Bankruptcy Choice with Endogenous Financial Constraints[*]

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Abstract

In this paper we study firm dynamics and industry equilibrium when firms under financial distress face a non-trivial choice between alternative bankruptcy procedures. Given limited commitment and asymmetric information, optimal financial contracts specify that some firms are financially constrained and that default occurs in equilibrium. Default leads to either liquidation or renegotiation. Furthermore, contracts allow for enhanced creditor monitoring upon renegotiation, resembling actual reorganization practices. We calibrate the model to match certain aspects of the data on bankruptcy and firm dynamics in the U.S. Our counterfactual experiments show that, compared with an economy with liquidation only, the rehabilitation of firms has a sizable negative effect on exit rates, and modest positive effects on average size and productivity.

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1 Introduction

Models of financial frictions and firm dynamics typically ignore the possibility that troubled firms are rehabilitated.[1] However, recent data from both developed and developing countries strongly suggest that alternatives to liquidation are important resolution mechanisms for financially distressed firms. In the U.S., according to data from bankruptcy courts and Dun & Bradstreet, about 25% of formal bankruptcy filings that followed the 2008 financial crisis were for Chapter 11,[2] while two thirds of overall business failures (including informal bankruptcy and private workouts) were resolved under some reorganization procedure (see Table B1 and Figure B1 in Appendix B). Moreover, the World Bank's 2012 Doing Business reports that 21 economies have introduced or improved (in- and/or out-of-court) reorganization or debt restructuring proceedings since 2005, including: Spain in 2009, Italy and Austria in 2010, Denmark and South Africa in 2011, and more recently, Germany.[3]

What are the potential consequences of this recent spread in the use of alternative bankruptcy procedures? What is the role of institutional factors in producing different outcomes from alternative bankruptcy procedures? This paper provides answers to such questions by extending a simple model of endogenous financial constraints with liquidation, to allow for the possibility of *default, renegotiation and reorganization*.

In our model, an entrepreneur is endowed with a risky project and signs a contract with a bank in order to invest and realize project returns. However, financial contracts are constrained by moral hazard and limited commitment. In particular, in every period the entrepreneur receives a random outside option and cannot credibly commit not to exercise it. A key feature of our model is that the outside option is observed only by the entrepreneur and therefore actual default occurs along the equilibrium path.

If the entrepreneur defaults, the parties may renegotiate the contract or liquidate the firm. Renegotiation is immediate and results in a redistribution of the social surplus. If the entrepreneur does not default, then she privately observes project returns and may divert cash flows at some cost. The cost of diversion in turn depends on the quality of the monitoring technology used, which is itself a decision variable. That is, in every period the parties choose between a high quality, high cost and low quality low cost monitoring technology.

The high quality monitoring technology shares some features with formal (e.g., Chapter 11)

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[1] Recent exceptions are Corbae and D'Erasmo (2014), Peri (2014) and Senkal (2013).

[2] Chapter 7 of the U.S. Bankruptcy Code governs the process of liquidation in which the assets of a corporation are sold either piecemeal or as a going concern. Alternatives to liquidation are Chapters 11 and 13 of the Code, under which an exchange of securities is formally proposed in a reorganization plan.

[3] In the sample of 35 economies used by Claessens and Klapper (2005) to assess the relative use of bankruptcy, 33 countries had laws permitting both liquidation and restructuring of distressed firms.

or informal reorganization procedures. First, the use of this alternative is costly as are all cases of reorganization, where dismissing management entails learning costs and payments to trustees, accountants or courts are made. Second, it allows creditors to exert tighter control over the firm's revenues, which is one of the purposes of most reorganization cases (see Section 2.2). Finally, in equilibrium, this costly financing technology will only be used when the firm is under financial distress -after experiencing a long sequence of bad revenue shocks- but before deciding on liquidation.

We embed the financial contracting model described above into a standard industry equilibrium framework and then calibrate it to conduct a quantitative analysis. The baseline calibration seeks to match certain features of the U.S. data on bankruptcy and firm finance. In particular, we match quite well the exit rate, the frequency of renegotiation (Chapter 11) relative to liquidation (Chapter 7), and the average size of firms in renegotiation relative to the average size of all firms in financial distress (including liquidation and reorganization).

We use the calibrated version of the model to conduct a simple counterfactual exercise: a comparison with an economy where liquidation is the only way to deal with financially distressed firms. Our results suggest that the richer bankruptcy framework that allows for renegotiation and reorganization significantly reduces firm exit (thus increasing firm age). It also increases average (and aggregate) firm size and decreases size dispersion.

The paper is organized as follows. The next section summarizes the empirical literature on dynamic contracts and bankruptcy procedures, whose main findings motivate our theory. Section three highlights our contribution to the literature. In section four we presents and solve the contracting problems which are then embedded into the industry equilibrium framework in section five. Section six contains the quantitative analysis, and section seven concludes.

2   An overview of the empirical evidence and related literature

Our theory of bankruptcy and firm finance is built on two premises: the use of dynamic credit arrangements and the importance of renegotiation and reorganization as alternatives to liquidation. In what follows we survey the main findings from the specialized literature devoted to each of these aspects. We also present a detailed description of where we stand in comparison with other contributions to the literature on financial frictions and firm dynamics.

2.1 Dynamic credit relationships

From a conceptual standpoint, long-term lending relationships can be an efficient way of overcoming informational asymmetries. In particular, the use of long-term contracts minimizes the cost of providing incentives for borrowers to reveal information through intertemporal transfers that are not available in short-term interactions (Boot (2000)).

Consistent with these theoretical arguments, available evidence for the U.S. does suggest that borrowers suffering from greater information asymmetry (e.g., small, non rated firms) are more likely to use long term credit relationships for funding their operation (Bharath, Dahiya, Saunders and Srinivasan (2007)). In fact, there is sound evidence that the duration of the bank-lender relationship positively affects the availability of credit, and, specially for young firms, is positively associated with intertemporal smoothing of contract terms (Petersen and Rajan (1994)). Overall, repeated borrowing seems to be a particularly attractive feature for firms with severe informational that would otherwise face higher financing costs (Bharath, Dahiya, Saunders and Srinivasan (2011)).

The recent financial crisis has provided another illustration of the benefits associated with enduring credit relationships. In a study of Italian firms, Bolton, Freixas, Gambacorta and Mistrulli (2013) report that banks specializing in long-term relationships offered more favorable continuation-lending terms to their client firms in response to the crisis. This is consistent with earlier evidence presented in Elsas and Krahnen (1998), according to which, German banks engaging in long-term lending relationships do provide some kind of liquidity insurance in situations of unexpected deterioration of borrower creditworthiness.

Given its above mentioned benefits, it is perhaps not surprising to find that long term financing is widely used by firms in the U.S. and elsewhere. In a panel of over 90,000 non-financial firms, collected by Custodio, Ferreira and Laureano (2013) for the period 1976-2008, the average fraction of debt with maturities over three and five years was 43% and 28%, respectively. Furthermore, the average maturity of syndicated loans for the period 1987-2008 was 4.15 years. But credit relationships often last longer than a single loan. For instance, in a sample of Belgian firms, Degryse and Cayseele (2000) report that the average duration of a loan is 2.4 years while the average length of relationship with the current lender is about 7.9 years.

The nature and purpose of long-term financing is directly related to the extent to which state contingency can be truthfully reported under asymmetric information. Inter-temporal transfers and information acquisition imply therefore that state contingency will indeed be a defining feature of dynamic credit relationships. In a recent paper, Roberts and Sufi (2009) provide compelling evidence that this is the case. For a sample of 1,000 credit agreements between U.S. firms and financial institutions, these authors show that 90% of long-term debt contracts

are renegotiated prior to maturity but less than 18% of these renegotiated episodes are associated with a covenant violation or payment default. Renegotiation of terms in the absence of severe financial distress or default suggests that long-term arrangements can be viewed as state-contingent contracts.[4]

2.2 Alternative bankruptcy procedures: costs and benefits

Alternatives to liquidation are important resolution mechanisms for financially distressed firms. In order to avoid liquidation, a firm under severe financial distress must do at least one of the following. Either it must raise cash through asset sales, operating improvements, and new financing; or it must negotiate with its creditors to reduce or postpone interest and principal payments on the debt. Of course, firms typically do both and the two are usually related to one another; as pointed out by Aghion and Bolton (1992) creditors are willing to renegotiate or write-off a fraction of their claims only if they can be credibly protected against borrowers' future opportunistic behavior.

Debt restructuring is a well known feature of bankruptcy procedures such as Chapter 11 in the U.S. The standard practice is that an exchange of securities is formally proposed by debtors[5] and may or may not be accepted by claimholders. Restructuring plans tend to have higher probability of success when the main creditors are banks, who appear to be more willing to renegotiate (lengthen) maturities and accept reductions of interest and principal (Gilson et al. (1990)). Furthermore, Roberts and Sufi (2009) show that renegotiations are more likely to result in favorable terms for the borrower when the latter has access to relatively inexpensive alternative sources of funds and thus the threat of exiting the relationship is credible. In their own words this illustrates how "outside options can generate surplus under the initial terms of the contract and lead to renegotiation".

Renegotiation and restructuring of debt are typically part of a broader reorganization plan aimed at improving operational efficiency and securing additional sources of cash. In fact, the most prominent way in which firms obtain new financing from creditors (and usually improve its operations in the process) is by realigning the interests of managers and creditors.[6] Detailed studies of reorganization cases provide ample support to the notion that under such arrangements

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[4]Naturally, long term contracts usually have quite some built-in state contingency whenever contract terms are linked to observables (e.g., adjustable rates, loan-to-value ratios).

[5]Filing for Chapter 11 is not always the exclusive right of stockholders. Creditors may file an "involuntary" Chapter 11 if they can demonstrate that the firm has been delinquent on its debt. For more details on the procedures under Chapter 11, see, for instance, Gilson, Kose and Lang (1990).

[6]Many other sources of operational improvements are used during formal corporate reorganizations (e.g., asset sales, layoffs, changes in business segment; see Wruck (1990)). We focus here in the agency problems of the firm since this is the key motivating factor behind our mdoelling choices.

creditors often condition renegotiations on the replacement of incumbent management and board of directors (Gilson (1990), Jostarndt and Sautner (2008), Ayotte and Morrison (2009)).[7]

In addition to management/directors dismissal, there is ample evidence that creditors exert control by influencing distressed firms' capital expenditures, leverage and shareholder payouts policies through stringent debt covenants (Nini, Smith and Sufi (2012)). Indeed, fresh financing provided under Chapter 11 usually comes with much stringent restrictions from creditors (Bharath, Panchapegesan and Werner (2013)).[8]

In practice, formal restructuring procedures offer several benefits to both debtors and creditors. First, provisions such as "debtor in possession" (DIP) offers a solution to the debt overhang problem, as new lenders are senior to all other claimants, except secured creditors. In fact, DIP lenders rarely fail to be fully repaid, which increases the troubled firm's chances of raising new cash (Gilson (2012)). In addition to DIP financing, automatic stay clauses ensure that the firm can continue its operations without the risk of "race against the assets" by its creditors. Recent evidence suggests that declines in operating income are reversed in the two quarters immediately following the Chapter 11 filing, and that this is mostly associated with improved investment policy (Kalay, Singhal and Tashjian (2007)).

