sticky as in Calvo (1983) and Yun (1996). A firm in sector j adjust its price with probability $1 - \alpha_j$ each period. Since only the fraction $1 - \alpha_j$ of all the prices in that sector is set anew while the remaining fraction α_j of prices is carried over from the previous period, the sectoral price level $P_{j,t}$ evolves as:

$$\begin{aligned} P_{j,t} &= \left[\frac{1}{n_j} \int_{\mathcal{I}_j^*} P_{j,t}^*(i)^{1-\theta} di + \frac{1}{n_j} \int_{\mathcal{I}_j - \mathcal{I}_j^*} P_{j,t-1}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \\ &= \left[\frac{1}{n_j} \int_{\mathcal{I}_j^*} P_{j,t}^*(i)^{1-\theta} di + \alpha_j P_{j,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \end{aligned} \tag{22}$$

where $P_{j,t}^*(i)$ is an optimal price chosen by type-i firm when $i \in \mathcal{I}_j^*$. The set $\mathcal{I}_j^* \subset \mathcal{I}_j$, with measure $n_j(1-\alpha_j)$, is a randomly chosen subset in which the firms get a chance to update their prices.

Firms that adjust their prices at time t set the optimal pirce, $P_{j,t}^*(i)$ maximizing expected discounted profits:

$$\max_{P_{j,t}^*(i)} E_t \sum_{k=0}^{\infty} \alpha_j^k q_{j,t,t+k}(i) \frac{\tilde{\Pi}_{j,t+k}(i)}{P_{t+k}},$$

where $q_{j,t,t+k}(i)$ is a type-i firm's real stochastic discount factor between time t and $t+k$, and $\Pi_{j,t+k}(i)$ is the firm's nominal profit at time $t+k$ given that the price chosen at time t is still being charged:

$$\Pi_{j,t+k}(i) = P_{j,t}(i) Y_{j,t+k}(i) - W_{j,t+k}(i) H_{j,t+k}(i).$$

When asset markets are incomplete and a firm is owned by more than one household, there is no unique way to determine a firm's stochastic discount factor. Each shareholder would want to use its own stochastic discount factor in maximizing the expected discounted profits. Therefore there is a conflict of interests among the shareholders. I here assume that a firm maximizes the weighted average of the different objective functions of its shareholders. Then type-i firm's discount factor can be expressed as

$$q_{j,t,t+k}(i) = \beta^k \left[(1-\chi) \left(\frac{\Gamma_t C_{j,t}(i)}{\Gamma_{t+k} C_{j,t+k}(i)} \right) + \chi \left(\sum_{l=1}^{J} \int_{\mathcal{I}_l} \left(\frac{\Gamma_t C_{l,t}(z)}{\Gamma_{t+k} C_{l,t+k}(z)} \right) dz \right) \right].$$

Alternatively one could assume that a household who has the largest voting right gets to choose the firm's discount factor. Under this assumption, the discount factor of type-i firm would be given by $\beta^k \left(\frac{\Gamma_t C_{j,t}(i)}{\Gamma_{t+k} C_{j,t+k}(i)} \right)$. Another yet alternative assumption that could be made is that a firm discounts its future profits in a non-state-contingent way using the real interest rates. In that case, $q_{j,t,t+k}(i)$ would no longer be firm-specific and be instead given by $q_{j,t,t+k}(i) = \prod_{l=0}^{k} R_{t+l}{}^{-1} \frac{P_{t+l+1}}{P_{t+l}}$. However, alternative choices of the discount factor do not make any difference quantitatively since only the steady state value of the discount factors enters the equilibrium conditions in the first order approximation. The steady state level of the shadow value of a dollar is identical across the households and it is equal to the steady state value of risk free real interest rate.[15]

[15] Pescatori (2006) has also made a similar argument.

The first order condition of type-i firm is given by[16]

$$0 = E_t \sum_{k=0}^{\infty} \alpha_j^k q_{j,t,t+k}(i) D_{j,t+k}^R Y_{t+k} \left(\frac{P_{j,t}^*(i)}{P_{j,t+k}}\right)^{-\theta} \left(\frac{P_{j,t+k}}{P_{t+k}}\right)^{-\eta} \left\{\left(\frac{P_{j,t}^*(i)}{P_{t+k}}\right) - \left(\frac{\theta}{\theta-1}\right) MC_{j,t+k}(i)\right\}, \tag{23}$$

where $MC_{j,t+k}(i) = \frac{W_{j,t+k}(i)}{A_{j,t+k}P_{t+k}}$ is type-i firm's real marginal cost at time $t+k$. The optimal prices chosen at time t, $\left\{P_{j,t}^*(i)\right\}_{i \in \mathcal{I}_j^*}$ that satisfy the first order condition (23) determine the equilibrium dynamics of the sectoral price level $P_{j,t}$ through (22). The aggregate price dynamics is then determined by aggregation of such sectoral prices through (16).

3.3 Government

The government budget constraint is

$$\frac{B_t - R_{t-1}B_{t-1}}{P_t} + \sum_{j=1}^{J} \int_{I_j} \tilde{\epsilon} \left(\frac{B_{j,t}(i)}{P_t Y_t}\right)^2 di = \tau_t + G_t, \tag{24}$$

where B_t is government bond supply and G_t is government expenditure at time t. The government collects the participation costs and redistributes them to the households as a transfer.

Monetary policy is characterized by a Taylor rule:

$$R_t = \beta^{-1} R_{t-1}^{\rho_m} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{(1-\rho_m)} \exp(\mu_t). \tag{25}$$

I assume a simple Ricardian fiscal policy:

$$G_t = 0 \quad \text{and} \quad B_t = 0. \tag{26}$$

Although the Ricardian fiscal policy is a nontrivial assumption for equilibrium as pointed out by Leeper (1992) and Sims (1994), I assume it for a direct comparison with many existing papers on sticky-price models as well as for simplicity.

3.4 Equilibrium

The equilibrium of the economy is characterized by the quntities and prices at micro level:

$$\left\{\{C_{j,t}(i), Y_{j,t}(i), H_{j,t}(i), B_{j,t}(i), P_{j,t}(i), W_{j,t}(i)\}_{i,j}\right\}_{t=0}^{\infty},$$

[16] Looking at type-i firm's stochastic discount factor, one might think that the firm can manipulate its discount factor by influencing $C_{j,t}(i)$. But it is not the case in this model. Just like each firm takes the industry wage as given, it also takes the discount factor as given. Recall that type-i firm and type-i household only represent respectively the infinitely many firms and households that participate in the same labor market. Therefore type-i firm's stochastic discount factor should in fact be interpreted as "industry-i stochastic discount factor."

outputs and prices at sectoral level:

$$\left\{ \{Y_{j,t}, P_{j,t}\}_j \right\}_{t=0}^{\infty},$$

and the three aggregate variables: output, price level, and nominal interest rate:

$$\{Y_t, P_t, R_t\}_{t=0}^{\infty}$$

that satisfy the household optimality conditions, (12) and (13), the household budget constraint, (11), the firm optimality conditions, (17), (20), and (23), the government budget constraint (24), the monetary and fiscal policies (25) and (26), the aggregators (15), (16), (18), and (19), and finally the market clearing conditions, $\sum_{j=1}^{J} \int_{\mathcal{I}_j} C_{j,t}(i)di = Y_t$ and $\sum_{j=1}^{J} \int_{\mathcal{I}_j} B_{j,t}(i)di = 0$, given $B_{j,-1}(i) = 0$, $\forall i \in [0,1]$.

Let me introduce some additional notations. In order to study equilibrium dynamics of an aggregate hour index, I define it as the sum of hours worked by all the households in the economy:

$$H_t \equiv \sum_{j=1}^{J} \int_{\mathcal{I}_j} H_{j,t}(i)di.$$

To study the aggregate equilibrium dynamics, it is useful to introduce the notations for household consumption and bond holding at the sectoral level. I use $C_{j,t}$ to denote sectoral consumption, $\int_{\mathcal{I}_j} C_{j,t}(i)di$ and $B_{j,t}$ to denote sectoral bond holding, $\int_{\mathcal{I}_j} B_{j,t}(i)di$. For any generic variable X_t, I use $X_{j,t}^R(i)$ and $X_{j,t}^R$ to denote "*relative*" variables:

$$X_{j,t}^R(i) \equiv \frac{X_{j,t}(i)}{n_j^{-1} X_{j,t}} \quad \text{and} \quad X_{j,t}^R \equiv \frac{n_j^{-1} X_{j,t}}{X_t}.$$

I log-linearize the equilibrium conditions around a symmetric steady state to solve the model. I use lowercase letter x_t to denote percentage deviation from the steady state, X (i.e. $x_t \equiv \ln X_t - \ln X$), excpet the case of the bond holdings where lowercase letters denotes deviation of nominal bond holdings from their steady-state level, $B = 0$, measured as a percentage of steady state nominal output (for example, $b_{j,t} \equiv \frac{B_{j,t} - B}{PY}$).

4 The Generalized New Keynesian Phillips Curve

The model considered here is necessarily more complicated than a standard representative-agent sticky-price model because there is the continuum of heterogeneous households in addition to the continuum of the firms. In principle, with heterogeneous households, the distributions of consumptions and of asset holdings across the households affect aggregate equilibrium dynamics, and therefore the evolution of the distributions should be computed. It is thus impractical to solve the model without an approximation scheme. For this reason and also for comparision with earlier studies, I take the same approximation strategy commonly employed in the NK literature. First, I assume the time-dependent pricing as in Calvo (1983) and Yun (1996). Second, I take a log-linear approximation of the model.

Thanks to those approximation schemes and symmetric nature of the model, it is unnecessary to compute time path of the household distributions in the single-sector case where the degrees of nominal rigidity are identical across the sectors. Moreover the form of the equations that characterize the equilibrium dynamics are identical between the $\mathcal{RH}$ and $\mathcal{HH}$ models. It however does not mean that the household heterogeneity have no first-order effect on equilibrium aggregate dynamics because the two models are characterized by different mappings between the structural parameters and the reduced-form parameters in the equilibrium conditions. More specifically, it turns out that the household heterogeneity change only one reduced-form parameter and thus affect aggregate dynamics: it decreases the slope of Phillips curve for a given nominal rigidity.

Unlike the single-sector case, the evolution of cross-sector distribution of houeseholds and firms must be computed to study equilibrium aggregate dynamics in multiple-sector model because it is no longer possible to utilize the symmetry properties across the sectors. As in the single-sector case, the household heterogeneity alters Phillips curve, but this time it not only decreses the slope but also add an endogenous shift term which can be calculated once cross-sector distribution of household consumption computed.

Since Phillips curve is the key equation through which the household heterogeneity influences aggregate dynamics, I detail a derivation of a generalized Phillips curve, which nests a standard Phillips curve as a special case, and then document some of its properties. It turns out that the household heterogeneity makes constructing Phillips curve somewhat more complicated. Those who are not interested in detailed derivation of the Phillips curve can skip most of the next subsection to Proposition 4 without loss of continuity.

4.1 Derivation of the Generalized Phillips Curve

One important difference from the standard representative household model in constructing Phillips curve is that optimal prices chosen at time t are not identical across the firms but firm-specific. This is because a firm's optimal price is a function of its worker's consumption, and the consumption then depends on $b_{j,t-1}(i)$, the bond holding carried over from the previous period. This feature of the model creates a complication similar to one that arises when the capital stocks are firm-specific. One cannot derive a Phillips curve with the conventional method, and should instead rely on the undetermined coefficient method suggested in Woodford (2005).