Bankruptcy procedures are, of course, subject to sizable costs that are often the deciding factor in the course of action. Bris, Welch and Zhu (2006) is the most recent attempt to measure bankruptcy costs in the U.S.[9] That paper shows that bankruptcy costs are largely dependent upon firm size, which is not surprising since the administrative protocols of Chapter 11 virtually guarantee that, in larger and more complex cases, such costs will rapidly escalate (Gilson (2012)).[10] Bris et al. (2006) also show that, taken together, direct and indirect bankruptcy costs result in recovery rates under liquidation that are on, average close to 50%, much lower than under reorganization where this average is closer to 80%. These results coincide remarkably well with the cross-country survey data presented in Djankov, Hart, McLiesh and Shleifer (2008) for

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[7]For the U.S., Gilson (1990) finds that in a sample of 111 firms that were formally or privately reorganized between 1979 and 1985, on average 55% and 57% of the incumbent board directors and CEOs, respectively, were replaced over the course of reorganization. Ayotte and Morrison (2009) find that in a sample of firms filing for Chapter 11 during 2001, about 70% of CEOs are replaced right before the start of a reorganization process. For a sample of 267 German firms in the 1996-2004 period Jostarndt and Sautner (2008) find that, following debt restructurings and informal reorganizations, ownership representation by outside investors doubles, and four years into the reorganization process only 14% and 22% of incumbent directors and CEOs remain in office. In his study of 94 publicly traded businesses that exited Chapter 11 in 2002, Baird and Rasmussen (2003) concludes that "creditors use their powers to remove managers in whom they have lost conffidence, [and] replace the board of directors".

[8]These restrictions are directly aimed at influencing the corporate governance of the firm in bankruptcy, and often include specific covenants relating to board seats, asset sales, and even the transfer of control.

[9]Earlier attempts include Warner (1977) and Altman (1984).

[10]A simple ilustration of this feature of Chapter 11 is that when a firm files for such bankruptcy procedure, it must pay a fixed filing fee for each and every one of its subsidiaries.

high income countries. The picture in low-and-middle income countries is more grim, as recovery rates are expected to be, on average, less than 25% and 48% for liquidation and reorganization, respectively.

2.3   Related literature: financial frictions and firm dynamics

This paper draws heavily on the insights from the literature on financial frictions and firm dynamics.[11] Among the first contributions to this literature is the paper by Cooley and Quadrini (2001) in which the external finance premium is used to explain certain empirical regularities of firm dynamics such as age and size dependence.[12] In that paper, however, the sources of financial frictions and firm exit are not modelled explicitly as we do in the current paper.

The papers by Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006) and Hopenhayn and Werning (2008, 2006) study the roles of limited commitment and moral hazard in generating financial constraints and producing non-trivial firm dynamics.[13] The microfounded financial frictions embedded in these models create endogenous liquidation and are able to produce firms whose size and profits increase with age while their growth and hazard rates endogenously decrease with it. Quadrini (2004) and Clementi and Hopenhayn (2006) study risky investment under asymmetric information but abstract from the possibility of default. Albuquerque and Hopenhayn (2004) considers the threat of default but perfectly symmetric information implies that actual default does not occur in equilibrium. Finally, Hopenhayn and Werning (2008, 2006) build models with limited commitment and unobservable outside options in which default occurs in equilibrium. In contrast, our model includes private information on both firm returns and outside options but also allows for the possibility of contract renegotiation and costly mitigation of agency problems.

Firm dynamics models have also been used to study the role financial market institutions in explaining cross-country differences in firm size, productivity and firm growth. Arellano, Bai and Zhang (2012) adopt an incomplete markets framework in which default-risk interacts with direct lending costs to explain the observed cross-country variations in average firm size, leverage and growth as a function of differences in the level of financial development. Exit in this model is completely exogenous and bankruptcy procedures are not part of the analysis. Rodriguez-

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[11]The flourishing of this literature owes a large debt to the seminal papers of Jovanovic (1982) and Hopenhayn (1992) which developed the basic theory of firm dynamics that we still use today.

[12]Size dependence corresponds to the observation that, conditional on age, the dynamics of firms (growth, volatility of growth, job creation, job destruction, and exit) are negatively related to the size of firms. On the other hand, age dependence refers to the observation that, conditional on size, the dynamics of firms are negatively related to the age of firms

[13]While Albuquerque and Hopenhayn (2004) consider the option of default, enforcement constraints are added to the contracting problem to ensure that default never occurs in equilibrium.

Delgado (2010) extends this same framework to consider costly liquidation and to show that high costs associated with liquidation not only increase the cost of external finance but also create significant drops in aggregate productivity. Alternatives to liquidation are not considered in this model, however.

Broadly speaking, the papers surveyed above consider one of the two following environments: firms under financial distress either default and renegotiate their debt, or they are liquidated and exit the market irreversibly. Recently Senkal (2013) and Corbae and D'Erasmo (2014) considered an environment in which liquidation and renegotiation coexist in an otherwise standard firm dynamics framework. In particular, these papers take an incomplete markets approach in which firms under distress can either file for liquidation or renegotiate their debt with creditors in order to continue operating. A similar approach is taken by Peri (2014) who models reorganization as the combination of debt renegotiation and an uncertain period in which the firm is unable to issue debt or distribute dividends. However, all three papers abstract from asymmetric information and, accordingly, the only goal of a restructuring procedure is to reduce the amount of firm debt. By contrast, our model captures both aspects of rehabilitation procedures summarized in the previous section: debt renegotiation and firm reorganization through reduced agency.

Finally this paper is also related to a large body of research on the issues of financial distress, security design and corporate control.[14] In the theoretical strand of this literature, White (1994) first introduced the notion of corporate bankruptcy as a filtering device: bankruptcy law should be designed so as to force inefficient firms into liquidation and allow efficient firms to be rehabilitated.[15] Recent country studies from developing countries which carried out bankruptcy reform have provided empirical support for this proposition (Gine and Love (2010), Araujo, Ferreira and Funchal (2012), and Lim and Hee Hahn (2004)). Taken together, these empirical studies show that lowering bankruptcy costs and enhancing creditor protection improves the separation between persistently underperforming firms (that should be liquidated) and firms with temporary setbacks (that should undergo reorganization).

3   The model economy

We now present a theory of firm finance in which contracting parties are presented with different alternatives to deal with the possibility of financial distress. After describing the environment and main workings of the model, we introduce ex-ante project heterogeneity and then embed the contracting model into a standard industry equilibrium framework.

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[14]A comprehensive survey of this literature can be found in Senbet and Wang (2012).
[15]However, White (1994, 1989) also points out that in the presence of imperfect information this filtering mechanism is deemed to operate with error at best.

3.1 Preferences and technology

The entrepreneur has access to a project characterized by a production technology $F : \{0,1\} \times R_+ \to R_+$ which combines working capital inputs, $k_t \in R_+$, with project-level productivity z_t. More specifically, $F(k_t, z_t) = z_t f(k_t)$, with $f(0) = 0, f' > 0, f'' < 0$ and $\lim_{k \to 0} f'(k) = \infty$, $\lim_{k \to \infty} f'(k) = 0$, while z_t is the realization of an i.i.d. random variable Z with support $\{z_L, z_H\} = \{0,1\}$ and $\Pr(Z = z_H = 1) = \pi$. The project requires, in addition to $k_{t \geq 0}$, an initial set-up cost I_0. The entrepreneur has wealth $M < I_0$ so, to operate the technology, she must enter a financial contract with a bank with deep pockets. We refer to projects that are successfully initiated as "firms". In each period in which the firm operates and returns z_t are realized, the bank expects a repayment from the entrepreneur, τ_t, per period.

Both the entrepreneur (e) and the bank (b) are risk-neutral, discount cash flows at a common rate $\beta \in (0,1)$, and seek to maximize the expected present value of dividends:

$$\left\{ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t^j \right\}$$

for $j = e, b$. The following assumption is introduced to guarantee the existence of a firm:[16]

Assumption 1 $\{\pi f[(f^{(-1)})'(1/\pi)] - [f^{(-1)}]'(1/\pi)\}(1-\beta)^{-1} > I_0$

3.2 First-best

Under symmetric information and perfect enforcement, this problem is trivial enough: the properties of $f(\cdot)$ imply that there exists a unique:

$$\tilde{k} = \arg\max_k [f(k) \mathbb{E}Z - k] = \arg\max_k [\pi f(k) - k],$$

which is referred to the first-best level of capital. A planner facing no information or commitment constraints, and concerned with maximizing social surplus only, will choose $k_t = \tilde{k} \ \forall \ t \geq 0$. Thus, the first-best value of the firm is given by $\tilde{W} = [\pi f(\tilde{k}) - \tilde{k}]/(1-\beta)$, with $\tilde{V} = \pi f(\tilde{k})/(1-\beta)$ being the lifetime expected value accruing to the entrepreneur. The solution to the first-best problem, therefore, implies that all entrepreneurs are able to borrow the first-best level of capital $\tilde{k}$ and, once started, the firm will never grow, shrink or exit.

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[16] In what follows, the terms "entrepreneur" and "firm" will be used interchangeably.

3.3 Contracts under private information and limited commitment

The problem becomes interesting when private information and limited commitment are introduced as follows. At the beginning of each period, the entrepreneur can leave the project and take an outside option s which is itself the realization of an i.i.d. random variable with support $S = [\underline{s}, \bar{s}]$ and differentiable cdf $G(s)$. As in Hopenhayn and Werning (2008), s is private information to the entrepreneur. The entrepreneur will therefore take her outside option ("default") whenever s is higher than the value that she can expect from continuing with the project given the current terms of the contract. Let $x_t = 1$ if the entrepreneur decided to default (after observing s_t) in period t and $x_t = 0$ otherwise.

Notice that the unobservable nature of the outside option implies that contract terms may not depend upon s as it does in other models of limited commitment (Thomas and Worrall (1990), Albuquerque and Hopenhayn (2004)). There, contracts are designed so as to avoid actual default. In contrast, given the current information structure, default will happen along the equilibrium path as in Hopenhayn and Werning (2008).[17]

If the entrepreneur defaults and leaves the relationship, she gets to keep s, and the firm is liquidated. In such case, the lender appropriates the scrap value of the project, Δ. Alternatively, the parties may pay verification and renegotiation costs, θ, and renegotiate the original contract. Let $\ell_t = 1$ if the firm was liquidated upon default and $\ell_t = 0$ if the parties renegotiated. We assume that in case of liquidation, both parties receive their payoff at the beginning of the following period.