Since the dynamics of relative consumption and bond holding play important roles in firms' pricing decisions, I first present household optimality conditions. Log-linearizing household Euler equation and budget constraint, and then expressing them in terms of the relative consumption, relative bond holding, and relative price yields

$$\begin{aligned} c_{j,t}^R(i) &= E_t\left[c_{j,t+1}^R(i)\right] + 2\epsilon b_{j,t}^R(i) \\ c_{j,t}^R(i) &= -\psi_1 b_{j,t}^R(i) + \beta^{-1}\psi_1 b_{j,t-1}^R(i) - \psi_2 p_{j,t}^R(i), \end{aligned}$$

where

$$\psi_1 \equiv \left[1 - \chi\left(\frac{\theta-1}{\theta}\right)\right]^{-1} \quad \text{and} \quad \psi_2 \equiv (\theta-1)\left\{\chi(1+\varphi) + (1-\chi)\right\}\psi_1,$$

and $\epsilon \equiv \tilde{\epsilon}/PY$.[17] Combining the first and the second equations, I can substitute out type-i household's relative consumption $c_{j,t}^R(i)$, which gives an equation that describes the dynamics of a household's relative bond holding given relative price:

$$E_t\left[b_{j,t+1}^R(i) + \left(\beta^{-1} - 1 - \frac{2\epsilon}{\psi_1}\right) b_{j,t}^R(i) + \beta^{-1} b_{j,t-1}^R(i)\right] = \frac{\psi_2}{\psi_1} E_t\left[p_{j,t+1}^R(i) + p_{j,t}^R(i)\right] \tag{27}$$

Turning to firm side, the log-linearized first order condition of a firm that sets its price at time t is

$$\hat{E}_t^i \sum_{k=0}^{\infty} (\alpha_j\beta)^k \left\{p_{j,t}^*(i) - p_{t+k}\right\} = \hat{E}_t^i \sum_{k=0}^{\infty} (\alpha_j\beta)^k mc_{j,t+k}(i).$$

The expectation operator, $\hat{E}_t^i$ must be distinguished from E_t as emphasized in Woodford (2005): $\hat{E}_t^i$ is type-i firm's expectation at time t on the condition that its own price is not revised for the entire future since time t. Because the households and firms are so small in size that they cannot affect aggregate or sectoral level variables, distinguishing the two expectation operators would be important only for micro level variables. After substituting out the relative consumption from marginal cost $mc_{j,t+k}(i)$ and then replacing $\hat{E}_t^i\left[p_{j,t+k}^R(i)\right]$ by $p_{j,t}^{*R}(i) - \sum_{s=1}^k E_t\pi_{j,t+s}$ in the marginal cost, the firm's log-linearized first order condition can be further written as

$$\begin{aligned} p_{j,t}^{*R}(i) &= \left(\frac{1-\alpha_j\beta}{1+\varphi\theta+\psi_2}\right)\sum_{k=0}^{\infty}(\alpha_j\beta)^k E_t\left[V_{j,t+k}\right] + \sum_{k=1}^{\infty}(\alpha_j\beta)^k E_t\left[\pi_{j,t+k}\right] \\ &\quad -\psi_1(1-\alpha_j)\left(\frac{1-\alpha_j\beta}{1+\varphi\theta+\psi_2}\right)\sum_{k=0}^{\infty}(\alpha_j\beta)^k \hat{E}_t^i\left[b_{j,t+k}^R(i)\right] + \beta^{-1}\psi_1\left(\frac{1-\alpha_j\beta}{1+\varphi\theta+\psi_2}\right) b_{j,t-1}^R(i), \end{aligned} \tag{28}$$

where

$$V_{j,t} \equiv (1+\varphi)\, y_t + \left(\varphi + \eta^{-1}\right) y_{j,t}^R + c_{j,t}^R - (1+\varphi)\, a_{j,t} - \varphi\xi_t - \eta^{-1} d_{j,t}^R$$

is the common factor across all the firms within a sector. The operator, E_t is used in the first two summations on the right hand side of (28) in place of $\hat{E}_t^i$ since those terms have only aggregate and sectoral variables.

Finally, the expected value of the firm's next-period price must be weighted average of current

[17] Recall that $x_{j,t}^R(i)$ denotes a percentage deviation of $X_{j,t}^R(i)$ from its steady state (which is equal to zero). Therefore it must be that $c_{j,t}^R(i) = c_{j,t}(i) - c_{j,t}$, $b_{j,t}^R(i) = b_{j,t}(i) - b_{j,t}$, and $p_{j,t}^R(i) = p_{j,t}(i) - p_{j,t}$.

price and next-period *optimal* price:

$$E_t\left[p^R_{j,t+1}(i)\right] = \alpha_j\left[p^R_{j,t}(i) - E_t\pi_{j,t+1}\right] + (1-\alpha_j)\,E_t\left[p^{*R}_{j,t+1}(i)\right]. \tag{29}$$

The three equations, (27), (28), and (29) together characterize the dynamics of micro level variables $\{b^R_{j,t}(i), p^R_{j,t}(i), p^{*R}_{j,t}(i)\}$, given the time path of the aggregate and sectoral level variables, $\{V_{j,t}, \pi_{j,t}\}$. The system of the linear difference equations is, however, hard to solve analytically. I thus take the undetermined coefficient method as in Woodford (2005). From equation (27), I posit that the time path of relative bond holding follows

$$b^R_{j,t}(i) = \delta b^R_{j,t-1}(i) + \upsilon p^R_{j,t}(i), \tag{30}$$

where δ and υ are some functions of the structural parameters. From (28) and (30), it then follows that a firm's optimal price satisfies:

$$p^{*R}_{j,t}(i) = p^{*R}_{j,t} + \lambda b^R_{j,t-1}(i), \tag{31}$$

where λ is again a function of the parameters, and $p^{*R}_{j,t}$ denotes the common component of optimal prices of the firms who set prices anew in sector j, which is a function of the aggregate and sectoral variables only. If the set of parameters, $\{\lambda, \delta, \upsilon\}$ and the common component, $p^{*R}_{j,t}$ were known, one could easily construct the Phillips curve.

The first step to determine $\{\lambda, \delta, \upsilon\}$ and $p^{*R}_{j,t}$ is substituting (31) into (29). I then obtain:

$$E_t\left[p^R_{j,t+1}(i)\right] = \alpha_j p^R_{j,t}(i) + \lambda(1-\alpha_j) b^R_{j,t}(i). \tag{32}$$

Note that (30), the posited time path of bond holding, should satisfy the difference equation (27) after $E_t\left[p^R_{j,t+1}(i)\right]$ is substituted out using (32). This is true if and only if $\{\lambda, \delta, \upsilon\}$ satisfy the following conditions:

$$\upsilon = \frac{(1-\alpha_j)\,\psi_2\delta}{\alpha_j\psi_1\delta - \beta^{-1}\psi_1} \tag{33}$$

$$\lambda = \frac{\beta^{-1} - \alpha_j\delta}{(1-\alpha_j)\,\psi_2}\left[\frac{2\epsilon}{\beta^{-1}-\delta} - \frac{\psi_1(1-\delta)}{\delta}\right]. \tag{34}$$

Note I have expressed λ and υ as a function of δ. One more relation is needed to determine $\{\lambda, \delta, \upsilon\}$ and the firm's first order condition (28) provides the additional relation. Based on (30), $\hat{E}^i_t\left[b^R_{j,t+k}(i)\right]$ can be expressed as

$$\hat{E}^i_t\left[b^R_{j,t+k}(i)\right] = \delta\hat{E}^i_t\left[b^R_{j,t+k-1}(i)\right] + \upsilon\hat{E}^i_t\left[p^R_{j,t+k}(i)\right] = \delta\hat{E}^i_t\left[b^R_{j,t+k-1}(i)\right] + \upsilon\left[p^{*R}_{j,t}(i) - \sum_{s=1}^{k} E_t\pi_{j,t+s}\right],$$

which implies the following equation:

$$\sum_{k=0}^{\infty} (\alpha_j\beta)^k \hat{E}_t^i \left[b_{j,t+k}^R(i)\right] = \left(\frac{\delta}{1-\delta\alpha_j\beta}\right) b_{j,t-1}^R(i) + \frac{\upsilon}{(1-\alpha_j\beta)(1-\delta\alpha_j\beta)} \left[p_{j,t}^{*R}(i) - \sum_{k=1}^{\infty} (\alpha_j\beta)^k E_t\left[\pi_{j,t+k}\right]\right].$$

Plugging this expression into the firm's first order condition, (28), I obtain:

$$\Psi p_{j,t}^{*R}(i) = \left(\frac{1-\alpha_j\beta}{1+\varphi\theta+\psi_2}\right) \sum_{k=0}^{\infty} (\alpha_j\beta)^k E_t\left[V_{j,t+k}\right] + \Psi \sum_{k=1}^{\infty} (\alpha_j\beta)^k E_t\left[\pi_{j,t+k}\right] + \Phi b_{j,t-1}^R(i), \tag{35}$$

where

$$\begin{aligned} \Psi &\equiv 1 - \frac{\psi_2 (1-\alpha_j)^2 \delta}{(1+\varphi\theta+\psi_2)(1-\alpha_j\beta\delta)\left(\beta^{-1}-\alpha_j\delta\right)} \\ \Phi &\equiv \frac{\psi_1 (1-\alpha_j\beta)\left(\beta^{-1}-\delta\right)}{(1+\varphi\theta+\psi_2)(1-\alpha_j\beta\delta)}. \end{aligned}$$

Comparing (35) and (31), one can solve for $p_{j,t}^{*R}$:

$$p_{j,t}^{*R} = \Psi^{-1}\left(\frac{1-\alpha_j\beta}{1+\varphi\theta+\psi_2}\right) \sum_{k=0}^{\infty} (\alpha_j\beta)^k E_t\left[V_{j,t+k}\right] + \sum_{k=1}^{\infty} (\alpha_j\beta)^k E_t\left[\pi_{j,t+k}\right], \tag{36}$$

and the coefficient λ satisfies the following equation:

$$\Psi\lambda = \Phi. \tag{37}$$

The three equations, (33), (34), and (37) jointly determine the coefficients $\{\lambda, \delta, \upsilon\}$ if a solution exists. The system of the equations is nonlinear in $\{\lambda, \delta, \upsilon\}$, and thus there could be more than one solution. Following Woodford (2005), I consider only a solution that would make the joint dynamics of relative price and relative bond holding be convergent so that the means and the variances remain bounded. I can rewrite equation (30) and (32) as the following system:

$$\begin{pmatrix} E_t\left[p_{j,t+1}^R(i)\right] \\ b_{j,t}^R(i) \end{pmatrix} = \begin{pmatrix} \alpha_j + (1-\alpha_j)\lambda\upsilon & (1-\alpha_j)\lambda\delta \\ \upsilon & \delta \end{pmatrix} \begin{pmatrix} p_{j,t}^R(i) \\ b_{j,t-1}^R(i) \end{pmatrix}. \tag{38}$$

The system is stable if and only if the eigenvalues of the coefficient matrix are inside the unit circle.

Lemma 1 *If* $\alpha_j\beta^{-1} \leq 1$, *then the system (38) is stable if and only if* $0 < \delta < \beta^{-1}$.

Based on Lemma 1, I focus only on the values of δ on the interval $\left(0, \beta^{-1}\right)$, and α_j on $(0, \beta)$ in what follows. A natural question to ask at this point might be if there exists such $\{\lambda, \delta, \upsilon\}$ that solve (33), (34), and (37) while satisfying the stability condition, $0 < \delta < \beta^{-1}$. Lemma 2 shows that there indeed exists a unique such set of $\{\lambda, \delta, \upsilon\}$ as long as ϵ is positive.

Lemma 2 *There exists a unique set of* $\{\lambda, \delta, \upsilon\}$ *that satisfies (33), (34), and (37), and* $0 < \delta < \beta^{-1}$ *if* $\epsilon > 0$.

As mentioned above, once I find the solution for $\{\lambda, \delta, \upsilon\}$, the generalized NK Phillips curve can be easily constructed by combining (35) that determines a firm's relative optimal price $p^{*R}_{j,t}(i)$ and (22) that determines dynamics of sectoral price level $p_{j,t}$. Log-linearizing (22) yields

$$p_{j,t} = \frac{1}{n_j} \int_{\mathcal{I}^*_j} p^*_{j,t}(i) di - \alpha_j p_{j,t-1}.$$

Substituting (31) into the equation above, one obtains

$$\alpha_j \pi_{j,t} = \frac{1}{n_j} \int_{\mathcal{I}^*_j} \left(p^{*R}_{j,t} + \lambda b^R_{j,t-1}(i) \right) di,$$

implying

$$p^{*R}_{j,t} = \frac{\alpha_j}{1-\alpha_j} \pi_{j,t}, \tag{39}$$

because $\int_{\mathcal{I}^*_j} b^R_{j,t-1}(i) di = 0$ holds due to the assumption of time-dependent pricing. Note that the time-dependent pricing is a crucial assumption that allows me to avoid keeping track of distribution of the household wealth. Substituting (39) into (36) gives the "sectoral Phillips curve":

$$\pi_{j,t} = g\left(\alpha_j, \epsilon, \chi\right) V_{j,t} + \beta E_t \left[\pi_{j,t+1}\right], \tag{40}$$

where

$$g\left(\alpha_j, \epsilon, \chi\right) \equiv \left\{ \frac{\left(1-\alpha_j \beta\right)\left(1-\alpha_j\right)}{\alpha_j} \right\} q\left(\alpha_j, \epsilon, \chi\right) \tag{41}$$

$$q\left(\alpha_j, \epsilon, \chi\right) \equiv \left[\frac{\left(1-\alpha_j \beta \delta\right)\left(\beta^{-1}-\alpha_j \delta\right)}{\left(1+\varphi\theta+\psi_2\right)\left(1-\alpha_j \beta \delta\right)\left(\beta^{-1}-\alpha_j \delta\right) - \psi_2 \left(1-\alpha_j\right)^2 \delta} \right]. \tag{42}$$

For further analysis, I have made it expicit in (42) that q depends on the two parameters, (ϵ, χ), that measure the financial market frictions as δ and ψ_2 are functions of the two parameters. The results obtained so far are summarized in Proposition 4.

Proposition 4 *Suppose the economy consists of multiple sectors indexed by* $j = 1, 2, \cdots, J$. *In each sector* j, *there is a continuum of firms whose prices are sticky as in Calvo, with the probability of price adjustment in each period is* $1 - \alpha_j$. *Then, a sectoral inflation,* $\pi_{j,t} \equiv p_{j,t} - p_{j,t-1}$, *can be described by the following sectoral Phillips curve:*

$$\pi_{j,t} = \beta E_t \left[\pi_{j,t+1}\right] + g\left(\alpha_j, \epsilon, \chi\right) \left[\left(1+\varphi\right) y_t + \left(\varphi + \eta^{-1}\right) y^R_{j,t} + c^R_{j,t} \right] - \zeta_{j,t}, \tag{43}$$

where $g\left(\alpha_j\right)$ *is given by (41), and* $\{\lambda, \delta, \upsilon\}$ *satisfy (33), (34), (37) and* $0 < \delta < \beta^{-1}$, *and* $\zeta_{j,t}$ *is a linear combination of exogenous processes. Consequently, the Phillips curve for aggregate inflation*

π_t is obtained by taking weighted sum of secrtoral Phillips curves:

$$\pi_t = \beta E_t \left[\pi_{t+1}\right] + \kappa y_t + \Theta_{c,t} + \Theta_{y,t} - \zeta_t, \tag{44}$$

where

$$\begin{aligned} \Theta_{c,t} &\equiv \sum_{j=1}^{J} n_j g\left(\alpha_j, \epsilon, \chi\right) c_{j,t}^R, \qquad \Theta_{y,t} \equiv \left(\varphi + \eta^{-1}\right) \sum_{j=1}^{J} n_j g\left(\alpha_j, \epsilon, \chi\right) y_{j,t}^R, \\ \kappa &\equiv (1+\varphi) \sum_{j=1}^{J} n_j g\left(\alpha_j, \epsilon, \chi\right), \qquad \zeta_t \equiv \sum_{j=1}^{J} n_j \zeta_{j,t}, \end{aligned}$$

where $g(\alpha_j, \epsilon, \chi)$ is a decreasing nonlinear function in each of α_j, ϵ and χ.

The exogenous process $\zeta_{j,t}$ is given by $g\left(\alpha_j, \epsilon, \chi\right)\left[(1+\varphi) a_{j,t} + \varphi \xi_t + \eta^{-1} d_{j,t}^R\right]$. From the Phillips curve, (44), one can see that the household heterogeneity affect aggregate dynamics by changing the standard Phillips curve in two ways: (i) it decreases the slope of the Phillips curve, κ, for a given degrees of nominal rigidity, $\{\alpha_j\}$, and (ii) it introduces an endogenous shift term, $\Theta_{c,t}$, to the Phillips curve. The endogenous shift terms, $\Theta_{c,t}$ and $\Theta_{y,t}$, are relevant only when the economy has multiple sectors with heterogeneous price stickiness. The shifters would disappear if the frequencies of price adjustments were identical across the sectors in the economy.

4.2 Slope of the Phillips Curve

The major determinant of the slope, κ, is the function $g(\cdot)$, which is convex and decreasing in the measure of nominal rigidity, α. The function is a product of two components:

$$g\left(\alpha, \epsilon, \chi\right) \equiv \underbrace{\left\{\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\right\}}_{\text{common term}} \times \underbrace{q\left(\alpha, \chi, \epsilon\right)}_{\text{measure of real rigidity}}$$

The first term, $\frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$, commonly appears in every NK model with Calvo pricing. It is therefore the second term, $q\left(\alpha, \chi, \epsilon\right)$, that makes a difference. The second term is often considered as a measure of real rigidity in the literature: when q is small, the real rigidity is larger.

The expressions for the "real rigidity function", $q(\cdot)$, are different in the $\mathcal{HH}$ and $\mathcal{RH}$ model:

$$\begin{aligned} q^{\mathcal{RH}}\left(\alpha, \chi, \epsilon\right) &= q^{\mathcal{RH}} \equiv \frac{1}{1+\varphi\theta} \quad \text{in } \mathcal{RH} \text{ model} \\ q^{\mathcal{HH}}\left(\alpha, \chi, \epsilon\right) &\equiv \left[\frac{(1-\alpha\beta\delta)\left(\beta^{-1}-\alpha\delta\right)}{(1+\varphi\theta+\psi_2)(1-\alpha\beta\delta)\left(\beta^{-1}-\alpha\delta\right) - \psi_2 (1-\alpha)^2 \delta}\right] \quad \text{in } \mathcal{HH} \text{ model} \end{aligned}$$

where I use $q^{\mathcal{RH}}$ and $q^{\mathcal{HH}}$ separately for the $\mathcal{RH}$ and $\mathcal{HH}$ model to distinguish the two real rigidity functions.[18] Not surprisingly, with representative household, the real rigidity does not depend on

[18] Since the real rigidity function, q, is well known in the case of representative household, I omit the derivation

the financial friction parameters, (χ, ϵ), while they play a role in determining the degree of real rigidity when the households are heterogeneous.

It is tedious but straightforward to show $q^{\mathcal{HH}}(\alpha, \chi, \epsilon)$ is smaller than $q^{\mathcal{RH}}$, that is, the $\mathcal{HH}$ model is characterized by a larger degree of real rigidity. Instead of showing this analytically, I present a contour map of $\frac{q^{\mathcal{HH}}(\alpha,\chi,\epsilon)}{q^{\mathcal{RH}}}$ in Figure 1 based on numerical calculation. In addition, the second and third panel in Figure 1 plot $q^{\mathcal{HH}}$ and $q^{\mathcal{RH}}$, while varying ϵ and χ respectively, for some alternative values of α. A few observations are worth mentioning.

First, Figure 1-A shows that $q^{\mathcal{HH}} < q^{\mathcal{RH}}$ for non-negative values of (χ, ϵ). The result implies that, for a fixed degree of nominal rigidity, α, the slope of Phillips curve is smaller with heterogeneous households. As a consequence, the price level and inflation respond less to economic shocks and output deviates more from the natural level of output.

Second, Figure 1-B shows $q^{\mathcal{HH}}$ is decreasing in ϵ: the heterogeneous household model is characterized by a larger degree of real rigidity when there is a larger friction in bond trading. This result is intuitive. When financial frictions are large, comovement between a household's consumption and its labor income becomes stronger. Then the idiosyncratic wealth effect on labor supply, which is the main driving force to generate real rigidity, becomes more important. As an additional note, one can observe $q^{\mathcal{HH}}$ is convex in ϵ, which implies that even a small financial friction can potentially affect aggregate dynamics substantially.

Third, Figure 1-C shows $q^{\mathcal{HH}}$ is also decreasing in χ: the heterogeneous household model is characterized by a smaller degree of real rigidity when household capital incomes are less diversified. When χ is small, that is, as type-i household collects a disproportionately larger profit from type-i firms relative to the other types of firms, the household's income and thus its conumption becomes less depedent on its labor income. Therefore the idiosyncratic wealth effect, that makes household labor supply less elastic, becomes less important. To see this clearly, it is useful to consider an extreme case. When $\chi = 0$, type-i household receives profit exclusively from type-i firm. In this case, the household's total income, the sum of labor and capital incomes, equals the firm's revenue and is independent of the industry wage rate because the household's labor income and the firm's labor cost are canceled out each other.[19] In this situation, a household consumption does not depend at all on industry wage rate and hours, at least directly, and hence the wealth effects on labor supply are weak.

4.3 Endogenous Shift Terms of the Phillips Curve

When the economy is divided into multiple secotors, besides decreasing the slope, the household heterogeneity introdeces an endogenous shift term. There are two endogenous shift terms, $\Theta_{c,t}$ and $\Theta_{y,t}$ in the Phillips curve, (44), both of which contribute to slower adjustments of price level. The effects of $\Theta_{y,t}$ on aggregate dynamics are extensively documented in Carvalho (2006), and the other

for brevity and refer the interested reader to Woodford (2003).

[19]The household total income, in this case, is given by $W_t(i)H_t(i) + \Pi_t(i)$. But $\Pi_t(i) = P_t(i)Y_t(i) - W_t(i)H_t(i)$. Therefore the labor income and the labor cost are canceled out.

term $\Theta_{c,t}$ is a new addition due to the household heterogeneity.

As documented in Carvalho (2006), when there are more than one sectors, strategic complementarity in price setting can arise between sectors: the sectoral prices have less tendency to deviate from the economy's average price level (i.e. aggregate price).[20] This implies two opposing effects on the aggregate price adjustments. On one hand, firms in "high-frequency sectors" (i.e. sectors with a small α_j) adjust their prices by a *smaller* amount than they would adjust in the single-sector model because their pricing decision is influenced by firms in "low-frequency sectors" (i.e. sectors with a big α_j). This imposes *more rigidity* on aggregate price level. By the same logic, however, firms in the low-frequency sectors adjust their prices by a larger amount, which provides *more flexibility* to aggregate price level. However the high-frequency sectors dominate in the process of aggregate price adjustments because firms from the high-frequncy sectors outnumber those from the low-frequency sectors, among the firms that adjust prices, at any given time. Therefore introducing multiple sectors leads to more rigid aggregate price and inflation.

Introducing heterogenous households reinforces this mechanism through the wealth effects. To see the mechanism more clearly, it is helpful to consider a particular example. Suppose the economy is hit by a contractionary monetary shock. Then, regardless of the household heterogeneity, a high-frequency sector price would decrease *more* than a low-frequency sector price in response to the shock, or equivalently a high-frequency sector output would decrease *less* than a low-frequency sector output. Moreoever, since consumption and output in the same sector respond in the same direction under incomplete asset markets, households in high-frequency sectors would enjoy a higher consumption level relative to households in low-frequency sectors. Since households in high-frequency sectors would then have less incentive to supply labors due to the wealth effects, the cost of the firms would increase. Therfore firms in high-frequency sectors do not decrease their prices as much as they would decrease if the asset markets were complete. The exact opposite happens in low-frequency sectors. As in the previous case, however, the high-frequncy sectors dominate in influencing adjustments of the aggregate price. Since this wealth effects are absent in the representative household model, the price adjustment is slower in the model with heterogeneous households.

The effects that I just mentioned are reflected by the shift terms in the Phillips curve. Note that $\Theta_{c,t}$ and $\Theta_{y,t}$, are weighted sums of the relative sectoral consumption gaps $c^R_{j,t}$ ($\equiv c_{j,t} - y_t$) and of the relative sectoral output gaps $y^R_{j,t}$ ($\equiv y_{j,t} - y_t$) respectively. The weights however are not eqaully assgined acoss the sectors. Since the function $g(\alpha, \epsilon, \chi)$ is decreasing and convex in α, disproportionately larger weights are placed on the high-frequency sectors, that is, high-frequency sectors have larger impact on a change in the aggregate price level as mentioned above. Again with a contractionary monetary shock, $c^R_{j,t}$ and $y^R_{j,t}$ are positive in high-frequency sectors and negative in low-frequency sectors. However, overall impacts on $\Theta_{c,t}$ and $\Theta_{y,t}$ would be positive as high-frequency sectors dominate. Solving the Phillips curve forward, the inflation can be written as a

[20] Nakamura and Steinsson (2009) have shown similar results in a multi-sector menu cost model.

weighted sum of expected future values of the shift terms and the output gap:

$$\pi_t = \sum_{k=0}^{\infty} \beta^k E_t \left[\kappa y_{t+k} + \Theta_{c,t+k} + \Theta_{y,t+k} - \zeta_{t+k} \right].$$

With a contractionary shock, output falls below its natural level of output. Inflation, however, does not fall as much because $\Theta_{c,t}$ and $\Theta_{y,t}$ are expected to rise for a time being after the shock. These endogenous Phillips curve shifters make the model "stickier" by making the response of the price level and of the inflation more sluggish.

5 Estimation and Model Comparisons

This section details estimation procedures and evaluates the two model economies, the $\mathcal{RH}$ and $\mathcal{HH}$ models, based on the estimated degree of nominal rigidity implied by each model, that is, estimated frequency of price adjustments and duration of price contracts implied by the frequency.

I estimate the model using Bayesian methods which provide a coherent way to deal with uncertainty about model parameters and to estimate them. A modeler's prior belief about the structural parameters $\boldsymbol{\omega}$ can be incorporated with estimation by specifying a prior distribution $f(\boldsymbol{\omega})$. Given a data set $\mathbf{X}^T$, a likelihood function $f(\mathbf{X}^T|\boldsymbol{\omega})$ can be obtained by exploiting restrictions imposed by general equilibrium of the model economy. The posterior distribution of $\boldsymbol{\omega}$, $f(\boldsymbol{\omega}|\mathbf{X}^T)$ is then determined by Bayes theorem: $f(\boldsymbol{\omega}|\mathbf{X}^T) = f(\mathbf{X}^T|\boldsymbol{\omega})f(\boldsymbol{\omega})/\int f(\mathbf{X}^T|\boldsymbol{\omega})f(\boldsymbol{\omega})d\boldsymbol{\omega}$. Since it is impossible to obtain the analytical solution for the posterior distribution, I simulate this distribution by Markov Chain Monte Carlo methods.