If the entrepreneur does not default, the parties then face an investment decision under asymmetric information since returns z_t are not observed by the bank. That is, the entrepreneur can misreport actual returns to the bank and divert cash flows at some cost ρ^i. Such cost in turn depends on the monitoring technology used which is a decision variable at the beginning of the period (after observing the default decision). In particular, $\rho^r > 0$ if the parties decide to use a high-quality monitoring technology which costs a fraction μ_r of the firm's returns and $\rho^u = 0$ if they use a low-quality monitoring technology which is costless. Let $\mathfrak{u}_t = 0$ if the parties decide to use the high-quality monitoring technology in t and $\mathfrak{u}_t = 1$ otherwise. In what follows -and for reasons that will become apparent shortly- we refer to the case in which $\mathfrak{u}_t = 0$ as "reorganization"[18] and label the case of $\mathfrak{u}_t = 1$ as "undistressed".

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[17]For a sovereign debt model with unobservable outside option and production shocks, see Aguiar and Amador (2014), section 4.1.

[18]As mentioned in the introduction, this costly hi-quality monitoring technology shares some features with formal or informal reorganization procedures. First, the use of this alternative is costly as are most (all) cases of reorganization, where dismissing management entails learning costs and payments to trustees, accountants or courts are made. Second, it allows creditors to exert tighter control over the firm's revenues which is one of the purposes of most reorganization cases. Finally, in equilibrium, this reduced agency path will only be taken when

Private information implies that the terms of the contract must depend upon the agent's report, $\hat{z}_t$, rather than on z_t itself. While the agent's reporting strategy may be arbitrarily complicated, the Revelation Principle can be invoked to identify the support of Z with the set of admissible reports. To avoid any indeterminacies and for simplicity it is assumed that when indifferent the agent will not divert cash flows and that costs θ, μ_r are borne by the bank.

To complete the formal statement of the problem, let $\mathfrak{s}_t = s_t$ if there is renegotiation ($x_t = 1$ and $\ell_t = 0$) and $\mathfrak{s}_t = \emptyset$ if $x_t = 0$ or $\ell_t = 1$. Let $\tilde{h}^{t-1} = (h^{t-1}, x_t)$ denote the interim public history after the default decision has been made. For notational convenience, we label choices made after observing $\tilde{h}^{t-1}$ with a subscript $i_t \in \{u, r\}$, where $i_t = r$ ("reorganization") if $\mathfrak{u}_t = 0$ so the firm is financed under the high-quality monitoring technology and $i_t = u$ ("undistressed") if $\mathfrak{u}_t = 1$ so the firm operates under the low-quality financing technology. That is, in what follows, we write, e.g., $k_t\left(h^{t-1}, 0, \mathfrak{u}_t\right)$ as $k_t^i\left(h^{t-1}\right)$. Furthermore, we adopt the convention $\hat{z}_t^i = k_t^i = \tau_t^i = \mathfrak{u}_t = \emptyset$ if $x_t = 1$, $\ell_t = \emptyset$ if $x_t = 0$. Public histories are given by $h^t = \{h^{t-1}, h_t\} \in H^t$ where $h_t = \left\{x_t, \mathfrak{u}_t, \ell_t, \mathfrak{s}_t, [k_t^i, \hat{z}_t^i, \tau_t^i]_{i=u,k}\right\}$. The timing of events is depicted in Figure 4.

A contract, $\boldsymbol{\sigma}$, is a collection of functions specifying a reorganization - undistressed policy, $\mathfrak{u}_t$, liquidation - renegotiation decisions, ℓ_t, capital advancements, k_t^i, and repayments to the bank, τ_t^i : $\boldsymbol{\sigma} = \left\{x_t\left(h^{t-1}\right), \mathfrak{u}_t(\tilde{h}^{t-1}), \ell_t(\tilde{h}^{t-1}, \mathfrak{s}_t), [k_t^i(\tilde{h}^{t-1}), \tau_t^i(\tilde{h}^{t-1})]_{i=u,r}\right\}_{t=0}^{\infty}$.[19] This contract implicitly defines an equity value for the firm V_t and the long-term debt level or value to the bank B_t. The equity value for the firm gives the discounted sum of future dividends whereas the long-term debt or value to the lender gives the discounted cash flows to the lender. Thus, the total asset value after history h^t is defined by $\mathsf{W}_t = V_t + \mathsf{B}_t$. As in Spear and Srivastava (1987), V_t effectively summarizes all information provided by history up to $t-1$, and can be used as state variable in a recursive formulation of the contracting problem. The points $(\mathsf{B}(V), V)$ trace the Pareto frontier and $\mathsf{W}(V) = \mathsf{B}(V) + V$ is usually referred to as the "value of the firm".

We will characterize contracts recursively by specifying value functions at the different decision stages within a period. Working backwards, consider first the problem of a firm which has not defaulted in the current period ($x_t = 0$) and is being financed under monitoring quality i. This problem can be written recursively as:

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the firm is under financial distress -after having experienced a long enough sequence of bad revenue shocks- but before deciding on liquidation.

[19] In a feasible contract, probabilities are well defined and the firm's limited liability is satisfied so that the entrepreneur's payoff is non-negative in each period.

Nature Draws z_t

undistressed
$\mathbf{u}_t = 1$

Bank lends
k_t^i

Entrepreneur
reports $\hat{z}_t^i$

Entrepreneur
repays τ_t^i

monitor
$\mathbf{u}_t = 0$

Bank pays μ_r

Continue
$x_t = 0$

Default
$x_t = 1$

Liquidate
$\ell_t = 1$

Renegotiate
$\ell_t = 0$

Bank receives $\beta\Delta$
Entrepreneur receives βs_t

Nature Draws
s_t
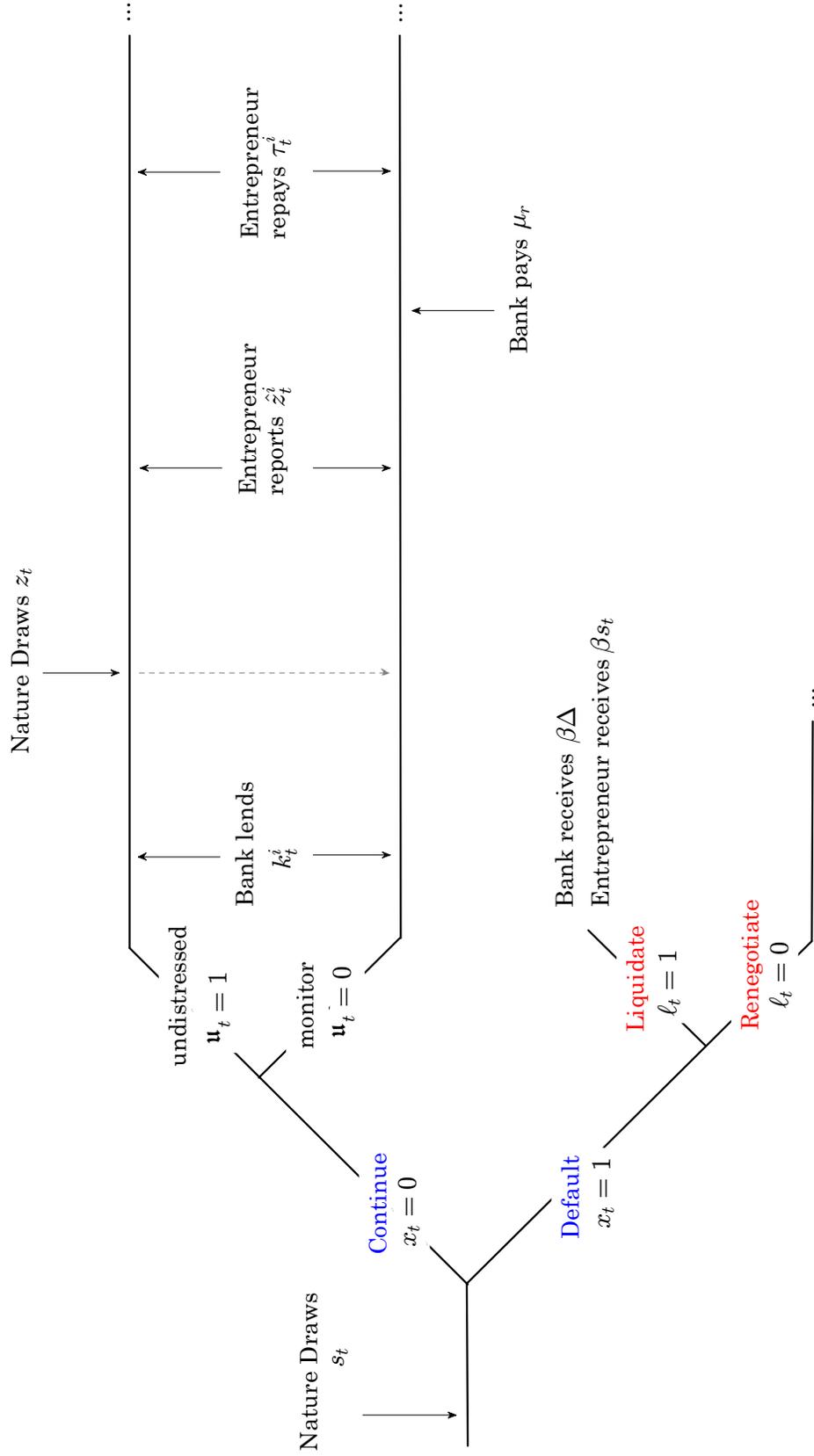
Figure 1: Timing of events within a period

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$$\widehat{W}_i\left(V_c^i\right) = \max_{k^i, \tau^i, V_H^i, V_L^i} \pi\left(1 - \mu_i\right) f\left(k^i\right) - k^i + \beta\left[\pi W\left(V_H^i\right) + (1 - \pi) W\left(V_L^i\right)\right] \qquad (\widehat{\mathbf{PP}})$$

$$s.t.: f\left(k^i\right) + \beta\left(V_H^i - V_L^i\right) \geq \tau^i + (1 - \rho^i) f\left(k^i\right),$$

$$V_c^i = \pi\left(f\left(k^i\right) - \tau^i\right) + \beta\left[\pi V_H^i + (1 - \pi) V_L^i\right],$$

$$f\left(k^i\right) \geq \tau^i \ \text{ and } \ V_H^i, V_L^i \geq 0$$

In $(\widehat{\mathbf{PP}})$, $\mu_u = 0$ and V_z^i, $z = H, L$, is the firm's value of equity beginning the following period after a revenue shock z has been reported. Moreover, this formulation of the problem already uses the fact that from limited liability $z_L^i = 0 \Rightarrow \tau_L^i = 0$ and hence τ_H^i can be written as τ^i. The first constraint in $(\widehat{\mathbf{PP}})$ requires that contracts are incentive compatible. Here, we make use of the fact that a realization of $z_L^i = 0$ will never result in the agent reporting z_H^i and thus only one incentive constraint is required. The second constraint in $(\widehat{\mathbf{PP}})$ imposes individual rationality (the so-called promise-keeping constraint) while constraints $f\left(k^i\right) - \tau^i, V_H^i, V_L^i \geq 0$ capture the entrepreneur's limited liability.