As mentioned in the previous paragraph, among the model parameters, I choose to focus on the parameters that measure nominal rigidity for two reasons. First, comparing the nominal rigidity implied by each model is a reasonable way to *quantify* the real rigidity endogenously delivered by the models. To account for the persistent aggregate dynamics, the models need certain amount of *total* rigidity that, roughly speaking, is a sum of the nominal and real rigidities. Since a model with greater real rigidity do not require large nominal rigidity, the estimated nominal rigidity in that model would necessarily be smaller. It then can be argued that the difference between the estimated nominal rigidities in the two models is the amount of endogenous stickiness generated by the model. The second reason is closely related to the first. It is documented that the baseline sticky-price DSGE models often require an implausibly large degree of nominal rigidity. Thus it would be interesting to see if the household heterogeneity is quantitatively meaningful in rectifying this problem.

Following the tradition in NK literature, I first estimate both the $\mathcal{RH}$ and $\mathcal{HH}$ models assuming $\alpha_j = \bar{\alpha}$, that is, the firms in the economy update their prices with the same frequency. I then consider a more complicated, yet more realistic case in which nominal rigidity is heterogeneous across the sectors.

5.1 Single Sector

There are two alternative ways to view the single-sector case. One can think that the economy is literally composed of one sector (i.e. $J = 1$). The other way to view the single-sector economy would be that the economy consists of multiple sectors (i.e. $J > 1$), yet the degrees of nominal rigidity are homogeneous across the sectors. The two different views give the same log-linearized equilibrium conditions.

As mentioned in a previous section, in the single-sector case, the Phillips curve is reduced to have the conventional form with no endogenous shift terms. The only difference from the standard Phillips curve, when the households are heterogeneous, is the expression for the slope. Thus, we do not need to keep track of the distributions of household consumption and wealth to compute the equilibrium dynamics of the aggregate variables. The system of the equations to be estimated looks much like the standard sticky-price models:

$$y_t = E_t[y_{t+1}] - (r_t - E_t[\pi_{t+1}]) + \left(\gamma_t - E_t\gamma_{t+1}\right) \tag{45}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t - \zeta_t \tag{46}$$

$$y_t = a_t + h_t \tag{47}$$

$$r_t = \rho_m r_{t-1} + (1-\rho_m)\left\{\phi_\pi \pi_t + \phi_y y_t\right\} + \mu_t. \tag{48}$$

The first equation (45), often called IS equation, is obtained by integrating the log-linearized household optimality conditions over all households and then imposing a market clearing condition; (46) is the Phillips curve; the aggregate production function (47) is obtained by integrating the production functions over all firms; the last equation (48) is log-linear approximation of the interest rate rule and it closes the model. Note that the slope of Phillips curve is given by $\kappa = (1+\varphi)\, g\left(\bar{\alpha}, \epsilon, \chi\right)$. The function $g(\cdot)$ is model-specific because of the different real rigidities, but other than that, the equilibrium conditions look identical between the $\mathcal{RH}$ and $\mathcal{HH}$ models.

I specify the stochastic exogenous processes following a convention in the DSGE literature. I assume that the two preferences shocks and the technology shock follow independent AR(1) processes whereas the monetary policy shock follows an i.i.d. process since interest rate smoothing term is already included in (48):

$$\begin{pmatrix} \gamma_t \\ \xi_t \\ a_t \\ \mu_t \end{pmatrix} = \begin{pmatrix} \rho_\gamma & 0 & 0 & 0 \\ 0 & \rho_\xi & 0 & 0 \\ 0 & 0 & \rho_a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{t-1} \\ \xi_{t-1} \\ a_{t-1} \\ \mu_{t-1} \end{pmatrix} + \begin{pmatrix} \sigma_\gamma & 0 & 0 & 0 \\ 0 & \sigma_\xi & 0 & 0 \\ 0 & 0 & \sigma_a & 0 \\ 0 & 0 & 0 & \sigma_\mu \end{pmatrix} \begin{pmatrix} \varepsilon_{\gamma,t} \\ \varepsilon_{\xi,t} \\ \varepsilon_{a,t} \\ \varepsilon_{\mu,t} \end{pmatrix}, \tag{49}$$

where $(\varepsilon_{\mu,t}\ \varepsilon_{a,t}\ \varepsilon_{\gamma,t}\ \varepsilon_{\xi,t})'$ is i.i.d $N(0_4,\ I_4)$.

The system of linear equations (45)-(49) characterizes the joint distribution of $\{\Delta y_t, \pi_t, r_t, \Delta h_t\}_{t=0}^{T}$. The model is taken to time series data for the United States. I use the real GDP and the GDP deflator to construct the growth rate of the aggregate output and of the price level, $\{\Delta y_t, \pi_t\}$; The effective federal funds rate measures the nominal interest rate r_t; Total hours from the nonfarm business sector are used to construct the growth rate of the aggregate hours. Since I have normalized the size of the model economy to one, I divide the real GDP and the hours by the total civilian non-institutional population over age of 16. I then demean every time series. The date are quarterly, and the sample period runs from 1954:Q3 to 2006:Q4.

Some parameters such as $(\beta, \theta, \varphi, \epsilon, \chi)$ are not well identified in the current system unless I impose strong priors on those parameters. Throughout the paper I fix (β, θ, φ) at some conventional values. I set the discount factor β to 0.99. The elasticity of substitution θ is fixed at 6 so that a firm's mark-up is 20 percent. I set φ to 1, which implies the Frisch elasticity of labor supply is 1. Up to my knowledge, no qunatitative analysis has been done on the parameters χ and ϵ in the literature of Bayesian DSGE models, and thus there is no general agreement as to what are appropriate values of the parameters for uses in sticky-price DSGE models. This paper therefore repeats the estimation for some altervative values of χ and ϵ in the single-sector case. In the next section, when the models have multiple sectors, the two parameters will be estimated along with the other parameters.

The prior and posterior distributions for the remaining parameters are summarized in Table 1. The prior distributions are mostly standard. However unlike many earlier papers, I assume a flat pior on α, instead of imposing an informative prior because α is the key parameter of interest in this paper. The models then have complete freedom to choose any degree of nominal rigidity that provides the best model fit.

Once the posterior distribution of α is obtained, I also construct a posterior distribution of duration of price contracts D, employing the relation:

$$D = -1/\log \alpha.$$

Table 2 presents the posterior means of α and D under the $\mathcal{RH}$ model and also under the $\mathcal{HH}$ model with different sets of values for (ϵ, χ).

In the $\mathcal{RH}$ model, the posterior mean of the duration is 4.65 quarters, with 3.32 and 6.80 being the lower and the upper bounds of 95% highest posterior density region (HPD). If the true duration is indeed less than 2 quarters, it may be reasonable to reject the representative household model on the basis of the estimated duration.

As expected, the $\mathcal{HH}$ model performs better than the $\mathcal{RH}$ model along this dimension. Especially, the model with a large value of χ appears to match the empirical frequency well. With $\chi = 1$ (i.e. when household captial income is well diversified across industries), the posterior mean is about 2.29 quarters and the 95% HPD interval is given by $[1.74Q,\ 3.12Q]$ when $\epsilon = 0.1$. Even a smaller value of ϵ can reduce the implied duration substantially. When $\epsilon = 0.01$ (and $\chi = 1$), the estimated duration of price contract is about 2.96 quarters, which is about 60% of the estimated duration in the $\mathcal{RH}$ model. This result is somewhat expected because the slope of Phillips curve is highly convex in ϵ as can be seen in Figure 1-B. Overall, Table 2 suggests that introducing heterogeneous households with some financial frictions can "potentially" decrease the estimated duration of price substantially, with a caveat that the model does not tell us what could be reasonable values of (χ, ϵ) that can be used for sticky-price DSGE models. In the next section, this paper attempts to address this issue with multiple-sector models. Here I "arbitarary" choose the case with $(\chi, \epsilon) = (1, 0.1)$ as the benchmark and draw posterior densities of in-frequncy and duration for that benchmark along with the $\mathcal{RH}$ model in Figure 3.

The main reason that the model fail to provide information on (χ, ϵ) is the two model economies are observationally equivalent. The only difference between the two models in the equilibrium conditions is the functional form of the slope of Phillips curve κ. Therefore, if we allow α to vary freely, which was the case here since I have imposed a flat prior for α, the model is unable to identify α, χ and ϵ separately since these parameters enters only one reduced from parameter, κ. This will no longer be the case with the models with multiple sectors.

As a final note, the models considered here are abstract from investment decision for simplicity. In order to be more consistent with the theretical construction of the models, I also use the Personal Consumption Expenditure (PCE) as a measure of aggregate output and PCE price index as a measure of aggregate price level. The estimattion results are almost identical with those reported in Table 1 and 2.

5.2 Multiple Sectors

I now relax the restriction that every firm updates price at an identical frequency. In this case, I can estimate the sectoral in-frequencies $\{\alpha_1, \alpha_{2,}..., \alpha_J\}$ and the implied durations $\{D_1, D_{2,}..., D_J\}$. An aggregate (or average) frequncy and/or duration then can be computed based on the estimated sectoral frequencies.

As seen earlier, in the case of multiple sectors, the Phillips curve contains two endogenous shift terms $\Theta_{c,t}$ and $\Theta_{y,t}$, which makes studying equilibrium aggregate dynamics much more challenging computationally. To compute the endogenous shift terms, we should keep track of the cross-sector distributions of consumptions and outputs, which in turn, can be computed only if we know the evolutions of sectoral inflations and bond holdings. Put differently, we have to know the time path of the sectoral variables $\left\{c^R_{j,t}, y^R_{j,t}, b^R_{j,t}, \pi_{j,t}\right\}_{j=1}^J$ and the sectoal weights $\{n_j\}_{j=1}^J$ to obtain the equilibrium dynamics of the aggregate variables, $\{y_t, \pi_t, r_t, h_t\}$.

It is tedious yet straightforward to show the following $4{+}(4 \times J)$ equations determine the equilibrium path of the aggregate variables $\{y_t, \pi_t, r_t, h_t\}$ and the sectoral variables $\left\{c^R_{j,t}, y^R_{j,t}, b^R_{j,t}, \pi_{j,t}\right\}_{j=1}^J$:

$$r_t = \rho_m r_{t-1} + (1-\rho_m)\left\{\phi_\pi \pi_t + \phi_y y_t\right\} + \mu_t \tag{50}$$

$$y_t = E_t[y_{t+1}] - (r_t - E_t[\pi_{t+1}]) + \left(\gamma_t - E_t \gamma_{t+1}\right) \tag{51}$$

$$y_t = \left(\sum n_j a_{j,t}\right) + h_t \tag{52}$$

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \kappa y_t + \left\{\sum n_j g(\alpha_j) c^R_{j,t}\right\} + \left\{\left(\varphi + \eta^{-1}\right)\sum n_j g(\alpha_j) y^R_{j,t}\right\} - \zeta_t \tag{53}$$

$$
\begin{aligned}
b_{j,t}^R &= \beta^{-1} b_{j,t-1}^R + \left\{\chi\left(1+\varphi\right)\left(\frac{\theta-1}{\theta}\right) + \left(1-\chi\right)\left(\frac{\eta-1}{\eta}\right)\right\} y_{j,t}^R && (54)\\
&\quad - \left\{1-\chi\left(\frac{\theta-1}{\theta}\right)\right\} c_{j,t}^R - \chi\left(1+\varphi\right)\left(\frac{\theta-1}{\theta}\right) a_{j,t}^R + \left(1-\chi\right)\frac{1}{\eta} d_{j,t}^R \\
c_{j,t}^R &= E_t[c_{j,t+1}^R] + 2\epsilon b_{j,t}^R, && (55)\\
y_{j,t}^R &= y_{j,t-1}^R - \eta\left(\pi_{j,t} - \pi_t\right) + \Delta d_{j,t}^R, && (56)\\
\pi_{j,t} &= \beta E_t\left[\pi_{j,t+1}\right] + g(\alpha_j)\left\{\begin{array}{c} \left(1+\varphi\right) y_t + \left(\varphi + \eta^{-1}\right) y_{j,t}^R + c_{j,t}^R \\ -\left(1+\varphi\right) a_{j,t} - \varphi \xi_t - \eta^{-1} d_{j,t}^R \end{array}\right\}. && (57)
\end{aligned}
$$

The first four equations (50)-(53) are identical to those in the single sector models except the presence of endogenous shift terms. The remaining $(4 \times J)$ equations follow to determine simultaneously the dynamics of the aggregate as well as the sectoral variables.