Next, given $x = 0$ the problem of choosing monitoring quality is given by:[20]

$$W^c\left(V_c\right) = \max_{V_c^u, V_c^r, \mathfrak{u}} \mathfrak{u} \widehat{W}_u\left(V_c^u\right) + (1 - \mathfrak{u})\widehat{W}_r\left(V_c^r\right) \qquad (\mathbf{PP})$$

$$s.t.: V_c = \mathfrak{u} V_c^u + (1 - \mathfrak{u}) V_c^r \text{ and } V_c^u, V_c^r \geq 0$$

where $\widehat{W}_i\left(V_c^i\right)$, $i = u, r$ satisfy $(\widehat{\mathbf{PP}})$. Now, consider the problem of the match when $x = 1$. At this point we assume that the entrepreneur will prefer to renegotiate and keep the firm if she gets at least her outside option in the renegotiated contract. That is, we assume that the bank has all the bargaining power. Then the decision whether to liquidate the firm or to continue with the relationship by renegotiating the original contract, for a given realization s, solves:

$$W_d\left(s\right) = \max_{\ell(s) \in \{0,1\}} \ell\left(s\right) W_\ell\left(s\right) + \left[1 - \ell\left(s\right)\right] (1 - \theta) \beta W\left(s\right) \qquad (\mathbf{PD})$$

where $W_\ell\left(s\right) = \beta\left(\Delta + s\right)$ and we have used the fact that, upon default, the entrepreneur is indifferent between renegotiation and liquidation, so that s is the appropriate argument for $W\left(\cdot\right)$.

---

[20] Hereafter, the dependence of the policy functions on equity is supressed and we write e.g., $x\left(s, V\right)$ as $x\left(s\right)$. Ocasionally we revert to e.g., $x\left(s, V\right)$ when characterizing these policies as functions of V.

Then the optimal default policy can be found as the solution to:

$$
W(V) \quad = \quad \max_{x(s)\in\{0,1\}} W^c\left(V_c\right) \int_{\mathcal{S}} \left[1 - x\left(s\right)\right] dG\left(s\right) + \int_{\mathcal{S}} x\left(s\right) W_d\left(s\right) dG\left(s\right) \qquad \text{(P)}
$$

$$
s.t. \quad : \quad V = V_c \int_{\mathcal{S}} \left[1 - x\left(s\right)\right] dG\left(s\right) + \beta \int_{\mathcal{S}} x\left(s\right) s dG\left(s\right)
$$

$$
: \quad x\left(s\right) = \begin{cases} 1, & \text{if } \beta s > V_c \\ 0, & \text{otherwise} \end{cases}
$$

where $W^c\left(V_c\right)$ and $W_d\left(s\right)$ in the objective function of (P) satisfy, respectively, (PP) and (PD) and the promise-keeping constraint already uses the fact that upon default the entrepreneur receives exactly the value of her outside option.

3.4   Optimal contracts and the value of the firm

We begin our characterization of optimal contracts by studying the solution to problems $(\widehat{\text{PP}})$-(PP). That is, we first consider optimal policies when $x = 0$. Notice that for each $i = u, r$, the problem in $(\widehat{\text{PP}})$ is virtually identical to that of Clementi and Hopenhayn (2006) (henceforth CH) and hence their main results apply. In particular, capital advancement policy satisfies $k^i\left(V^i\right) < \tilde{k}^i = \arg\max_k \left[\pi\left(1 - \mu_i\right) f\left(k^i\right) - k^i\right]$ as long as $V^i \leq \tilde{V}^i = \pi\left(1 - \rho_i\right) f(\tilde{k}^i)/\left(1 - \beta\right)$; that is, the firm is borrowing constrained. Along with risk neutrality, this implies that allowing the equity value V^i to reach the threshold $\tilde{V}^i$ in the shortest possible time is optimal, i.e., $V^i \leq \tilde{V}^i$ implies $f\left(k^i\right) = \tau^i$. This allows capital to increase with equity values so that the endogenous financing constraints tend to relax as the firm's equity grows. Finally, for $V^i \leq \tilde{V}^i$ future equity values satisfy $V_L^i\left(V^i\right) < V^i < V_H^i\left(V^i\right)$, implying that the firm's equity value increases with a good shock and decreases with a bad shock ("cash-flow sensitivity"). These results allow us to reduce $(\widehat{\text{PP}})$ to unidimensional maximization problems:

$$
\widehat{W}_i\left(V_c^i\right) \quad = \quad \max_{k^i} \pi\left(1 - \mu_i\right) f(k^i) - k^i + \beta \left\{ \pi W\left[\frac{V_c^i + \left(1 - \pi\right)\left(1 - \rho^i\right)f(k^i)}{\beta}\right] \right. \qquad (\widehat{\text{P3}})
$$
$$
\left. + \left(1 - \pi\right) W\left[\frac{V_c^i - \pi(1 - \rho^i)f(k^i)}{\beta}\right] \right\}
$$

Notice that for $i = u$, and using the notation and definitions of the previous paragraph, one has that $\tilde{k} = \tilde{k}^u$, $\tilde{W} = \tilde{W}^u$ and $\tilde{V} = \tilde{V}^u$. Denote by $\tilde{W}^r$ the value of a firm that is currently under "reorganization" and operated at scale $\tilde{k}^r$. For the reminder of the paper, and given that $\mu_u = \rho^u = 0$, we write $\mu = \mu_r$ and $\rho = \rho^r$. Then:

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Lemma 1 $\mu > 0 \Rightarrow \tilde{W}^r < \tilde{W}$.

Proof. See appendix A. ■

Lemma 1 establishes that for large enough values of equity, leaving the firm undistressed is optimal. Finding conditions under which $\mathfrak{u} = 0$ is optimal requires some more work:

Proposition 1 *There exist* $\Delta, \mu, \rho \in (0,1)$ *with* $\mu_r < \rho^r$ *such that for some* $0 < V_{\mathcal{R}} < \tilde{V}^r$, $V_c < V_{\mathcal{R}} \Rightarrow W^c(V_c) = \widehat{W}_r(V_c^r)$ *and* $V_c \geq V_{\mathcal{R}} \Rightarrow W^c(V_c) = \widehat{W}_u(V_c^u)$

Proof. See Appendix A. ■

Heuristically, Proposition 1 says that reorganization is optimal for intermediate values of equity and the firm is left undistressed if equity is large enough. The content and intuition for Proposition 1 can be seen graphically in the left panel of Figure 2, where the function $W^c(V_c)$ is shown as the upper envelope of the functions $\widehat{W}_u(V_c^u)$ and $\widehat{W}_r(V_c^r)$. An immediate consequence of the proposition is that for some combinations of parameters, the value of the firm is higher when reorganization is an option than when only liquidation is available as in CH (see right panel of Figure 3).

Figure 2 traces the value of the firm as a function of continuation (i.e., no-liquidation) equity V_c. In the region to the right of $V_{\mathcal{R}}$ the firm is undistressed but may be financially constrained. In turn, the equity region in which the firm is financially distressed (left of $V_{\mathcal{R}}$) can be divided into liquidation and reorganization. The right panel of Figure 2 compares the value of the firm under a contract which allows for costly hi quality monitoring, with a contract in which only the low quality monitoring technology is available (i.e., the contract found in CH).
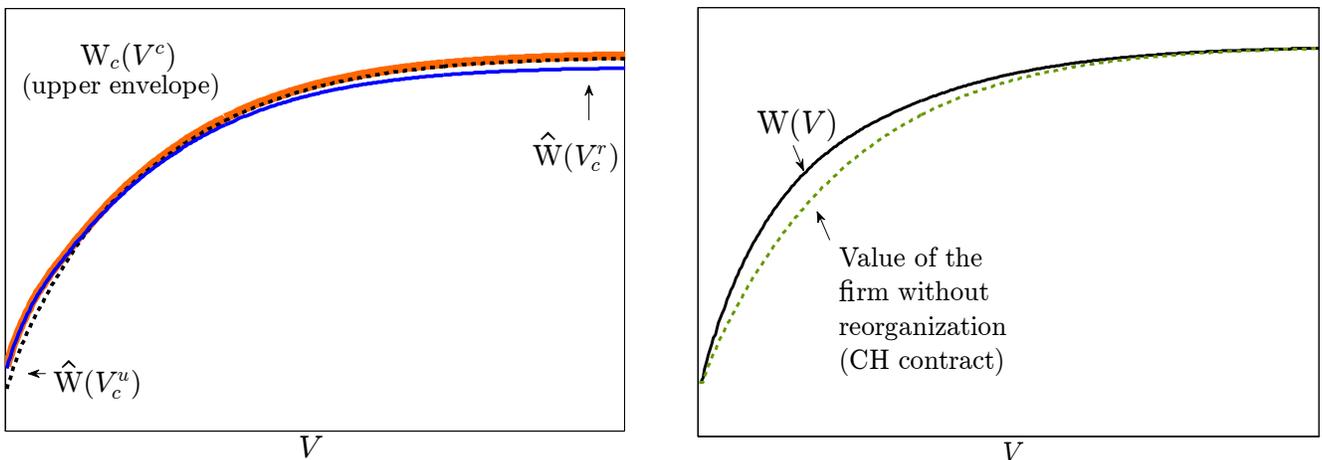

Figure 2. The reorganization option and the value of the firm

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Next we add the following assumption in order to provide a sharper characterization of the default and liquidation policies:

Assumption 2 *Suppose now that $s \sim \mathcal{U}[0, \bar{s}]$ with $\bar{s} < \tilde{W}^u$*

The final statement in Assumption 2 implies that the outside opportunity is never more valuable than the total value of the project, if the latter were operated at full efficiency. Confronted with any contract, the entrepreneur will employ a reservation policy, taking any outside opportunities above some threshold s_d and rejecting the rest. That is, for each V, $s \geq s_d \Rightarrow x(s, V) = 1$ and $s < s_d \Rightarrow x(s, V) = 0$. The entrepreneur's lifetime utility evolves according to:

$$V = \int_0^{\bar{s}} \max\{\beta s, V_c\} \, dG(s) = V_c \int_0^{s_d} dG(s) + \beta \int_{s_d}^{\bar{s}} s \, dG(s)$$

It is easy to see that if $V \geq \beta \bar{s}$ the entrepreneur will be better off by staying in the relationship regardless of the realization of s. For $V < \beta \bar{s}$ the following proposition characterizes the default policy in the optimal contract:

Proposition 2 *Suppose that Assumption 2 is satisfied. Then in the optimal contract, for $V < \beta \bar{s}$, the default threshold, $s_d(V)$, is weakly increasing in V. Moreover, there exists $V^+ > \beta E(s)$ such that for $V \geq V^+$, $s_d(V)$ is strictly increasing in V.*

Proof. See Appendix A. ∎

The intuition of Proposition 2 is straightforward: as the value delivered by the original contract increases, the entrepreneur requires a higher realization of the outside option to be tempted to default. However, the truncation of the equity domain introduced by the risk of default imposes a lower bound on the default threshold, hence the weak qualification at the beginning of the statement. An immediate result from this proposition is:

Corollary 1 *There exists $V^- > \beta E(s)$ such that for $V \geq V^+$ the optimal working capital policy is $k = 0$*

Proof. See Appendix A. ∎

That is, for small enough values of equity, the firm can survive into the following period without capital advancement (inaction). This will happen if the realization of the outside option is very low and the entrepreneur does not default even knowing that the contract promises her

little equity. Again, because the lack of commitment truncates the equity domain from below, no positive level of capital can satisfy the incentive constraint (recall that $k > 0$ requires $V_L < V$).

We now turn to characterizing the liquidation decision. Recall that the entrepreneur is indifferent between the liquidation/renegotiation outcome as she is assured a payoff βs. If the contract is renegotiated, the value of the firm (after proportional renegotiation costs are paid) is given by $\beta (1 - \theta) \mathrm{W}(s)$. In such case, the bank receives $\beta (1 - \theta) \mathrm{W}(s) - \beta s$, while if the firm is liquidated, the bank receives $\beta \Delta$. Thus, the liquidation threshold solves:

$$\mathrm{W}(s_\ell) = \frac{\Delta + s_\ell}{1 - \theta} \qquad (1)$$

Naturally, if θ is too large, equation (1) will not have a solution which, once again, points to the role of bankruptcy costs in shaping renegotiation/liquidation decisions. Unfortunately, low renegotiation costs are not enough to find s_ℓ as equation (1) may not have a unique solution. The following assumption introduces a sufficient condition for s_ℓ to be unique and allows us to provide a sharper characterization of the liquidation decision:

Assumption 3 $W(\bar{s}) > \frac{\Delta + \bar{s}}{1 - \theta}$

When assumption (3) is satisfied, the bank will find it optimal to renegotiate if s is sufficiently large and liquidate otherwise. We summarize our previous discussion in the following proposition:

Proposition 3 *Suppose that assumptions 2 and 3 are satisfied. Then $s_\ell(V)$ is unique and satisfies $s_\ell(V) \geq s_d(V)$.*

This result is in line with the evidence discussed in section 2 according to which firms with better outside options in the form of alternative financing (e.g., through equity) are more likely to successfully renegotiate their contracts with creditors. With the results from propositions (2)-(3) at hand, the problem in (**P**) can be conveniently reformulated, for $V \in [V_+, \bar{s}]$, as:

$$\mathrm{W}(V) = \mathrm{W}^c(V_c) \int_0^{s_d} dG(s) + \beta \int_{s_d}^{s_\ell} [\Delta + s] \, dG(s) + \beta (1 - \theta) \int_{s_\ell}^{\bar{s}} \mathrm{W}(s) \, dG(s) \qquad (\mathbf{PU})$$

$$= \frac{1}{\bar{s}} \left\{ s_d \mathrm{W}^c(V_c) + \beta \Delta (s_\ell - s_d) + \frac{\beta}{2} (s_\ell^2 - s_d^2) + \beta (1 - \theta) \int_{s_\ell}^{\bar{s}} \mathrm{W}(s) \, dG(s) \right\}$$

where, again, $\mathrm{W}^c(V_c)$ solves (**PP**), s_ℓ solves (1) and $s_d = \max\{\beta E(s), \sqrt{2\bar{s}V\beta^{-1} - \bar{s}^2}\}$. The results from Propositions (2)-(3) can be seen graphically in the left panel of Figure 3

where we have depicted the optimal default, liquidation and renegotiation policies for a given parametrization, as well as the value of the firm when there is the possibility of default and renegotiation.
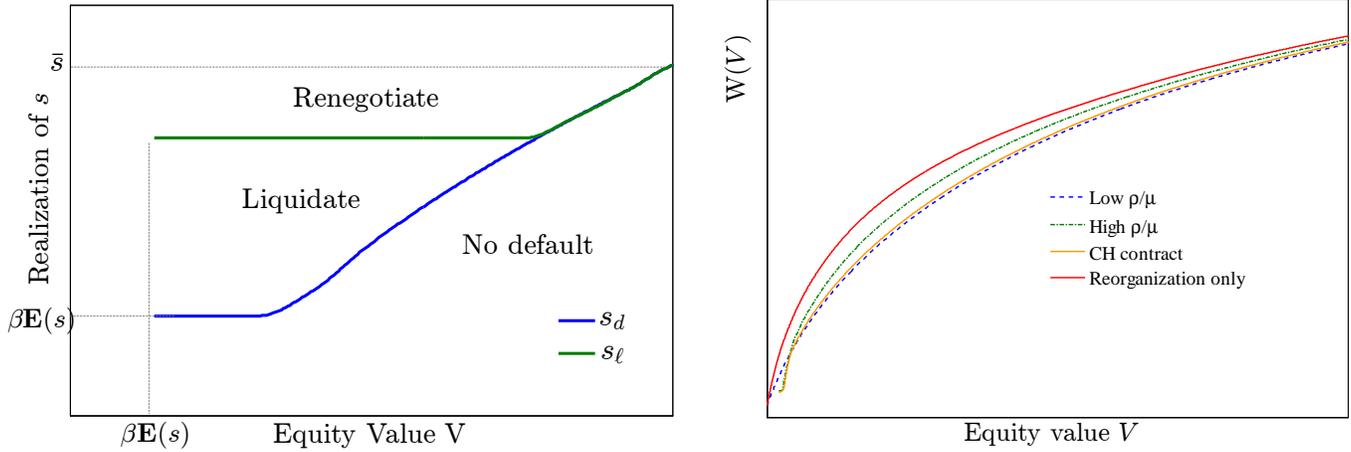

Figure 3. Liquidiation-renegotiation policies and the value of various contracts

The right panel of Figure 3 illustrates how the default risk affects the value of the firm. Importantly for our purposes the figure shows that, while the contract with the reorganization option strictly dominates the CH contract, this may or may not be true for the contract with a default option. In particular, the CH contract may dominate the contract with default if the ratio ρ/μ is too high; i.e., if the benefit of reorganization is low relative to its cost.

3.5 Heterogeneous projects

Suppose now that in every period a continuum of entrepreneurs are born, each of which has access to exactly one project of average productivity π (recall $\pi = \Pr(Z = 1)$), which is drawn from a time-invariant distribution $\Gamma(\pi)$ with support Π. After project types are drawn, they become public information so that banks offer each entrepreneur a contract indexed by her type π. Accordingly, the value and policy functions are now written e.g., $W(V; \pi)$.

As a first approximation to the effect of project heterogeneity, consider the simple case in which there is neither default nor reorganization (i.e., $\bar{s} = 0$ and $\mu_r = \infty$), but the firm can be liquidated at the beginning of each period (the CH contract). A deterministic liquidation rule will specify a cutoff value of equity $V_\ell(\pi)$ such that for $V \leq V_\ell(\pi)$ the firm is liquidated. We will next show that even if there is neither renegotiation nor reorganization, a financial contract that

relies on intertemporal incentives for truthful reporting induces selection along the productivity dimension:

Proposition 4 *Suppose that $\pi' > \pi$. Then $V_\ell(\pi') < V_\ell(\pi)$.*

Proof. See Appendix. ∎

Figure 6 illustrates the content of Proposition 4: A project with higher average productivity will face a smaller liquidation set. Since a firm reaches the liquidation set only after experiencing enough bad realizations of the revenue shock, and such low realizations are more likely with lower π, firms with higher average productivity face a lower unconditional probability of exit. In other words, the contract exhibits financial selection.
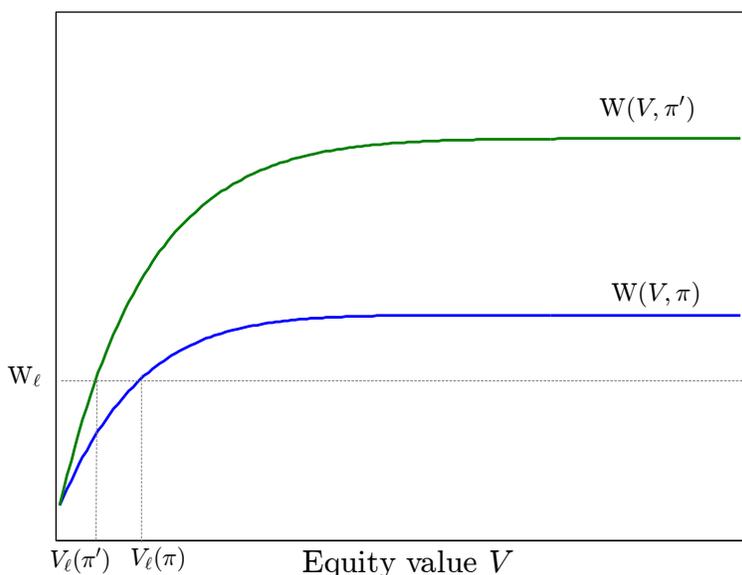

Figure 4. Financial Selection

This intuition can be carried over to the contracting problem of section 3.3. As shown in Figure 3, the reorganization option increases the slope of the value function at the origin. With heterogeneous projects, this effect is compounded so that projects with higher average productivity disproportionately benefit from the reorganization option and financial selection is enhanced. This issue will be pursued further in the quantitative exercise (see Figure 6).

3.6 Industry equilibrium

In order to conduct a meaningful quantitative exercise, the contracting problems studied above are embedded into a standard industry equilibrium framework. The details of the industry follow

closely those found in Li (2010).

Incumbent firms behave competitively, taking prices in output (p) market as given. Aggregate demand for the product is given by the inverse demand function, $p = D(Q)$, where the function D is continuous, strictly decreasing, and satisfies $\lim_{Q \to \infty} D(Q) = 0$ and $0 < \lim_{Q \to 0} D(Q) < \infty$. Notice that the output price is now a state variable and therefore value functions and policy functions depend upon it, for instance, $W(V; p)$.

We assume that entrepreneurs must pay the setup cost I_0 before drawing a project from the invariant distribution $\Gamma(\pi)$. Therefore, the value entitlement to an entrepreneur upon entry, V_0, is such that -given competition- banks break even in expectation:

$$\max_{V_0 \geq 0} V_0 \qquad \textbf{(P0)}$$

$$s.t. \int_\Pi B(V_0, \pi; p) \, d\Gamma(\pi) \geq I_0 - M \qquad (2)$$

That is, entrepreneurs are offered the "average" initial equity though all other contract terms are project specific.[21] If a solution to **(P0)** exists,[22] denoted as $V_0(p)$, (2) is binding, i.e., $\int B(V_0(p), \pi; p) \, d\Gamma(\pi) = \int W(V_0(p), \pi; p) \, d\Gamma(\pi) - V_0(p) = I_0 - M$. Once the initial equity value $V_0(p)$ is determined, the evolution of a new firm's equity value during its life cycle is completely regulated by the (project-specific) optimal financial contract.