In the system of equilibrium condition above, the $\mathcal{HH}$ model differs from the $\mathcal{RH}$ model at two places. First, as in the single-sector case, the slope of the Phillips curve is differnt. As shown before, the slope κ has a smaller value in the $\mathcal{HH}$ model for a given nominal rigidity. Second, in the $\mathcal{RH}$ model, (54) and (55) become irrelavant as it trivially holds that $c_{j,t}^R = b_{j,t}^R = 0$.

As in the single-sector case, I make distibutional assumptions for the sectoral demand and supply shocks to complete the models. Similarly to the aggregate shocks, the sectoral shocks are assumed to follow independent AR(1) processes:

$$
\begin{aligned}
a_{j,t} &= \rho_{a,j} a_{j,t-1} + \sigma_{a,j}\varepsilon_{a,j,t}\ ; \qquad \varepsilon_{a,j,t} \overset{i.i.d}{\sim} N(0,1) \\
d_{j,t} &= \rho_{d,j} d_{j,t-1} + \sigma_{d,j}\varepsilon_{d,j,t}\ ; \qquad \varepsilon_{d,j,t} \overset{i.i.d}{\sim} N(0,1)
\end{aligned}
$$

Before estimating the models, I present impulse responses to a monetary shock, at some fixed parameter values, in Figure 2, to study the dynamic properties of the models. The aim is to investigate what differeces does introducing the household heterogeneity create in the dynamics of the aggregate and sectoral variables, if the two models are parameterized at the same values, especially the frequency of price changes.[21] The impulse responses confirm the theoretical results summarized in Proposition 2 and 3: (i) the aggregate output responds more as inflation responds less to a monstary shock in the $\mathcal{HH}$ model than in the $\mathcal{RH}$ model; (ii) the $\mathcal{HH}$ model generates stronger co-movements among the sectoral outputs and inflations. The stronger co-movements are due to the wealth effects: since the households in "wealthy" sectors would have less incentive to supply labors, and this increases a firm's cost, "wealthy" sectors do not produce as much as they would produce if the asset markets were complete, and vice versa for "poor" sectors. Consequently, the sectoral outputs tend to move together more.[22]

I take the models to sectoral and aggregate time series data for the United States. In addition to the aggregate time series $\{\Delta y_t, \pi_t, r_t, \Delta h_t\}$, I also include in the set of observables the growth

[21] For this exercise, I divide the economy into 5 sectors with same size. The Calvo parameters are assigned as $\{0.1, 0.3, 0.5, 0.7, 0.9\}$. I use the following numerical values: I set all the persistent parameter...to be added

[22] Impulse responses to other shocks shows the same dynamics qualitatively, and thus they are omitted for brevity.

rate of sectoral outputs and of sectoral price indices $\{\Delta y_{j,t}, \pi_{j,t}\}_{j=1}^{J}$. I use PCE as a measure of output and the corresponding price deflator as a measure of price index. The PCE is divide into 3 categories: durable goods, nondurable goods, and services, and they are further disaggregated to 13 sectors, each of which are then further divided into smaller sub-sectors. Although it might be more interesting to estimate the models with more disaggregated variables as in Boivin et al. (2007), I choose to estimate the models with the 13 sectors for computational reasons. Table 3 presents those 13 sectors. The sectoral weights in the table are the expenditure weights averaged over the sample period of 1954:Q3-2006:Q4.[23]

The prior distributions are summarized in Table 4. Since no qunatitative analysis has been done on the parameters χ and ϵ in the literature of Bayesian DSGE models, the prior distributions for χ and ϵ are chosen based on the empirical exercise in the single-sector case. The pior mode and mean for χ are set to 0.875 and 0.7 respectively. A relatively large value for χ is based on the result that a larger value of χ, in the single-sector $\mathcal{HH}$ model, is necessary to match the empirically reasonable frequency of price adjustments. Given the large values of χ, values around 0.1 work best for ϵ in matching the empirical frequncy in the single sector model. I thus set the pior mode and mean for ϵ to 0.09 and 0.1 respectively. Since this choice of prior distributions has been made solely based on the dynamic properties of the single-sector $\mathcal{HH}$ model and the multiple-sector $\mathcal{HH}$ model has different dynamics, it will not be necessarily the case that similar values of χ and ϵ do a good job also in the multiple-sector model. The AR coefficient and innovation parameters of sectoral shocks have the same prior distributions as those of the aggregate shocks. I set the prior mean of the cross-sector elasticity substitution η to 1 so that final good producers have a Cobb-Douglas production function. As in the single sector case, the key parameters of interest are the Calvo parameters. I thus aussume a flat prior for each α_j. Finally, the sectoral weights $\{n_j\}$ are parameterized to the values in Table 3.

The posterior means and 95% HPD of the model parameters, except the sectoral frequencies, are presented also in Table 4. The means and 95% HPD of the Calvo parameters $\{\alpha_j\}$ and durations of price contracts, that are implied by the esimated $\{\alpha_j\}$, are presented in Table 5. In addition, Table 6 and 7 compare the nominal rigidities estimated in the models to the empirical counterparts. The model-implied durations in Table 6 are constructed in the following way. For the mean duration, I first obtain the posterior distribution of $\bar{\alpha} = \sum_{j=1}^{13} n_j \alpha_j$, the weighted mean of in-frequencies of price changes. With the posterior distribution of $\bar{\alpha}$, the posterior distribution of the mean duration $\bar{D}$ is then obtained employing the relation $\bar{D} = -1/\log(\bar{\alpha})$. The posterior distributions of the durations in durable, non-durable, and service sectors are computed in a similar way.[24]

[23] Note that the model imposes the structural relationships :$y_t = \sum_{j=1}^{J} n_j y_{j,t}$ and $\pi_t = \sum_{j=1}^{J} n_j \pi_{j,t}$. This suggests that, in estimating the models, the aggregate variables y_t and π_t are redundant if I include all the sectoral counterparts in the observables. Hence I drop the two aggregate variables in actual estimation.

[24] For instance, the posterior distribution of the durable-sector duration can be obtained by taking posterior draws of $-1/\log(\tilde{\alpha})$, where

$$\tilde{\alpha} = \frac{\sum_{j=1}^{3} n_j \alpha_j}{\sum_{j=1}^{3} n_j}.$$

The empirical counterparts are based on BK, which are denoted by D^{BK}.[25] To be consistent, the empirical durations are computed in the same way as above: I first take $\bar{\alpha}$, the weighted mean of the sectoral in-frequencies reported in BK, and then compute the corresponding duration by $\bar{D} = -1/\log(\bar{\alpha})$. Figure 4 is a graphical representation of Table 6, presenting the posterior densities of the model-implied durations for 3 broad sectors as well as for the whole economy along with their empirical counterparts. Table 7 reports the model-implied and empirical durations for more disaggregated sectors. BK and NS have used the consumption categories constructed by the Bureau of Labor Statistics, which do not exactly match the consumption categories in PCE. There are however some comparable categories, and they are reported in Table 7.[26]

Some observations from Figure 4, Table 6 and Table 7 are worth mentioning. First, allowing different degrees of nominal rigidity across the sectors has non-trivial implications for inference of frequency of price changes, even when one's interest is mainly on the *aggregate frequency/duration* as in many earlier papers in the literature of real rigidity. In both the $\mathcal{RH}$ and $\mathcal{HH}$ models, the estimated mean durations are much smaller relative to the single-sector case: they are 1.74 and 1.39 quarters respectively. Second, the $\mathcal{HH}$ model is broadly consistent with empirical evidence of frequency of price changes not only at the aggregate but also at the *sectoral* levels. Moreover, along this dimension, Figure 4 and Table 6 suggest that the $\mathcal{HH}$ model performs better than the $\mathcal{RH}$ model. However, the $\mathcal{RH}$ model is not far worse. With the multiple sectors, the $\mathcal{RH}$ model becomes much closer to the $\mathcal{HH}$ model in matching the empirical frequencies. The finding, that the sticky-price DSGE models can match reasonably well the empirical cross-sector distribution of frequencies while fitting the major U.S. time series data, is somewhat surprising given that the models are highly stylized with many strong and implausible assumptions. Perhpas not surprsingly, however, the models performs relatively poorly in capturing the dynamics of the durable sector with empirically plausible price stickiness. This suggests that treating household consumption behavior symmetrically for the durable and non-durable goods might not be a good modeling strategy if one wants to study the dynamics of disaggregate variables.

There are two main reasons that the multiple-sector models do a better job than their single-sector counterparts in accounting jointly for persistent aggregate dynamics and for relatively flexible prices. One comes from a statistical property of the time series, and the other is from a theoretical property of the sticky-price DSGE models. First of all, sectoral inflations are far less persistent than aggregate inflation as shown in Figure 5, in which autocorrelation function of aggregate inflation is presented in a dotted black line and those of sectoral inflations are presented in solid lines. The aggregate inflation is more persistent because the indiosyncratic components in sectoral inflations are averaged out through aggregation. Since I include the sectoral time series in the obervables in estimating the multiple-sector models, it may not be too surprising, if not expected, that the estimated *sectoral* nominal rigidities in the models are small, based on the statistical property

[25] D^{BK} (eis) denotes the estimated durations, excluding observations with item substitutions.

[26] The empirical durations from BK and NS in Table 7 and 8 are the ones estimated *including* observations with temporary sales. There is no consensus yet whether the temporary sales should be included for macroeconomic analyses. Therefore, D^{BK} and D^{NS} in Table 7 and 8, can be a conservative criteria.

of the sectoal inflations.[27] Unlike the sectoral inflations, however, the aggregate inflation is very persistent as mentioned above. Then a natural question to ask would be why the models do not need large nominal rigidities to account for the persistent aggregate dynamics? An answer to this question is discussed in a previous section. Introducing multiple sectors with different price stickiness endogenously increase the persistence of *aggregate* variables by creating a shift term in Phillips curve. Moreover introducing hetrogeneous households, on top of the multiple sectors, amplifies this mechanism by adding another shift term. Thanks to this theoretical property of the models, there is no need for a large degree of nominal rigidity at the *sectoral* level to account for persistent *aggregate* dynamics. The estimation scheme used in this paper effectively take the theoretical property into account since it utilizes the whole general equilibrium effects of the models. Figure 6 is a theoretical counterpart of Figure 5, presenting autocorrelation of aggregate and sectoral inflations implied by the models at estimated parameters (the parameters are set to their posterior means). It shows that the models are able to explain joint behaviors of persistent aggregate inflation and less persistent sectoral inflations.

5.3 Some Additional Observations for Multiple-Sector Models

Unlike the single-sector cases, the $\mathcal{RH}$ and $\mathcal{HH}$ models are not observationally equivalent with respect to the observables. The log marginal likelihoods of the $\mathcal{RH}$ and $\mathcal{HH}$ models are given as $\log f_{\mathcal{RH}}(\mathbf{X}^T) = -8335.2$ and $\log f_{\mathcal{HH}}(\mathbf{X}^T) = -8291.4$ respectively. Then the posterior odd ratio (or Bayes factor) can be computed as:

$$\frac{f_{\mathcal{HH}}(\mathbf{X}^T)}{f_{\mathcal{RH}}(\mathbf{X}^T)} = \exp(43.8),$$

which suggests that the heterogeneous household model is better at explaining the joint dynamics of aggregate and sectoral U.S. time series data. The magnitude of the posterior odd ratio is not small statistically. However, the difference *may not be economically significant*. Investigating if the $\mathcal{HH}$ model is systematically better in any economic sense and studying what feature of the $\mathcal{HH}$ model is responsible for the improved fit might be potentially important, but it is beyond the scope of this paper. I leave that as a future research.

There are some common features of the estimated multiple-sector DSGE models worth mentioning. First, the sectoral shocks seem to be more volatile than aggregate shocks on average. Second, many sectoral shocks are as persistent as the aggregate shocks. Third, there is a positive correlation between the volatilities of sectoral demand shocks and the sectoral frequencies of price changes. This suggests that firms in the sectors with more volatile demand shocks tend to adjust prices more frequently. All three features may not be surprising, if not expected. On the other hand, there is a negative correlation between the volatilities of sectoral supply shocks and the sectoral frequencies

[27] While the fact, that sectoral inflations are not persistent, might explain the overall small estimated values for the model-implied durations at the sectoral level, it does not necessarily explain the estimated sector-by-sector model-implied durations match well the empirical counterparts.

of price changes, implying that firms in the sectors with less volatile supply shocks tend to change prices more frequently, which is somewhat counter-intuitive. I suspect that the data fail to correctly identify the sectoral technology shocks $a_{j,t}$ as the average labor productivities since the obervables lack the sectoral labor data. The shocks therefore serve only as a residual in the sectoral Phillips curves without much economic meaning (see the equation (43)). However, since the coefficient on $a_{j,t}$ is given by $g(\alpha_j, \epsilon, \chi)(1+\varphi)$, which is decreasing in α_j, $a_{j,t}$ in a low-frequncy sector has a small coefficient. To compensate this, the residual $a_{j,t}$ happens to fluctuate more widely in low-frequncy sectors, which leads to the negative correlation. This issue deserves a further investigation in the future.

Another parameters of interest, besides the frequency of price changes, are the financial friction paprameters (χ, ϵ), which are potentially identifiable as the parameters are associated with the maginitute of the comovement of the sectors. The parameter χ appears to be well identified, and its poterior mean is 0.12, which suggests that a smaller value of χ is needed in multiple-sector case, because the heterogeneous sectors have already generated large real rigidity and a larger value of χ would create too strong comovements of the sectors. On the other hand, the data is not much informative for ϵ,[28] which suggests an additional source of identification is needed.[29] Having the household comsumption and wealth data at the sectoral level would slove the identification problem. I leave it for a future research as well.

6 Conclusion

This paper shows that relaxing the representative-household assumption in sticky-price DSGE models can affect the equilibrium aggregate dynamics nontrivially by increasing the degree of real rigidity through the wealth effects on labor supply. To quntify the importance of the household heterogeneity in amplifying the price stickiness, I estimate and compare the representative household and heterogeneous household models based on the estimated frequency of price changes. The quantitative exercise shows that the introduced household heterogeneity can quantitatively help the sticky-price model be more consistent with the empirical evidence on nominal rigidity documented from micro studies.

One of interesting future studies based on this paper would be to investigate if the household heterogenity introduced here can also solve another well-known puzzle between macro and micro observations: large elasticity of aggregate labor supply vs. small elasticity of individual labor supply. This paper has shown the idiosyncratic wealth effects lead to a smaller individual labor supply elasticity. The idiosyncratic wealth effects, however, would cancel each other out through aggregation across households. Therefore aggregate labor supply elasticity needs not be identical to individual labor supply elasticity, and most likely the former would be larger than the later. If

[28] The posterior mean of ϵ is a little larger than its prior mean, but the standard error in the posterior is also larger.

[29] When I fix χ to one, ϵ is well identified as it becomes the main parameter that controls the sectoral comovement. As expected, the estimated ϵ turns out to be very small (around at 0.001) for the same reason that estimated χ is small when χ is not fixed.

this is indeed the case, the household heterogeneity would provide a single mecahnism that can simultaneously solve two important puzzles between micro and macro obervations on price rigidity and labor supply elasticity.

The model developed here provide a testable prediction: other things being equal, the business cycles of a country with larger financial frictions should be more persistent and volatile. The model thus proposes an explanation as to why developing countries often experience more prolonged and severe business cycles than developed countries. A careful cross-country empirical analysis would be an interesting future research.

Finally, the model developed here can provide a tool to study implications of imperfect consumption insurance among heterogeneous households for optimal monetary policy. Investigating policy and welfare implications would be another area for future reseach.

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A Tables

Table 1: Prior and Posterior Distributions (Single-Sector)

	prior distribution	**prior mean (std)**	**posterior mean & 95% HPD**
ϕ_π	*Gamma*	1.3 (0.2)	1.5645 [1.3884, 1.7632]
ϕ_y	*Gamma*	0.125 (0.1)	0.0701 [0.0363, 0.1111]
ρ_μ	*Beta*	0.75 (0.15)	0.7595 [0.7199, 0.7945]
ρ_a	*Beta*	0.6 (0.2)	0.9150 [0.8819, 0.9437]
ρ_γ	*Beta*	0.6 (0.2)	0.9481 [0.9226, 0.9715]
ρ_ξ	*Beta*	0.6 (0.2)	0.8329 [0.7696, 0.8926]
σ_μ	*Inverse Gamma*	0.25 (0.2)	0.2677 [0.2444, 0.2933]
σ_a	*Inverse Gamma*	3 (3)	1.2995 [1.1996, 1.4078]
σ_γ	*Inverse Gamma*	3 (3)	3.3827 [2.3751, 4.9798]
σ_ξ	*Inverse Gamma*	3 (3)	11.0105 [6.2723, 16.7307]
α	$Uniform\ (0,1)$	0.5 (0.25)	see Table 2

Table 2: Posterior Distribution of In-frequency and Duration (Single-Sector)

	α, In-Frequncy	**D, Duration (quarters)**
$\mathcal{RH}$	**0.8065** [0.7393, 0.8632]	**4.65 Q** [3.32, 6.80]
$\mathcal{HH}$ ($\epsilon = 0.1, \chi = 1$) benchmark	**0.6464** [0.5630, 0.7259]	**2.29 Q** [1.74, 3.12]
$\mathcal{HH}$ ($\epsilon = 0.01$, $\chi = 1$)	**0.7134**	**2.96 Q**
$\mathcal{HH}$ ($\epsilon = 0.1$, $\chi = 1$)	**0.6464**	**2.29 Q**
$\mathcal{HH}$ ($\epsilon = 1$, $\chi = 1$)	**0.5530**	**1.69 Q**
$\mathcal{HH}$ ($\epsilon = 10$, $\chi = 1$)	**0.5155**	**1.51 Q**
$\mathcal{HH}$ ($\epsilon = 0.01$, $\chi = 0$)	**0.7699**	**3.81 Q**
$\mathcal{HH}$ ($\epsilon = 0.1$, $\chi = 0$)	**0.7555**	**3.56 Q**
$\mathcal{HH}$ ($\epsilon = 1$, $\chi = 0$)	**0.7497**	**3.47 Q**
$\mathcal{HH}$ ($\epsilon = 10$, $\chi = 0$)	**0.7487**	**3.45 Q**

Table 3: Sectors and Weights

j	**Sectors**	**Weights (n_j)**
1	Motor vehicles and parts	4.91
2	Furniture and household equipment	2.52
3	Other durable goods	1.71
4	Food	18.94
5	Clothing and shoes	3.69
6	Gasoline, fuel oil, and other energy goods	4.21
7	Other nondurable goods	7.96
8	Housing	16.18
9	Household operation	5.63
10	Transportation	4.19
11	Medical care	14.37
12	Recreation	2.91
13	Other services	12.77
	Total	100%

Table 4: Prior and Posterior Distributions (Multiple Sectors)

	prior distribution	prior mean (std)	$\mathcal{RH}$ model posterior mean & 95% HPD	$\mathcal{HH}$ model posterior mean & 95% HPD
ϕ_π	*Gamma*	1.3 (0.2)	1.4544 [1.3248, 1.5980]	1.4446 [1.3150, 1.5929]
ϕ_y	*Gamma*	0.125 (0.1)	0.0174 [0.0062, 0.0312]	0.0148 [0.0074, 0.0241]
ρ_μ	*Beta*	0.75 (0.15)	0.6576 [0.6078, 0.7098]	0.6521 [0.6021, 0.7051]
ρ_γ	*Beta*	0.6 (0.2)	0.9753 [0.9621, 0.9848]	0.9771 [0.9638, 0.9869]
ρ_ξ	*Beta*	0.6 (0.2)	0.9908 [0.9834, 0.9975]	0.9930 [0.9861, 0.9981]
σ_μ	*Inverse Gamma*	0.25 (0.25)	0.2960 [0.2658, 0.3268]	0.3014 [0.2750, 0.3335]
σ_γ	*Inverse Gamma*	3 (3)	3.0341 [2.0664, 4.1045]	3.1918 [2.1870, 4.4487]
σ_ξ	*Inverse Gamma*	3 (3)	2.0948 [1.9111, 2.3306]	2.0631 [1.9052, 2.2456]
η	*Gamma*	1 (0.25)	0.7664 [0.6982, 0.8402]	1.3068 [1.1796, 1.4030]
χ	*Beta*	0.7 (0.2)	NA	0.1202 [0.0262, 0.1792]
ϵ	*Gamma*	0.1 (0.03)	NA	0.1464 [0.0204, 0.2859]
$\rho_{a,1}$	*Beta*	0.6 (0.2)	0.8682 [0.8155, 0.9211]	0.9156 [0.8764, 0.9540]
$\rho_{a,2}$	*Beta*	0.6 (0.2)	0.9835 [0.9674, 0.9951]	0.9881 [0.9781, 0.9972]
$\rho_{a,3}$	*Beta*	0.6 (0.2)	0.8354 [0.7665, 0.9070]	0.8732 [0.8073, 0.9341]
$\rho_{a,4}$	*Beta*	0.6 (0.2)	0.9086 [0.8731, 0.9396]	0.9311 [0.8996, 0.9599]
$\rho_{a,5}$	*Beta*	0.6 (0.2)	0.8569 [0.8076, 0.9058]	0.8263 [0.7631, 0.8864]
$\rho_{a,6}$	*Beta*	0.6 (0.2)	0.9757 [0.9552, 0.9929]	0.9821 [0.9630, 0.9947]
$\rho_{a,7}$	*Beta*	0.6 (0.2)	0.9454 [0.9109, 0.9747]	0.9627 [0.9384, 0.9862]
$\rho_{a,8}$	*Beta*	0.6 (0.2)	0.9824 [0.9583, 0.9954]	0.9886 [0.9775, 0.9966]
$\rho_{a,9}$	*Beta*	0.6 (0.2)	0.8931 [0.8343, 0.9553]	0.8891 [0.8370, 0.9380]
$\rho_{a,10}$	*Beta*	0.6 (0.2)	0.9696 [0.9403, 0.9925]	0.9733 [0.9519, 0.9927]
$\rho_{a,11}$	*Beta*	0.6 (0.2)	0.9517 [0.9271, 0.9779]	0.9588 [0.9377, 0.9793]
$\rho_{a,12}$	*Beta*	0.6 (0.2)	0.8893 [0.8351, 0.9457]	0.9112 [0.8658, 0.9690]
$\rho_{a,13}$	*Beta*	0.6 (0.2)	0.8398 [0.7831, 0.8892]	0.8543 [0.8101, 0.8949]
$\rho_{d,1}$	*Beta*	0.6 (0.2)	0.7177 [0.6573, 0.7819]	0.6668 [0.6053, 0.7241]
$\rho_{d,2}$	*Beta*	0.6 (0.2)	0.9753 [0.9556, 0.9933]	0.9608 [0.9327, 0.9855]
$\rho_{d,3}$	*Beta*	0.6 (0.2)	0.9240 [0.8856, 0.9596]	0.9275 [0.8939, 0.9588]
$\rho_{d,4}$	*Beta*	0.6 (0.2)	0.9758 [0.9537, 0.9937]	0.9834 [0.9675, 0.9959]
$\rho_{d,5}$	*Beta*	0.6 (0.2)	0.8712 [0.8156, 0.9229]	0.9799 [0.9596, 0.9965]
$\rho_{d,6}$	*Beta*	0.6 (0.2)	0.9760 [0.9561, 0.9934]	0.9828 [0.9644, 0.9961]
$\rho_{d,7}$	*Beta*	0.6 (0.2)	0.9871 [0.9763, 0.9967]	0.9862 [0.9766, 0.9964]
$\rho_{d,8}$	*Beta*	0.6 (0.2)	0.9776 [0.9556, 0.9957]	0.9751 [0.9414, 0.9939]
$\rho_{d,9}$	*Beta*	0.6 (0.2)	0.9442 [0.9127, 0.9769]	0.9619 [0.9358, 0.9854]
$\rho_{d,10}$	*Beta*	0.6 (0.2)	0.9466 [0.9018, 0.9817]	0.9367 [0.8896, 0.9766]
$\rho_{d,11}$	*Beta*	0.6 (0.2)	0.9908 [0.9816, 0.9975]	0.9865 [0.9703, 0.9967]
$\rho_{d,12}$	*Beta*	0.6 (0.2)	0.9685 [0.9478, 0.9878]	0.9649 [0.9367, 0.9861]
$\rho_{d,13}$	*Beta*	0.6 (0.2)	0.9420 [0.9103, 0.9728]	0.9040 [0.8517, 0.9543]
$\sigma_{a,1}$	*Inverse Gamma*	3 (3)	5.9218 [4.9913, 7.0129]	6.9584 [6.0268, 7.9933]
$\sigma_{a,2}$	*Inverse Gamma*	3 (3)	4.3581 [3.2713, 5.8067]	4.4731 [3.5550, 5.5924]
$\sigma_{a,3}$	*Inverse Gamma*	3 (3)	8.9397 [6.3958, 11.6060]	9.2567 [6.8493, 13.1550]
$\sigma_{a,4}$	*Inverse Gamma*	3 (3)	1.8594 [1.5873, 2.1313]	1.6962 [1.4690, 2.0037]
$\sigma_{a,5}$	*Inverse Gamma*	3 (3)	5.3915 [4.1478, 6.8178]	5.0128 [4.0896, 6.2935]
$\sigma_{a,6}$	*Inverse Gamma*	3 (3)	3.2209 [2.8756, 3.6376]	1.9048 [1.7545, 2.0786]
$\sigma_{a,7}$	*Inverse Gamma*	3 (3)	2.8096 [2.1465, 3.6233]	2.3937 [1.8842, 2.9691]
$\sigma_{a,8}$	*Inverse Gamma*	3 (3)	2.0141 [1.6857, 2.5392]	1.8656 [1.6229, 2.1155]
$\sigma_{a,9}$	*Inverse Gamma*	3 (3)	3.2699 [2.5499, 4.1379]	3.6611 [2.9248, 4.3750]
$\sigma_{a,10}$	*Inverse Gamma*	3 (3)	1.2322 [1.1138, 1.3573]	1.2955 [1.1786, 1.4270]
$\sigma_{a,11}$	*Inverse Gamma*	3 (3)	2.0037 [1.6736, 2.3966]	2.0924 [1.8156, 2.4322]
$\sigma_{a,12}$	*Inverse Gamma*	3 (3)	5.1175 [3.5623, 7.5629]	5.1796 [4.0211, 6.6388]
$\sigma_{a,13}$	*Inverse Gamma*	3 (3)	2.2275 [1.8899, 2.6483]	2.2759 [1.9704, 2.6387]
$\sigma_{d,1}$	*Inverse Gamma*	3 (3)	6.2365 [5.7256, 6.8163]	6.2837 [5.8251, 6.7771]
$\sigma_{d,2}$	*Inverse Gamma*	3 (3)	1.4525 [1.3399, 1.5678]	1.4302 [1.3262, 1.5470]
$\sigma_{d,3}$	*Inverse Gamma*	3 (3)	2.1954 [2.0034, 2.3916]	2.1920 [2.0190, 2.3753]
$\sigma_{d,4}$	*Inverse Gamma*	3 (3)	0.6440 [0.5812, 0.7078]	0.8447 [0.7507, 0.9459]
$\sigma_{d,5}$	*Inverse Gamma*	3 (3)	1.0735 [0.9873, 1.1632]	1.1080 [1.0083, 1.2177]
$\sigma_{d,6}$	*Inverse Gamma*	3 (3)	3.3325 [2.9423, 3.7816]	5.9144 [5.1244, 6.8166]
$\sigma_{d,7}$	*Inverse Gamma*	3 (3)	0.7561 [0.6785, 0.8468]	0.7867 [0.7136, 0.8699]
$\sigma_{d,8}$	*Inverse Gamma*	3 (3)	0.5444 [0.4870, 0.5987]	0.6104 [0.5427, 0.6866]
$\sigma_{d,9}$	*Inverse Gamma*	3 (3)	1.4792 [1.3667, 1.5960]	1.6268 [1.4946, 1.7637]
$\sigma_{d,10}$	*Inverse Gamma*	3 (3)	2.0892 [1.9064, 2.2694]	2.9030 [2.6296, 3.2066]
$\sigma_{d,11}$	*Inverse Gamma*	3 (3)	0.9742 [0.9004, 1.0499]	1.0492 [0.9547, 1.1435]
$\sigma_{d,12}$	*Inverse Gamma*	3 (3)	1.4949 [1.3759, 1.6245]	1.4817 [1.3552, 1.6184]
$\sigma_{d,13}$	*Inverse Gamma*	3 (3)	1.1212 [1.0135, 1.2236]	1.2046 [1.1053, 1.3013]