To generate an invariant distribution of firms, one must allow for some exogenous exit. Otherwise, for each cohort of new entrants there will be some firms that reach the unconstrained status and never exit. This would imply that the total mass of firms grows over time without bound. Hence, it is hereby assumed that, in every period, entrepreneurs are assumed to face a time-invariant exogenous probability $1 - \eta$ of surviving into next period. For simplicity, we assume that if the entrepreneur dies exogenously, the firm becomes obsolete and the bank's payoff is zero.[23]

The state of the industry can be described by the distribution of firms over equity-type pairs (V, π). Let $\psi_t(V, \pi; p)$ denote the distribution of incumbent firms after (endogenous and exogenous) liquidation has taken place and let E_{t+1} stand for the mass of new entrants at the beginning of $t + 1$. The law of motion for ψ_t can be written as:

$$\psi_{t+1} = \mathcal{T}^*(\psi_t, E_{t+1}; p) \qquad \textbf{(D)}$$

---

[21] That is, after paying the initial set-up cost, the entrepreneur's project type becomes common knowledge.

[22] A solution to this problem may not exist if $I_0 - M$ is too large.

[23] This way, exogenous exit merely implies a lower discount rate $\hat{\beta} = \beta(1 - \eta)$ and does not require modifying the contracting problems of the previous sections.

The expression for the mapping $T^{*}(\cdot)$ is derived in Appendix A for the economy with default and renegotiation. This is the more general economy and definitions are easily obtained for the other cases by applying suitable changes. We are now in a position to define an equilibrium:

Definition 1 *A stationary competitive equilibrium for the industry consists of output Q^{*} and price $p^{*} \geq 0$; policy functions $[\ell(s, V, \pi; p^{*}), x(s, V, \pi; p^{*})]_{s \in \mathcal{S}}$, $\mathfrak{u}(V, \pi; p^{*})$, $V_{c}(V, \pi; p^{*})$, $V_{\ell}(V, \pi; p)$, and $[V_{c}^{i}(V, \pi; p^{*}), k^{i}(V_{c}^{i}, \pi; p^{*}), \tau^{i}(V_{c}^{i}, \pi; p^{*}), V_{H}^{i}(V_{c}^{i}, \pi; p^{*}), V_{L}^{i}(V_{c}^{i}, \pi; p^{*})]_{i=r,u}$, as well as value functions $\mathsf{W}(V, \pi; p^{*})$, $\mathsf{W}_{d}(s, V_{\ell}, \pi; p^{*})$, $\mathsf{W}^{c}(V_{c}, \pi; p^{*})$, $[\widehat{\mathsf{W}}_{i}(V_{c}^{i}, \pi; p^{*}))]_{i=r,u}$; a measure of incumbent firms ψ^{*} and a mass of entrants E^{*} such that:*

(i). The value and policy functions solve ($\widehat{\textcolor{red}{\mathbf{PP}}}$), ($\textcolor{red}{\mathbf{PP}}$), ($\textcolor{red}{\mathbf{PD}}$), and ($\textcolor{red}{\mathbf{P}}$)[24]

(ii). $p^{*} = D(Q^{*})$ and $Q^{*} = \int \pi f(k(V, \pi; p^{*})) \psi(dV, d\pi; p^{*})$

(iii). $V_{0}(p^{*}) \leq M$ solves ($\textcolor{red}{\mathbf{P0}}$) with equality if $E^{*} > 0$

(iv). $\psi_{t} = \psi^{*}$ and $E_{t} = E^{*}$ for all t solve ($\textcolor{red}{\mathbf{D}}$)

Condition (i) states that all players must optimize while condition (ii) requires goods market clearing. Condition (iii) is the free entry condition for firms; when $E^{*} > 0$ then $V_{0}(p^{*}) = M$ and $\int \mathsf{W}(V_{0}(p^{*}), \pi; p^{*}) d\Gamma(\pi) = I_{0}$, which pins down p^{*} in a stationary equilibrium with positive entry.

4 Quantitative analysis

We now calibrate the model presented above to match some salient features of the U.S. economy. With a reasonably realistic calibration at hand, we then conduct counterfactual experiments aimed at assessing the quantitative importance of reorganization and renegotiation.

4.1 Baseline calibration

Our baseline calibration is aimed at matching certain aspects of the U.S. economy. Given the nonlinearities of our model, it is not possible to precisely match parameters to moments. However, the mechanics of the model clearly indicates which are the key parameters for each set of moments. Below is a brief description of our calibration strategy and Table 1 presents a summary of the parameter values.

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[24]With suitable changes in notation to include π, p.

The model period is a quarter and the price of output is normalized to unity. The discount factor is set at $\beta = 0.97$ so as to match the average annual return of the S&P 500 over the period 1928-2013, which is 11.5%. As much of the finance literature, we view this as a better measure of the bank's opportunity cost than the risk-free interest rate often used in the RBC literature. We parametrize the production function as $f(k) = Ak^\alpha$ and choose $\alpha = 0.88$ for the returns to scale parameter following the calibration for the U.S. economy found in Li (2010). In his numerical exercises, Quadrini (2004) uses a value of 0.85 citing empirical work by Atkeson, Khan and Ohanian (1996), while Clementi and Palazzo (2013) use a value of 0.8.

<div align="center">

Table 1. Baseline calibration

parameter	description	value
β	Discount factor	0.97
$f(k)$	Production function	Ak^α
α	Returns to scale	0.88
Δ	Liquidation value (bank)	3.0
$\bar{s}$	Outside option upper bound	2.5
θ	Renegotiation costs	0.34
$\Gamma(\pi)$	Distribution of project types	$1 - R(\pi)^{-\zeta}$
Π	Support of $\Gamma(\cdot)$	$[0.5, 0.58]$
ζ	Tail parameter for $\Gamma(\cdot)$	4.45
V_0	Initial equity of new firms	3.728
η	Exogenous exit	0.01
μ	Cost of reducing agency	0.05
ρ	Cost of diverting cash flows	0.4

</div>

Following the literature on financial frictions and heterogeneous firms, we parametrize the distribution of project types as a Pareto distribution: $\Gamma(\pi) = 1 - R(\pi)^{-\zeta}$ where $R(\pi) \geq 1$ denotes the rank of π in the distribution. For the tail parameter we use $\zeta = 4.45$. In quantitative exercises similar to ours, Antunes, Cavalcanti and Villamil (2008) and Buera and Shin (2013) use $\zeta = 4.43$ and $\zeta = 4.15$, respectively. The support of the distribution is a less straightforward choice. In their numerical computations Clementi and Hopenhayn (2006) use a value of $\pi = 0.5$ while Li (2010) calibrates this parameter to 0.58. We therefore choose a Π that spans the entire interval between these two values found in the literature; that is, we set $\Pi = [0.5, 0.58]$.

Next, given α, Π, ζ and β, the upper bound for the total value of a liquidated firm, $\bar{s} + \Delta$, is

the key determinant (along with renegotiation costs) of the endogenous exit rate since it drives both incentives to liquidate and the initial equity of new firms, V_0. We choose $\bar{s} + \Delta = 5.5$ and take the exogenous exit rate $\eta = 0.01$ from Quadrini (2004) so as to match the average annual exit rate of firms in the U.S., estimated to be between 5.5% and 4.5% by Lee and Mukoyama (2013) and Dunne, Roberts and Samuelson (1988), respectively. In turn, we set $\Delta = 3$ so that the lender receives, on average 65% of the total value of the liquidated firm in line with available evidence on the fraction of bankruptcy assets that are distributed to creditors in Chapter 7 cases (see Bris et al. (2006)).

Given our assumption about the financing of new firms (i.e., problem (**P0**)), the choices made so far help us pin down the initial amount to be financed $I_0 - M = 4.3$, which gives us $V_0 = 3.728$. This in turn results in a ratio of working capital of new firms relative to incumbent firm of 18.6% which lies between the 17.4% reported by OECD (2001) for entering firms in manufacturing, and the 19.5% found in Haltiwanger, Jarmin and Miranda (2013) for young firms (1-10 years). The resulting stationary distribution of constrained firms (along the working capital dimension) bears some resemblance with the U.S. distribution of firms by employment (see Figure B2 in Appendix B).[25]

Table 2. Moments and data

moment	value	data
Exit rate	5.1%	4.5%-5.5%
Relative size of entrants (working capital)	18.6%	17.5%-19.8%
Assets of Ch.11 / assets of all distressed	63.4%	66.2%
Frequency of Ch.11 relative to Ch. 7	21.7%	24.5%
Ch. 11 recovery rate (pre-fees)	110.9%	107%
Ch. 11 recovery rate (post-fees)	74%	69%-90%

Finally, the key parameters of interest -those associated with bankruptcy procedures- are chosen so as to match some features of the data reported in empirical studies of financially distressed firms. The cost of renegotiation, θ, is set to 0.34 so we can approximate the frequency of Chapter 11 relative to Chapter 7. Next, given the data presented in the introduction (see also table B1 in Appendix B) showing that out-of-court reorganizations account for a large fraction of total business failures, we want to allow for reorganization to be optimal even beyond the

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[25] The distribution of U.S. firms is notorious for exhibiting siginficant positive skeweness (i.e., toward small firms) and a long right tail; see e.g., Luttmer (2007).

Chapter 11 (renegotiation+reorganization) option. Accordingly, the net benefit of reorganization, captured by the ratio ρ/μ is set to approximate the ratio of total assets $(B_t + V_t)$ of firms in Chapter 11 to all firms under reorganization which Chatterjee, Dhillon and Ramirez (1996) report to be close to 66%.[26]

As seen in Table 2, the results from our benchmark calibration also help us approximate some features of the U.S. corporate bankruptcy outcomes found in the data. In particular, we compute Chapter 11 recovery rates as the bank's value after renegotiation, $B(s_t)$, divided by the bank's value prior to default, $B(V_{t-1})$. This gives us $B(s_t)/B(V_{t-1}) = 110.9\%$ before bankruptcy costs, and of $(1-\theta)B(s_t)/B(V_{t-1}) = 74\%$ after costs. These figures conform well with those reported by Bris et al. (2006) of 107% and 69%-90.1%, respectively.