Table 5: Posterior Distribution of In-frequency and Duration of Price Contracts

	$\mathcal{RH}$ model		$\mathcal{HH}$ model	
Sectors	α_j	$D_j^{\mathcal{RH}}$	α_j	$D_j^{\mathcal{HH}}$
Motor vehicles and parts	0.6334 [0.6034, 0.6629]	**2.23** Q [1.98, 2.51]	0.5922 [0.5620, 0.6377]	**1.96** Q [1.74, 2.19]
Furniture and household equipment	0.8076 [0.7675, 0.8466]	**5.26** Q [3.81, 7.01]	0.7449 [0.6969, 0.7939]	**3.70** Q [2.86, 4.63]
Other durable goods	0.7712 [0.7407, 0.8012]	**4.00** Q [3.42, 4.63]	0.7246 [0.6827, 0.7730]	**3.31** Q [2.82, 3.88]
Food	0.4465 [0.3933, 0.4947]	**1.26** Q [1.08, 1.46]	0.3347 [0.2880, 0.3979]	**0.98** Q [0.84, 1.13]
Clothing and shoes	0.6795 [0.6379, 0.7229]	**2.82** Q [2.30, 3.38]	0.5733 [0.5288, 0.6181]	**1.98** Q [1.66, 2.34]
Gasoline, fuel oil, and other energy goods	0.0132 [0.0010, 0.0349]	**0.23** Q [0.15, 0.31]	0.0047 [0.0004, 0.0117]	**0.19** Q [0.12, 0.22]
Other nondurable goods	0.7065 [0.6576, 0.7496]	**2.96** Q [2.45, 3.48]	0.5920 [0.5296, 0.6513]	**2.09** Q [1.69, 2.56]
Housing	0.7408 [0.7036, 0.7748]	**3.48** Q [2.86, 4.16]	0.6364 [0.5986, 0.6747]	**2.42** Q [2.03, 2.84]
Household operation	0.5830 [0.5349, 0.6333]	**1.94** Q [1.64, 2.23]	0.5181 [0.4720, 0.5639]	**1.58** Q [1.36, 1.81]
Transportation	0.0278 [0.0106, 0.0486]	**0.28** Q [0.22, 0.33]	0.0668 [0.0507, 0.0849]	**0.37** Q [0.34, 0.41]
Medical care	0.6486 [0.6097, 0.6912]	**2.39** Q [2.02, 2.78]	0.5694 [0.5278, 0.6169]	**1.88** Q [1.64, 2.14]
Recreation	0.6981 [0.6401, 0.7486]	**2.87** Q [2.32, 3.49]	0.6386 [0.5848, 0.6821]	**2.33** Q [1.90, 2.85]
Other services	0.4475 [0.3985, 0.4941]	**1.27** Q [1.11, 1.44]	0.3703 [0.3231, 0.4198]	**1.06** Q [0.94, 1.19]

Table 6: Model-implied vs. Empirical durations I

	$D^{\mathcal{RH}}$	$D^{\mathcal{HH}}$	D^{BK}	D^{BK} (eis)
Mean	**1.74** Q [1.61, 1.88]	**1.39** Q [1.30, 1.53]	**1.10** Q	**1.24** Q
-Durable	**3.01** Q [2.68, 3.31]	**2.49** Q [2.24, 2.78]	**0.94** Q	**1.24** Q
-Nondurable	**1.38** Q [1.25, 1.53]	**1.07** Q [0.98, 1.19]	**0.94** Q	**1.04** Q
-Service	**1.87** Q [1.71, 2.04]	**1.53** Q [1.40, 1.67]	**1.44** Q	**1.55** Q

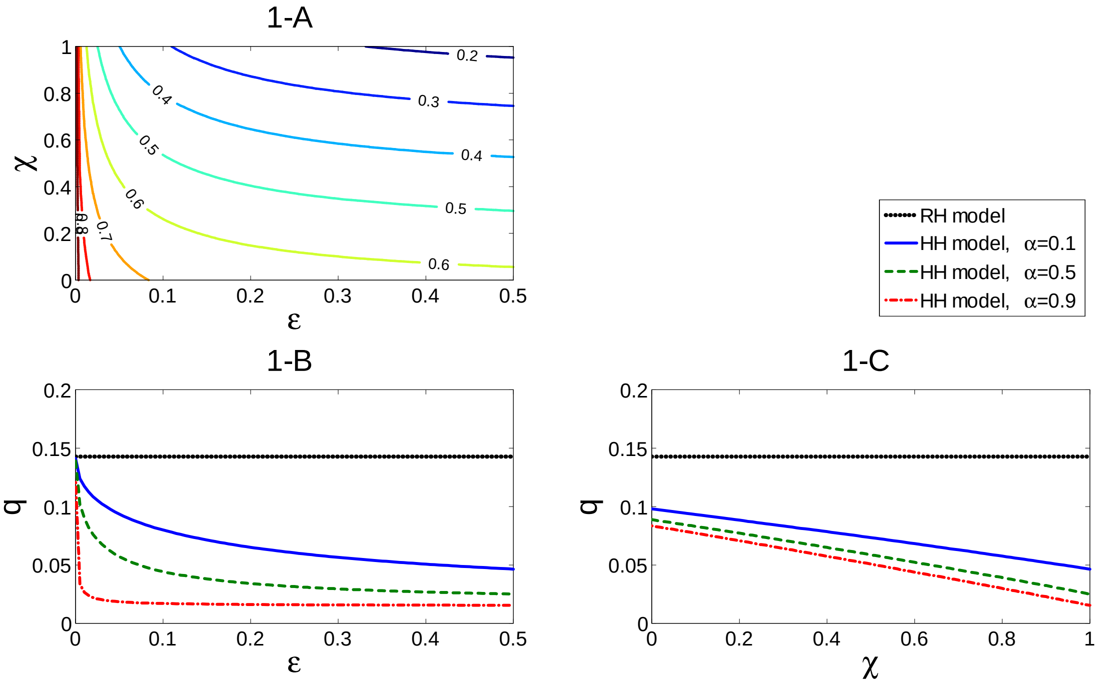
Table 7: Model-implied vs. Empirical durations II

j	Sectors	$D^{\mathcal{RH}}$	$D^{\mathcal{HH}}$	D^{BK}	D^{NS}
2	Furniture and household equipment	**5.26 Q** [3.81, 7.01]	**3.70 Q** [2.86, 4.63]	**1.09 Q***	**1.5 Q***
4	Food	**1.26 Q** [1.08, 1.46]	**0.98 Q** [0.84, 1.13]	**1.14 Q**	**0.81 Q****
5	Clothing and shoes	**2.82 Q** [2.30, 3.38]	**1.98 Q** [1.66, 2.34]	**0.97 Q**	**0.93 Q**
6	Gasoline, fuel oil, and other energy goods	**0.23 Q** [0.15, 0.31]	**0.19 Q** [0.12, 0.22]		**0.17 Q*****
10	Transportation	**0.28 Q** [0.22, 0.33]	**0.37 Q** [0.34, 0.41]	**0.67 Q**	**1.33 Q**
11	Medical care	**2.39 Q** [2.02, 2.78]	**1.88 Q** [1.64, 2.14]	**3.38 Q**	**4.44 Q**
12	Recreation	**2.87 Q** [2.32, 3.49]	**2.33 Q** [1.90, 2.85]	**2.78 Q**	**2.26 Q**

Note: *Home Furnishing, **Weighted average of processed and unprocessed food, ***Vehicle Fuel
$D^{\mathcal{RH}}$: durations of price contracts estimated in the $\mathcal{RH}$ model.
$D^{\mathcal{HH}}$: durations of price contracts estimated in the $\mathcal{RH}$ model.
D^{BK} : durations of price contracts reported in Bills and Klenow (2004).
D^{NS} : durations of price contracts reported in Nakamura and Steinsson (2008).

B Figures

Figure 1: Comparison of real rigidities

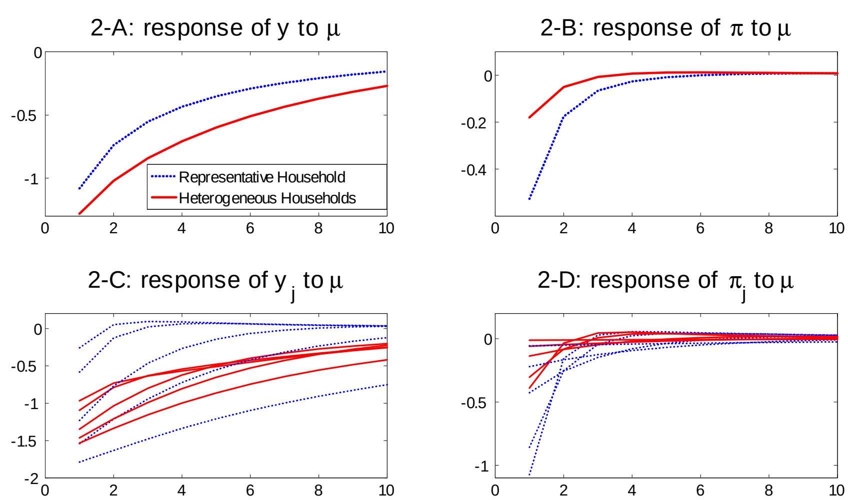


Notes: 1-A plots a contour map of $\frac{q^{\mathcal{HH}}(\alpha,\chi,\epsilon)}{q^{\mathcal{RH}}}$.