Firm dynamics

We now describe the model's implications for firm growth and survival, and compare them with the empirical evidence. The dynamics of firms resulting from the baseline calibration are presented in Figure 5.
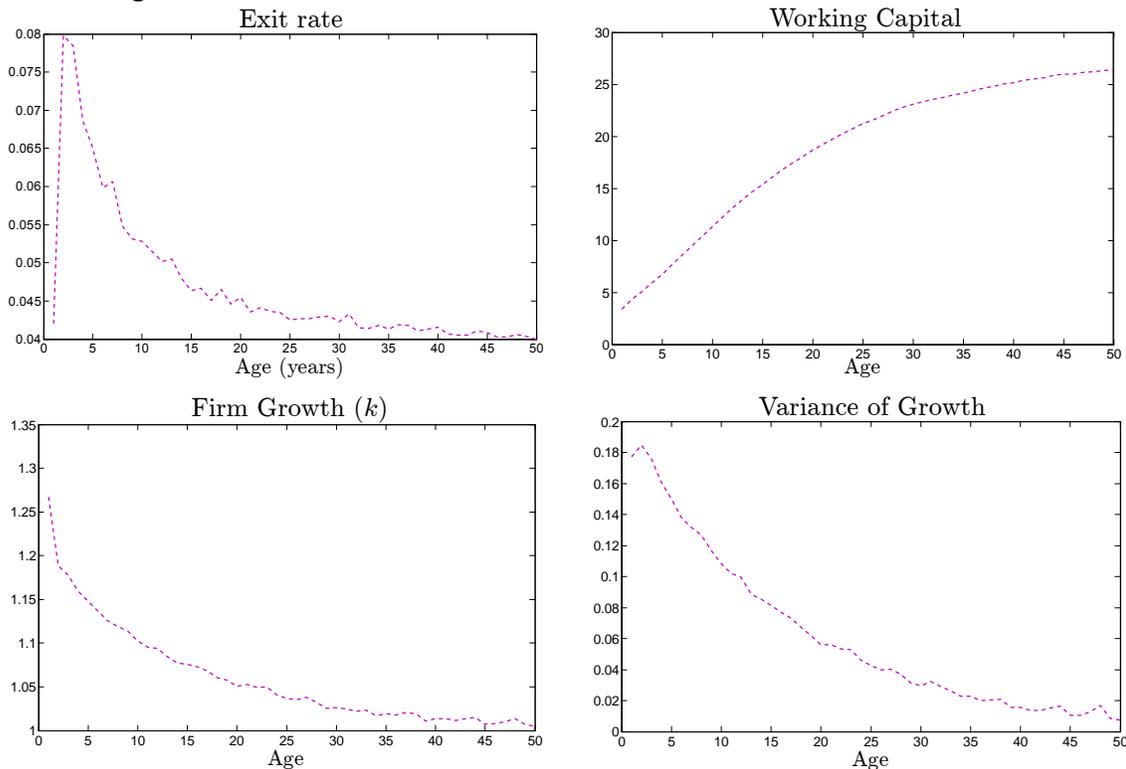

Figure 5. Firm dynamics

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[26] Unfortunately, we cannot match the frequency of Chapter 11 relative to out-of-court reorganization since our parametrization implies that new firms have equity below the reorganization threshold.

The first thing to notice is that the model studied here is consistent with the observation made elsewhere that firm age is positively correlated with size (lower-right panel) and negatively correlated with the variance of growth (lower-left panel). Secondly, the model is also consistent with the observation that exit hazard rates increase at the early stages of the firm and decrease systematically thereafter (Bruderl, Preisendorfer and Ziegler (1992)). To give a sense of the quantitative performance of the model, it is useful to note that estimates for the U.S. imply that around 75% of firms survive through their first three years of operation, while almost half of them have failed by their sixth year (Headd and Kirchhoff (2009)).

4.2   Counterfactual experiment

The counterfactual exercise that we conduct is aimed at quantifying the effects of allowing for renegotiation and reorganization as an alternative to liquidation. To this end, we use the CH contract as the data generation process for a large number of firms and compare the results with those obtained under the contract with renegotiation and reorganization. Table 3 presents the results of this comparison.

Table . Counterfactual exercise

moment	baseline	Liquidation only
Exit rate	5.1%	6.36%
Relative size of entrants (working capital)	18.6%	19.3%
Average firm age (years)	40.6	36.2
Average firm productivity	0.51	0.505
Average firm size (capital)	6.65	6.54
Size dispersion (Coeff. of variation)	0.74	0.77

The most noteworthy consequence of not having the option to rehabilitate troubled firms is that the exit rate increases by over a quarter. This is perhaps not surprising as some of the firms that would otherwise renegotiate and survive are liquidated. This has a corresponding effect on age; the average firm in the liquidation-only economy is over four years younger. Table 3 also shows that in the economy with renegotiation and reorganization the average firm is modestly larger and the dispersion of asset size is moderately lower. The mechanism behind this result can be seen in Figure 6 which tracks the average firm productivity in both economies. While there is some selection under both contracts (as shown by Proposition 4), selection is modestly

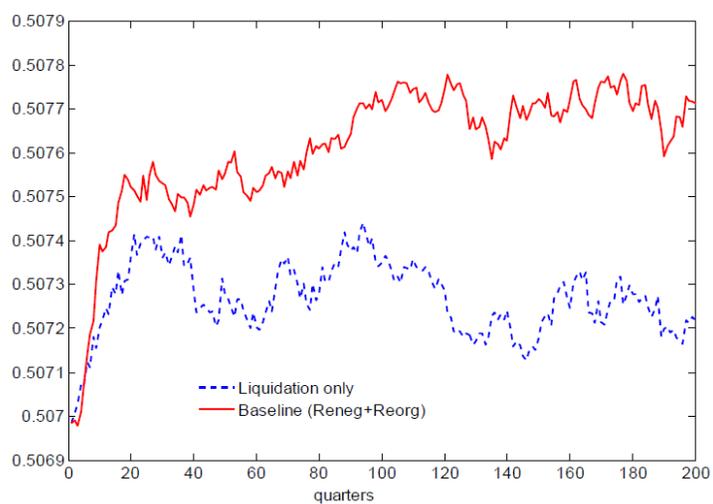enhanced by the contract with renegotiation and reorganization.

Figure 6. Average productivity and financial selection

5 Concluding remarks

6 Appendix A: proofs and derivations

Proof of Lemma 1. A firm currently under reorganization, and operating at $\tilde{k}^r$ can either become undistressed next period with probability π, or remain under reorganization and have the chance to become undistressed in the following period:

$$
\begin{aligned}
\tilde{W}_r &= \pi (1-\mu) f(\tilde{k}^r) - \tilde{k}^r + \beta \left[\pi \tilde{W} + (1-\pi) \tilde{W}_r \right] \tag{3}\\
&= \frac{\beta \pi \tilde{W} + \pi (1-\mu) f(\tilde{k}^r) - \tilde{k}^r}{1 - \beta (1-\pi)}
\end{aligned}
$$

since $\frac{\beta \pi}{1-\beta(1-\pi)} < 1$, it suffices to show that:

$$
\frac{\pi (1-\mu) f(\tilde{k}^r) - \tilde{k}^r}{1 - \beta (1-\pi)} < \left[1 - \frac{\beta \pi}{1 - \beta (1-\pi)} \right] \tilde{W}
$$

some tedious algebra and $\tilde{W} = \frac{\pi f(\tilde{k}) - \tilde{k}}{1-\beta}$ reduces this to showing that:

$$
\pi f(\tilde{k}) - \tilde{k} > \pi (1-\mu) f(\tilde{k}^r) - \tilde{k}^r
$$

which holds given the strict concavity of $f(\cdot)$ and $\mu > 0$. ∎

Proof of Proposition 1. First notice that $\widehat{W}_i(V^i) > \widehat{W}_{j \neq i}(V^{j \neq i})$ implies $V = V^i$ and $W(V) = \widehat{W}_i(V^i) = \widehat{W}_i(V)$. Now, $\widehat{W}_u(V) > \widehat{W}_r(V)$ for V large enough follows from continuity and Lemma 1, while $\widehat{W}_u(V) > W_\ell$ is true by Assumption 1 and continuity. Next, to show that $\widehat{W}_r(V) > \widehat{W}_u(V)$ for some V, first define $W_u^*(V)$ as the value of a firm that cannot be liquidated or reorganized and $W_r^*(V)$ as the value of a firm that always operates under reorganization and cannot be liquidated. Results analogous to those of the CH contract hold for each of these sub-problems, so one has that $\tau_*^i = f(k_*^i) = \frac{\beta(V_{H*}^i - V_{L*}^i)}{1 - \rho^i}$, and therefore $V = \beta \left[\pi V_{H*}^i + (1-\pi) V_{L*}^i \right]$ for $i = u, r$. Now $W_u^*(V), W_r^*(V)$ can be written:

$$
\begin{aligned}
W_u^*(V) &= \max_{V_*^{1H}, V_*^{1L} \geq 0} V - \beta V_{L*}^u - f^{-1} \left(\frac{V - \beta V_{L*}^u}{\pi} \right) + \beta \mathbb{E} W_u^*(V) \tag{4}\\
s.t. &: \quad V = \beta \left[\pi V_{H*}^i + (1-\pi) V_{L*}^u \right] = \beta \mathbb{E} V_*^u
\end{aligned}
$$

and:

$$
\begin{aligned}
W_r^*(V) &= \max_{V_*^{0H}, V_*^{0L} \geq 0} \frac{(1-\mu)(V - \beta V_{L*}^r)}{1 - \rho} - f^{-1} \left[\frac{V - \beta V_{L*}^r}{(1 - \rho_0)\pi} \right] + \beta \mathbb{E} W_r^*(V) \tag{5}\\
s.t. &: \quad V = \beta \left[\pi V_{H*}^r + (1-\pi) V_{L*}^r \right] = \beta \mathbb{E} V_*^r
\end{aligned}
$$

clearly the functions $W_u^*(V), W_r^*(V)$ are increasing, concave and differentiable so the Envelope Theorem applies and:

$$\frac{dW_u^*(V)}{dV} = 1 - \frac{1}{\pi}\left[f^{-1}\left(\frac{V - \beta V_*^{1L}}{\pi}\right)\right]'$$

where $\left[f^{-1}(y)\right]' = \frac{df^{-1}(y)}{dy}$, while:

$$\frac{dW_r^*(V)}{dV} = \frac{1-\mu}{1-\rho} - \frac{1}{(1-\rho)\pi}\left[f^{-1}\left(\frac{V - \beta V_{L*}^r}{(1-\rho)\pi}\right)\right]'$$