1-B plots $q^{\mathcal{RH}}$ and $q^{\mathcal{HH}}(\alpha,\epsilon,\chi)$ for $\epsilon \in (0, 0.5)$ while χ is fixed at 1, and for three different degrees of nominal rigidity

1-C plots $q^{\mathcal{RH}}$ and $q^{\mathcal{HH}}(\alpha,\epsilon,\chi)$ for $\chi \in (0, 1)$ while ϵ is fixed at 0.1, and for three different degrees of nominal rigidity

Figure 2: Impuse responses to a contractionary monetary shock



Notes: 2-A and 2-B show impulse responses of aggregate output and inflation to a monetary shock

2-C and 2-F show impulse responses of sectoral outputs and inflations to a monetary shock

Figure 3: Posterior density of α and D in single-sector economies.

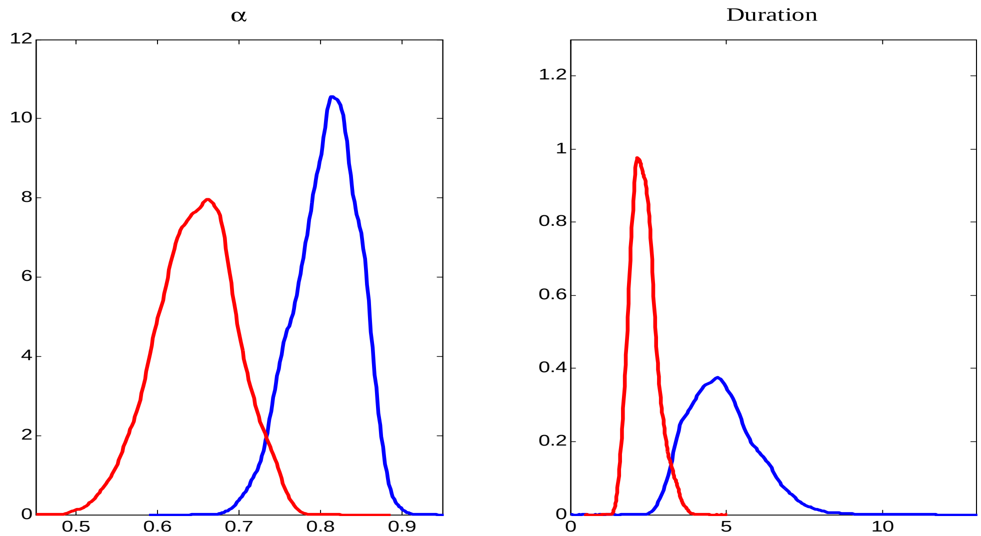


Figure 4: Posterior density of D in multiple-sector economies.

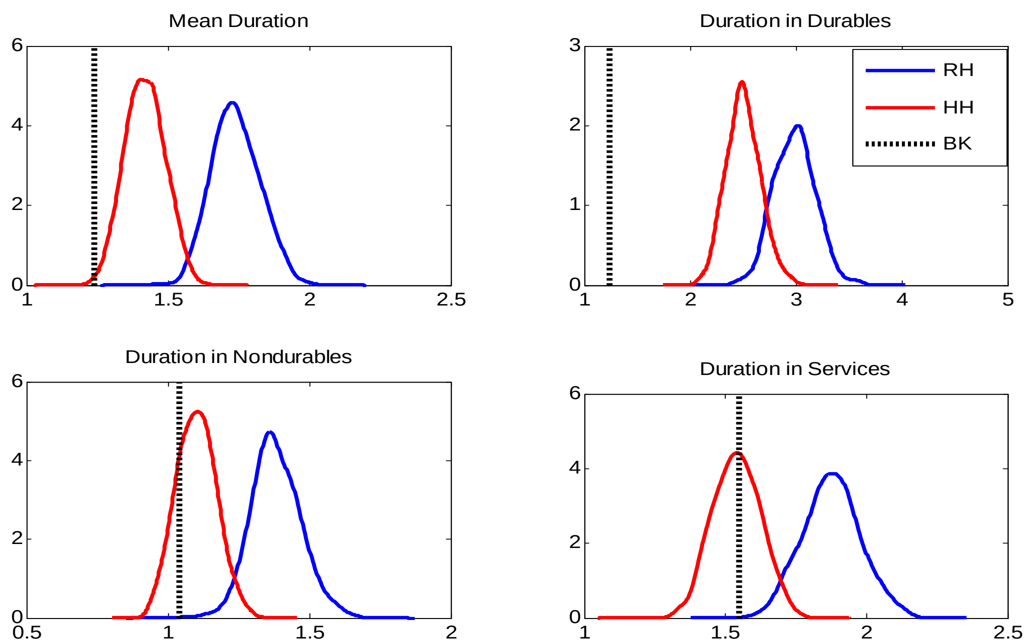
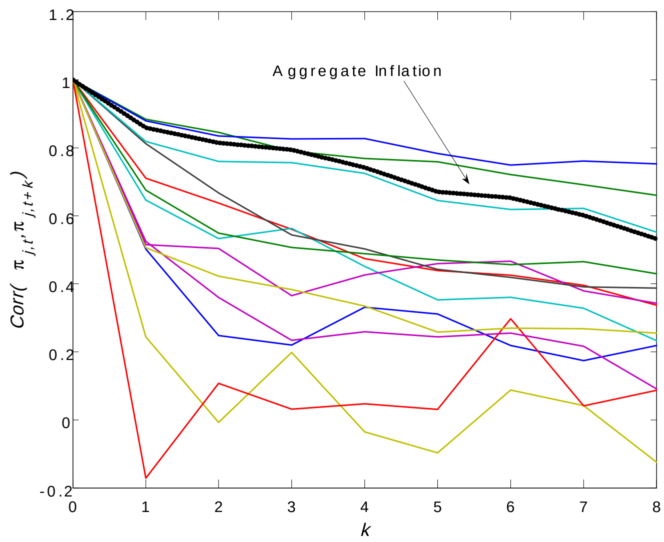
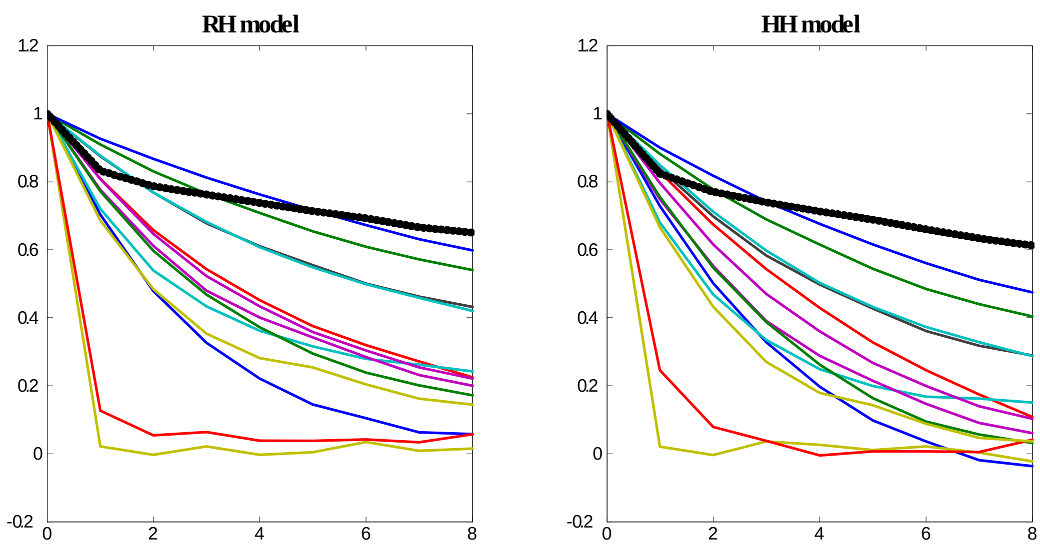


Figure 5: Autocorrelations of aggregate and sectoral inflations



Notes: autocorrelation functions on the vertical axis and lags on the horizontal axis.

Figure 6: Estimated autocorrelations of aggregate and sectoral inflations implied by the models



Notes: autocorrelation functions on the vertical axis and lags on the horizontal axis.

autocorrelation functions are plotted at the posterior mean.

C Proofs

C.1 Proof of Proposition 1

The equilibrium conditions can be reduced to

$$\frac{P(i)}{P} = \delta C(i), \quad (58)$$

$$C(i) = \lambda Y + (1-\lambda)\left(\frac{P(i)}{P}\right)^{1-\theta} Y, \quad (59)$$

$$P = \left(\int_0^1 P(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \quad (60)$$

$$M = PY. \quad (61)$$

(58) is a firm's optimality condition that equates the firm's price with marginal cost multiplied by mark-up; (59) is a household budget constraint after substituting out its incomes with other optimality conditions. Combining (58) and (59) gives

$$P^R(i) = \lambda\delta Y + (1-\lambda)\,\delta Y P^R(i)^{1-\theta}, \quad (62)$$

where $P^R(i) \equiv \frac{P(i)}{P}$ is firm i's relative price. Note (62) is the same equation as (10) in section 2, and the equation should hold for all i. Thus for any arbitrary i_1 and i_2 in $[0,1]$, it must be true that

$$P^R(i_1) - P^R(i_2) = (1-\lambda)\,\delta Y \left\{ \left(\frac{1}{P^R(i_1)}\right)^{\theta-1} - \left(\frac{1}{P^R(i_2)}\right)^{\theta-1} \right\}. \quad (63)$$

Note that both $(1-\lambda)\,\delta Y$ and $(\theta - 1)$ are positive. Therefore, it is not possible that either $P^R(i_1) > P^R(i_2)$ or $P^R(i_1) < P^R(i_2)$ while satisfying (63). The only case consistent with the equation (63) is $P^R(i_1) = P^R(i_2)$, and hence $P(i_1) = P(i_2)$. Then it should be that $P(i) = P$, $\forall i \in [0,1]$ from (60) and that $C(i) = Y$ from (59). Finally from (58) and (61), it can be obtained that $Y = 1/\delta$ and $P = \delta M$.

C.2 Proof of Proposition 2

Let $M > \bar{M}$. If $P_{IC} \geq \delta M$, then we have

$$\begin{aligned} P_{IC} &= \left(n\left[[\delta M]^{\frac{1}{\theta}} P_{IC}^{\frac{\theta-1}{\theta}}\right]^{1-\theta} + (1-n)\left[\delta\bar{M}\right]^{1-\theta}\right)^{\frac{1}{1-\theta}} \\ &\leq \left(nP_{IC}^{1-\theta} + (1-n)\left[\delta\bar{M}\right]^{1-\theta}\right)^{\frac{1}{1-\theta}} < \left(nP_{IC}^{1-\theta} + (1-n)\,P_{IC}^{1-\theta}\right)^{\frac{1}{1-\theta}} \qquad \left(\because \bar{M} < M \leq \frac{P_{IC}}{\delta}\right) \\ &= P_{IC}, \end{aligned}$$

which cannot be true. Therefore it must be that $P_{IC} < \delta M$. Then we have

$$\begin{aligned} P_{IC} &= \left(n\left[[\delta M]^{\frac{1}{\theta}} P_{IC}^{\frac{\theta-1}{\theta}}\right]^{1-\theta} + (1-n)\left[\delta\bar{M}\right]^{1-\theta}\right)^{\frac{1}{1-\theta}} < \left(n\left[(\delta M)^{\frac{1}{\theta}}\left[\delta M\right]^{\frac{\theta-1}{\theta}}\right]^{1-\theta} + (1-n)\left[\delta\bar{M}\right]^{1-\theta}\right)^{\frac{1}{1-\theta}} \\ &= \left(n\left[\delta M\right]^{1-\theta} + (1-n)\left[\delta\bar{M}\right]^{1-\theta}\right)^{\frac{1}{1-\theta}} = P_C < \left(n\left[\delta M\right]^{1-\theta} + (1-n)\left[\delta M\right]^{1-\theta}\right)^{\frac{1}{1-\theta}} = \delta M = P_F. \end{aligned}$$

Therefore, it has been shown that

$$P_{IC} < P_C < P_F.$$

From $Y = M/P$, it is also true that

$$Y_{IC} > Y_C > Y_F.$$

C.3 Proof of Proposition 3

The result in this proposition is a direct implication of Proposition 2. Taking log in the demand function (3), we get

$$\log Y_1 - \log Y = -\theta(\log P_1 - \log P) \quad \text{and} \quad \log Y_2 - \log Y = -\theta(\log P_2 - \log P),$$

where P_1 and P_2 are common prices, set by firms, that correspond to Y_1 and Y_2 respectively. Subtracting the second eqaution from the first, and then taking absolute values on both sides, we get $|\log Y_1 - \log Y_2| = \theta\,|\log P_1 - \log P_2|$. Since P_2 responds less to a shock under incomplete markets, $|\log P_1 - \log P_2|$ is smaller, and so is $|\log Y_1 - \log Y_2|$. Therefore $|Y_1 - Y_2|$ is also smaller under incomplete markets.