Now $\rho > \mu$, $V = 0 \Rightarrow V_{L*}^i = 0$ and $\left[f^{(-1)}(0)\right]' = \frac{df^{-1}(0)}{dk} = 0$ together imply that $\frac{dW_*^r(0)}{dV} > \frac{dW_u^*(0)}{dV}$. By continuity of the value functions, $\exists!\ V^{++}$ such that $\frac{dW_u^*(V)}{dV} < \frac{dW_r^*(V)}{dV}\ \forall\ V \in (0, V^{++})$. Given $W_u^*(0) = W_r^*(0) = 0$, it can be concluded that $W_u^*(V) < W_r^*(V)\ \forall\ V \in (0, V^{++})$. Next, let $W_u^\Delta(V)$ denote the value of a firm that is currently undistressed and can be liquidated with scrap value Δ, but cannot be reorganized. Define $W_r^\Delta(V)$ analogously for a firm currently under reorganization. Clearly, $\lim_{\Delta \to 0} W_u^\Delta(V) = W_u^*(V)$ and $\lim_{\Delta \to 0} W_r^\Delta(V) = W_r^*(V)$. Then continuity ensures that $\exists!\ \Delta^S$ and V^S such that $\Delta \in (0, \Delta^S) \Rightarrow W_r^\Delta(V) > W_u^\Delta(V)$ for $V \in (0, V^S)$. It remains to show that $W_r^\Delta(V) > W_u^\Delta(V)$ for some V is sufficient for $\widehat{W}_r(V) > \widehat{W}_u(V)$ to hold for some V. To see this, suppose otherwise (and find a contradiction). That is, suppose $W_r^\Delta(V) > W_u^\Delta(V)$ for some V but $\widehat{W}_r(V) \le \widehat{W}_u(V)\ \forall\ V$. Then $\widehat{W}_u(V) \ge \widehat{W}_r(V)\ \forall\ V \Rightarrow \widehat{W}_u(V) = W_u^\Delta(V)$. On the other hand, it must be true that $\widehat{W}_r(V) \ge W_r^\Delta(V)$ since a policy of never leaving the firm undistressed is clearly feasible and incentive compatible for the problem in $\widehat{W}_r(V)$. In other words, $\widehat{W}_u(V) = W_u^\Delta(V) \ge \widehat{W}_r(V) \ge W_r^\Delta(V)\ \forall\ V$, a contradiction since we have already shown that $W_r^\Delta(V) > W_u^\Delta(V)$ for some V. $\blacksquare$

Proof of Proposition 2. Notice that the entrepreneur will default if and only if $\beta s > V_c$, so the indifference point is $\beta s_d = V_c$. Notice also that $V \ge \beta E(s) = \frac{\beta \bar{s}}{2}$ since a strategy of defaulting for all realizations of s is always feasible for the entrepreneur, and such strategy gives her exactly $\beta E(s)$. This obviously implies $V_L^i, V_H^i \ge \beta E(s)$. Since $V_c \ge V_c^i = \beta\left[\pi V_H^i + (1-\pi)V_L^i\right]$, it follows that $V_c \ge \beta^2 E(s)$. Thus we require $\beta s_d \ge \beta^2 E(s) \Rightarrow s_d \ge \beta E(s)$ so we can find $s_d = \max\{\beta E(s), \sqrt{2\bar{s}V\beta^{-1} - \bar{s}^2}\}$. Obviously, $V = \beta E(s)$ implies $2\bar{s}V\beta^{-1} - \bar{s}^2 = 0$ in which case $s_d = \beta E(s)$. Hence, $\exists\ V^+ > \beta E(s)$ such that $s_d = 2\bar{s}V\beta^{-1} - \bar{s}^2$ and the second part of the proposition follows. $\blacksquare$

Proof of Corollary 2. From the last proof, we know that if $V \le \frac{\beta \bar{s}}{2}\left(\frac{\beta^2}{4} + 1\right)$ then $s_d = \beta E(s)$. This in turn implies that $\beta^2 E(s) = V_c$ which in turn means that $V_L^i = V_H^i = \beta^2 E(s)$. Thus, the binding ICC $\beta(V_H^i - V_L^i) = (1-\rho^i)f(k^i)$ implies $k^i = 0$. $\blacksquare$

Proof of Proposition 4. First, notice that using the result $f(k) = \tau$ it follows that for $V < \tilde{V}(\pi)$:

$$\frac{\partial \widehat{W}(V;\pi)}{\partial \pi} = f(k) + \beta\pi\left[W(V_H;\pi) - W(V_L;\pi)\right](V_H - V_L) \geq 0$$

where the inequality is ensured by $W(V_H;\pi) \geq W(V_L;\pi)$ and $V_H \geq V_L$. Next, notice that $\frac{\partial \widehat{W}(V;\pi)}{\partial V}$ is increasing in π. To see this, differentiate () w.r.t. its first argument:

$$\frac{\partial \widehat{W}(V;\pi)}{\partial V} = 1 - \frac{1}{\pi}\left[f^{-1}\left(\frac{V - \beta V^L}{\pi}\right)\right]' = 1 - \frac{1}{\pi}\left\{\frac{1}{f'\left[f^{-1}\left(\frac{V-\beta V^L}{\pi}\right)\right]}\right\}$$

where $\left[f^{-1}(y)\right]' = \frac{df^{-1}(y)}{dy}$ and the last equality is by the Inverse Function Theorem. Now, f increasing implies f^{-1} is increasing. Therefore, $f^{-1}\left(\frac{V - \beta V^L}{\pi}\right)$ decreases with π. Moreover, f concave implies that $f'\left[f^{-1}\left(\frac{V-\beta V^L}{\pi}\right)\right]$ increases with π, which in turn means that the term in braces decreases with π. Summarizing, one has that $\pi' > \pi$ implies $\widehat{W}(V;\pi') > \widehat{W}(V;\pi)$ and $\frac{\partial \widehat{W}(V;\pi')}{\partial V} > \frac{\partial \widehat{W}(V;\pi)}{\partial V}$. Since $\widehat{W}(0;\pi') > \widehat{W}(0;\pi) = \beta\Delta$, this establishes that $V_\ell(\pi') < V_\ell(\pi)$. ∎

Invariant distribution of firms

Let $\phi(V,\pi) = 1$ if $V \in E$ and $\pi \in Q$, and zero otherwise. Then ψ_t satisfies the law of motion:

$$
\begin{aligned}
\psi_{t+1}(\mathcal{E}, \mathcal{Q}; p) = {} & (1-\eta)\int \pi \sum_{i=u,r}\sum_{j=u,r}\left\{\phi\left[V_c^j\left(V_H^i(V,\pi;p),\pi;p\right),\pi\right]\times\right. \\
& \left[1 - s_\ell\left(V_H^i(V,\pi;p),\pi;p\right)\left(1 - s_d\left(V_H^i(V,\pi;p);p\right)\right)/\bar{s}\right]\right\}\psi_t(dV,d\pi;p^*) \\
& + (1-\eta)\int(1-\pi)\sum_{i=u,r}\sum_{j=u,r}\left\{\phi\left[V_c^j\left(V_L^i(V,\pi;p),\pi;p\right),\pi\right]\times\right. \\
& \left[1 - s_\ell\left(V_L^i(V,\pi;p),\pi;p\right)\left(1 - s_d\left(V_L^i(V,\pi;p);p\right)\right)/\bar{s}\right]\right\}\psi_t(dV,d\pi;p^*) \\
& + (1-\eta)\int\sum_{j=u,r}\left\{\phi\left[V_c^j\left(V_\ell(V,\pi;p),\pi;p\right),\pi\right]\times\right. \\
& \left[1 - s_\ell\left(V_\ell(V,\pi;p),\pi;p\right)\left(1 - s_d\left(V_\ell(V,\pi;p);p\right)\right)/\bar{s}\right]\right\}\psi_t(dV,d\pi;p^*) \\
& + E_{t+1}\int\phi\left[V_0(p),\pi\right]d\Gamma(\pi)
\end{aligned}
$$

The first four lines add up all the firms that have not defaulted in t, and who survive exogenous exit and endogenous liquidation in $t+1$. The next two lines add up all the firms that defaulted

(and renegotiated) in t and survive exogenous exit and endogenous liquidation in $t+1$. The final line accounts for new entrants (whose type is drawn from $\Gamma(\pi)$). Notice that V_{t+1} for the firms that did not default depends on the realization of z_t while for the defaulted firms does not (i.e., $V_{t+1} = V_\ell$). Notice also that we have used the fact that $s_d(V, \pi; p) = s_d(V; p)$ which follows directly from Proposition 2.
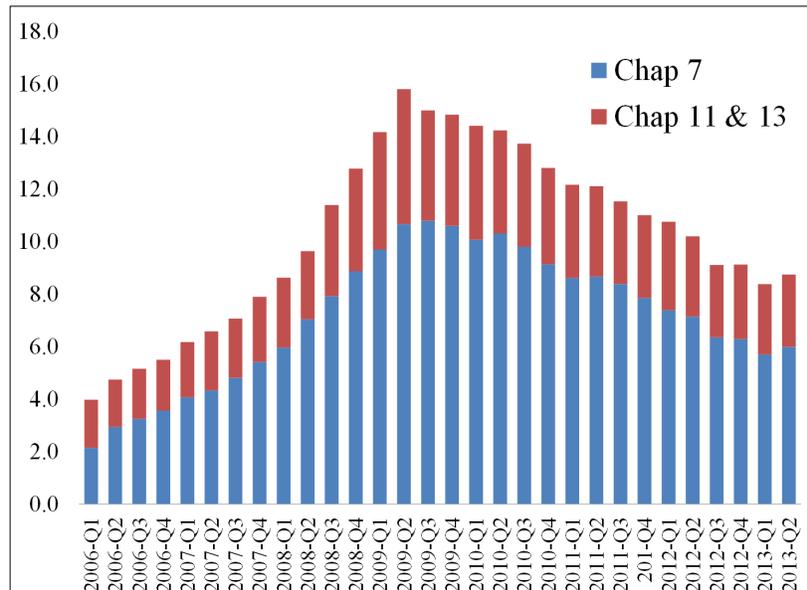
7 Appendix B: additional figures

Table B1. U.S. Business failures Jul/2008–Jun/2009

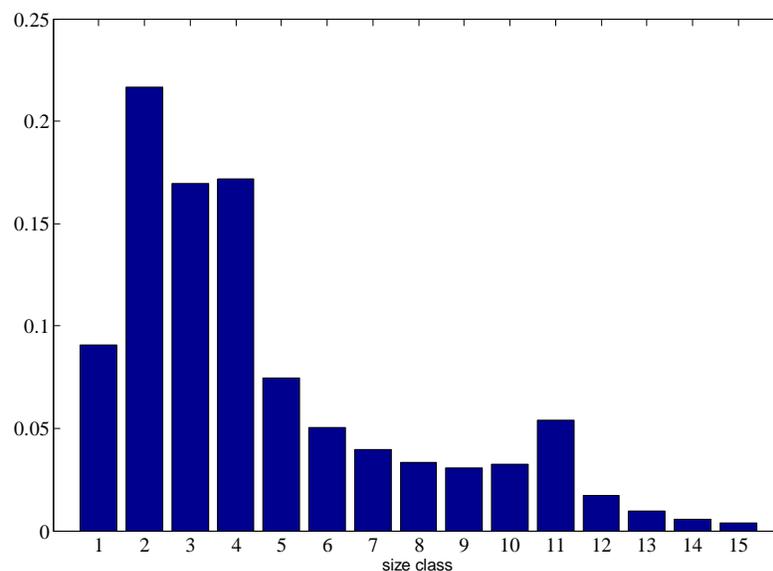| Liquidations	Reorganizations	
Chapter 7	Chapter 11	Private workouts
37,174	14,480	42,835

Source: uscourts.gov and Dun & Bradstreet

Figure B1. Quarterly formal bankruptcy filings in the U.S.

Source: uscourts.gov

Figure B2. Distribution of constrained firms by size (k_t)

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